

Supplementary Information

Without loss of generality, the general expressions for the charge and heat currents can be written when the external field E and temperature gradient ∇T are directed along \hat{x} according to Fig. 1. When the fluctuation-induced field points to the right, the Fermi levels of the left and right reservoirs are respectively $\mu_L = \mu - ew(E_T + E)$ and $\mu_R = \mu$. Assuming $E_T > E$, when the fluctuation-induced field points to the left, the Fermi levels of the left and right reservoirs are respectively $\mu_L = \mu$ and $\mu_R = \mu - ew(E_T - E)$.

After making the observation that both types of orientations are equally probable, the respective currents are expressed as

$$j = \int_{-\infty}^{\infty} dE [f(E, T + \Delta T) - f(E + ew(E_T + E), T)] M(E_T + E, E) + \int_{-\infty}^{\infty} dE [f(E + ew(E_T + E), T + \Delta T) - f(E, T)] \quad (\text{A.1})$$

$$j_L^q = \frac{1}{e} \int_{-\infty}^{\infty} dE [f(E, T + \Delta T) - f(E + ew(E_T + E), T)] [E - \mu + \dots] \quad (\text{A.2})$$

$$j_R^q = \frac{1}{e} \int_{-\infty}^{\infty} dE [f(E, T + \Delta T) - f(E + ew(E_T + E), T)] [E - \mu] M \quad (\text{A.3})$$

where $M(E_T \pm E, E)$ is given in Eq. 6. The heat current traveling through the junction is $j^q = (j_L^q + j_R^q)/2$. Applying linear response theory on the charge and heat current, one finds

$$j = L_{11}E + L_{12}\nabla T \quad (\text{A.4})$$

$$j^q = L_{21}E + L_{22}\nabla T \quad (\text{A.5})$$

where

$$L_{11} = \frac{\partial}{\partial E} (L_1(E_T + E, T) - L_1(E_T - E, T))_{E=0}, \quad L_{12} = w \frac{\partial L_2(E_T, T)}{\partial T} \quad (\text{A.6})$$

$$L_{21} = \frac{\partial}{\partial E} (L_1^q(E_T + E, T) - L_1^q(E_T - E, T))_{E=0}, \quad L_{22} = w \frac{\partial L_2^q(E_T, T)}{\partial T} \quad (\text{A.7})$$

$$L_1(E_T, T) = \int_{-\infty}^{\infty} dE \left(f\left(E - \frac{ewE_T}{2}, T\right) - f\left(E + \frac{ewE_T}{2}, T\right) \right) M(E_T, E) \quad (\text{A.8})$$

$$L_2(E_T, T) = \int_{-\infty}^{\infty} dE \left(f\left(E - \frac{ewE_T}{2}, T\right) + f\left(E + \frac{ewE_T}{2}, T\right) \right) M(E_T, E) \quad (\text{A.9})$$

$$L_1^q(E_T, T) = \frac{1}{e} \int_{-\infty}^{\infty} dE \left(f\left(E - \frac{ewE_T}{2}, T\right) - f\left(E + \frac{ewE_T}{2}, T\right) \right) [E - \mu] M(E_T, E) \quad (\text{A.10})$$

$$L_2^q(E_T, T) = \frac{1}{e} \int_{-\infty}^{\infty} dE \left(f\left(E - \frac{ewE_T}{2}, T\right) + f\left(E + \frac{ewE_T}{2}, T\right) \right) [E - \mu] M(E_T, E) \quad (\text{A.11})$$

It is noted that the Eqs. (A.6, A.7) can be recast in the following equivalent form used to obtain the analytical expressions in Table I:

$$L_{11} = 2 \frac{\partial L_1}{\partial E_T}, \quad L_{12} = w \frac{\partial L_2}{\partial T}, \quad L_{21} = 2 \frac{\partial L_1^q}{\partial E_T}, \quad L_{22} = w \frac{\partial L_2^q}{\partial T} \quad (\text{A.12})$$