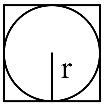
Supplementary information

Appendix A. Relation between uncertainty of sieve and mass

Considering a spherical particle of radius r which passes through the sieve square mesh of side l = 2r (case of circumference inscribed in a square).



Variations of volume are related to variation of radius as follows

$$dV = 4\pi r^2 dr \qquad \text{a.1}$$

mass and volume are related through density (ρ)

$$m = \rho V \qquad \text{a.2}$$

As a consequence variations of mass are related to variation of radius

$$dm = \rho \; dV \qquad \text{a.3}$$

combining equations a.1 and a.3 we obtain

$$dm = \rho \; 4\pi r^2 dr\,. \qquad \text{a.4}$$

Finally the uncertainty of the sieve can be expressed as:

$$\mathrm{u}_{sieve} = \frac{dm}{m} = 3\frac{dr}{r} \qquad \text{a.5}$$

assuming a regular separations of the mesh equation a.5 is equivalent to

$$\mathrm{u}_{sieve} = 3\frac{dL}{L}$$

where L is the sieve length

Appendix B. Calculation of the standard deviation for a function of *q* variables

Let $f(x_1,x_2,....,x_q)$ a function of q variables x_i each associated to an error given by the associated standard deviation σ_{x_i} of the corresponding measurement procedure. Where the standard definitions of σ_X is given by:

$$\sigma_X = \sqrt{\frac{1}{n-1}\sum_i^n \left[X_i - \overline{X}\right]^2} \qquad \text{b.1}$$

where X_i and $\overline{X}$ are the i-th determination and the mean value of the variable X, n the number of the evaluation of the variable. Generally the analytical form of the function f is unknown and a practical approximation is to replace it with the truncated first-order expansion of the corresponding Taylor series since is assumed that the displacements of the x_i variables respect to their corresponding means values are small. Under these hypothesis the expression of f becomes:

$$f(x_1',x_2',\ldots,x_q') \cong f(x_1,x_2,\ldots,x_q) + \left.\frac{\partial f}{\partial x_1'}\right|_{x_1}(x_1' - x_1) + \left.\frac{\partial f}{\partial x_2'}\right|_{x_2}(x_2' - x_2) + \cdots + \left.\frac{\partial f}{\partial x_q'}\right|_{x_q}(x_q' - x_q)\,. \qquad \text{b.2}$$

With these approximation we have that:

$$\overline{f(x_{1_i}',x_{2_i}',\ldots,x_{q_i}')} = f(\overline{x}_1,\overline{x}_2,\ldots,\overline{x}_q) \text{ and } \sigma_f = \sqrt{\frac{1}{n-1}\sum_i^n\left[f(x_{1_i}',x_{2_i}',\ldots,x_{q_i}') - f(\overline{x}_1,\overline{x}_2,\ldots,\overline{x}_q)\right]^2}$$

and for sake of simplicity, using the variance (σ_f^2) of f, we derive the relations between the standard deviation of f and those of the variables x_i.

$$\sigma_f^2 = \frac{1}{n-1}\sum_i^n \left[f(x'_{1_i}, x'_{2_i}, \ldots, x'_{q_i}) - f(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_q) \right]^2$$

$$= \frac{1}{n-1}\sum_i^n \left[\left.\frac{\partial f}{\partial x'_1}\right|_{x_1} (x'_{1_i} - \bar{x}_1) + \left.\frac{\partial f}{\partial x'_2}\right|_{x_2} (x'_{2_i} - \bar{x}_2) + \cdots + \left.\frac{\partial f}{\partial x'_q}\right|_{x_q} (x'_{q_i} - \bar{x}_q) \right]^2$$

$$= \frac{1}{n-1}\left[\left(\left.\frac{\partial f}{\partial x'_1}\right|_{x_1} \right)^2 \sum_i^n (x'_{1_i} - \bar{x}_1)^2 + \left(\left.\frac{\partial f}{\partial x'_2}\right|_{x_2} \right)^2 \sum_i^n (x'_{2_i} - \bar{x}_2)^2 + \ldots + \left(\left.\frac{\partial f}{\partial x'_q}\right|_{x_q} \right)^2 \sum_i^n (x'_{q_i} - \bar{x}_q)^2 \right] +$$

$$+ \frac{2}{n-1}\left[\left.\frac{\partial f}{\partial x'_1}\right|_{x_1} \left.\frac{\partial f}{\partial x'_2}\right|_{x_2} \sum_i^n (x'_1 - \bar{x}_1)(x'_2 - \bar{x}_2) + \ldots + \sum_{\substack{i \\ r \neq s}}^n \left.\frac{\partial f}{\partial x'_r}\right|_{x_r} \left.\frac{\partial f}{\partial x'_s}\right|_{x_s} (x'_{r_i} - \bar{x}_r)(x'_{s_i} - \bar{x}_s) + \ldots \right.$$

$$\left. + \sum_i^n \left.\frac{\partial f}{\partial x'_{q-1}}\right|_{x_{q-1}} \left.\frac{\partial f}{\partial x'_q}\right|_{x_q} (x'_{(q-1)_i} - \bar{x}_{q-1})(x'_{q_i} - \bar{x}_q) \right]$$

where using the standard deviation definition (a.1) and those of the covariance between two variables X, Y:

$$\sigma_{X,Y} = \frac{1}{n-1}\sum_i^n \left[X_i - \bar{X} \right] \cdot \left[Y_i - \bar{Y} \right] \qquad \text{b.3}$$

we obtain that:

$$\sigma_f^2 = \sum_k^q \left(\left.\frac{\partial f}{\partial x'_k}\right|_{x_k} \right)^2 \sigma_{x_k}^2 + 2 \sum_{\substack{r,s \\ r \neq s}}^{[q(q-1)/2]} \left.\frac{\partial f}{\partial x'_r}\right|_{x_r} \left.\frac{\partial f}{\partial x'_s}\right|_{x_s} \sigma_{x_r, x_s} \; . \qquad \text{b.4}$$

Equation a.4 show the relations that we are looking for between the variance of the *f* values respect to those of the single variables x_i, and of the covariance from the couples (x_i, x_j). If the errors associated to the two variables are randomly distributed and not correlated the covariance σ_{x_i, x_j} results to be equal to zero on the other hand if the two distributions are strongly correlated we have that $\sigma_{x_i, x_j} = \pm \sigma_{x_i} \sigma_{x_j}$ [1] and more generally holds that:

$$\left| \sigma_{x_i, x_j} \right| \leq \sigma_{x_i} \sigma_{x_j} \qquad \text{b.5}$$

independently from the correlations and distributions of the x_i and x_j errors.

So we could have two possibly limit values of the *f* variance which easily follows from equation a.4:

Electronic Supplementary Material (ESI) for Analytical Methods
This journal is © The Royal Society of Chemistry 2012

$$\sigma_f = \sqrt{\sum_k^q \left(\left. \frac{\partial f}{\partial x_k^{'}} \right|_{x_k} \right)^2 \sigma_{x_k}^2} \quad \textit{if the } x_k \textit{ variables are uncorrelated}$$

$$\sigma_f = \sum_k^q \left| \left. \frac{\partial f}{\partial x_k^{'}} \right|_{x_k} \right| \sigma_{x_k} \quad \textit{if the } x_k \textit{ variables are strongly correlated}$$

Now we could consider as example the following case where the function f is defined as:

$$f(x,y) = \frac{x}{y}$$

and the x and y standard deviations are respectively: $\sigma_x = \sqrt{\sigma_a^2 + \sigma_y^2}$ and σ_y. According to equation b.4 we have that:

$$\sigma_f^2 = \left(\frac{1}{y}\right)^2 \sigma_x^2 + \left(-\frac{x}{y^2}\right)^2 \sigma_y^2 + 2\left(\frac{1}{y}\right)\left(-\frac{x}{y^2}\right)\sigma_{x,y}$$

and using the inequality a.5 and dividing for f^2 easily follows that:

$$\left(\frac{\sigma_f}{f}\right)^2 \le \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2 + 2\left|\left(\frac{\sigma_x \sigma_y}{xy}\right)\right|$$

moreover if $\sigma_a > \sigma_y$ and assuming the strong correlation we obtain:

$$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_a}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2 + 2\left|\left(\frac{\sigma_a \sigma_y}{xy}\right)\right| \cong \left(\frac{\sigma_a}{x}\right)^2 + 2\left|\left(\frac{\sigma_a \sigma_y}{xy}\right)\right|$$

see equation 8 of main text.

If σ_y it is negligible we have

$$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_a}{x}\right)^2$$

see equation 10 of main text