Supporting Material

Modification of the Shear Velocity in the Capillary Embedded & Extended Step Microchannel

The shear velocity in the channel determines the shear force. In our previous work, the shear velocity in straight channels was calculated relying on the projected area in the flow direction. But in the step channel, the 3D orifice leads to the 3D distribution of flow rates at the orifice. A modification of shear velocity is called for the calculation of the Ca_c as well as the analysis of droplet formation process.

The area in the flow direction is first needed to be analyzed. Fig S1 gives us a sketch view of the orifice structure in the direction of flowing and in the direction of capillary extending. In this sketch, a 2D coordinate system is established on the underside of the capillary and the origin of the coordinate is right in the position where the jet is formed (downstream the capillary underside). We assume the inner 10% (S1) of the orifice to be the shear region and the average velocity in this region to be the calculated shear velocity. The rest axisymmetric 90% of the region would be labelled as S2.

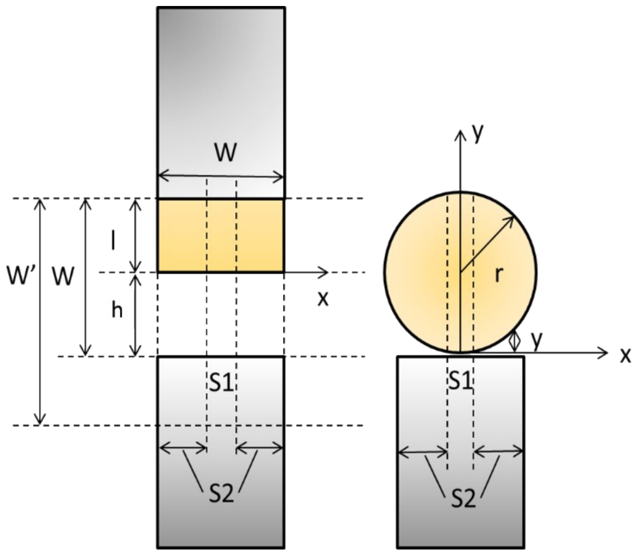

Figure S1 (Color online) Sketch view of the orifice structure. Front view and bottom view are presented above. Yellow regions represent the capillary and gray ones represent the microchannel. W', W, l, h and r are respectively the width of the main microchannel upstream, the width of the main microchannel downstream, the extending length of the capillary, the width of the orifice and the radius of the capillary.

The distance from the edge point of the capillary underside to the plane of the step perpendicular to the flow direction would be calculated by the Pythagorean theorem:

$$y = r - \sqrt{r^2 - x^2} \quad (S1)$$

Similarly, the distance from the edge point of the capillary underside to the edge of the step would be calculated as:

$$l' = \sqrt{h^2 + y^2} = \sqrt{h^2 + \left(r - \sqrt{r^2 - x^2}\right)^2} \quad (S2)$$

$l^{'}$ is actually the width of the orifice region in upstream direction. The element of the orifice upstream area can be obtained:

$$ds = l^{'}dx = \sqrt{h^2 + \left(r - \sqrt{r^2 - x^2}\right)^2}dx \quad (S3)$$

Considering the axisymmetric structure of the orifice, we would calculate the flow process in unilateral side. The areas of S1 and S2 region can be calculated by integration:

$$S_1 = \int_0^{0.05W} \sqrt{h^2 + \left(r - \sqrt{r^2 - x^2}\right)^2}dx \quad (S4)$$

$$S_2 = \int_{0.05W}^{0.5W} \sqrt{h^2 + \left(r - \sqrt{r^2 - x^2}\right)^2}dx \quad (S5)$$

The wet periphery lengths and equivalent diameters of both regions are:

$$l_1 = 0.2r, d_1 = 4S_1/l_1 \quad (S6)$$

$$l_2 = 0.4W + 0.5\pi r, d_2 = 4S_2/l_2 \quad (S7)$$

where $W = 2r$ in this channel.

As shown above, the Re is in the range of 0.01 to 40. When Re<<1, the fluid is in the slow creeping flow when across the sharp edge of the step. When Re is in the range of 1 to 50, eddy current appears after the sharp edge of the step and physical resistance dominates the process, which would call for resistance coefficient method to analysis the fluid resistance. It should be noted that the fluid resistance is made mainly by the sharp edges of capillary cross section and step structure of the channel, and thus the wet periphery lengths eliminate the channel wall.

The resistance distributions influence the distributions of the flow rates, inducing different velocities in different positions of the orifice. To simplify the calculation, we assume the resistance (namely the pressure drop) amasses in the orifice region and the pressures downstream the orifice are exactly equal in different positions of the fluid. In this work, we can regard the regions of S1 and S2 as parallel pipelines, where both inlet pressures and both outlet pressures are equal in all the directions of the fluid.

The pressure drop across the edge of the capillary cross section in region S1 can be obtained by resistance coefficient method:

$$h_f = 1/2C_d\rho_c u^2 \quad (S8)$$

where C_d is the resistance coefficient, and for this work (Re<50):

$$C_d = \frac{C}{Re} \quad (S9)$$

where C is a constant value with a fixed channel structure, and $Re = \dfrac{\rho_c du}{\mu}$.

Based on the assumption, the pressure drops are equal in both S1 and S2 regions:

$$0.5C_{d1}\rho u_1^2 = 0.5C_{d2}\rho u_2^2 \quad (S10)$$

Combining eqn (S8)-(S10):

$$u_2 = \frac{C_1 d_2}{C_2 d_1} u_1 \quad (S11)$$

For the case of $2V\ (m^3/s)$ as the total continuous phase flow rates (and V is the flow rate in the unilateral side), mass conservation is considered as a restrict condition:

$$u_1 S_1 + u_2 S_2 = V \quad (S12)$$

Furthermore, the shear velocity of the region S1 is presented as:

$$u_1 = V/(S_1 + \frac{C_1 d_2}{C_2 d_1} S_2) \quad (S13)$$

Combining eqn (S4)-(S7), u_1 would be obtained.

In our analysis, C_1 and C_2 are respectively constant values for S1 and S2, reflecting the influence of the structures on fluid resistances. The S1 region faces directly the upstream fluid and the S2 one faces obliquely the fluid. As a result, the value of C_1 would be larger than C_2, and the ratio of C_1 to C_2 should be in the range of 1 to $csc45°$. In this work, C_1/C_2 is selected as 1.2 to satisfy the condition that both calculated velocities by this model and by projected areas are equal, when capillary extending length is zero (see Fig 7).

We measured the continuous phase flow fields by micro-PIV, but unfortunately it cannot track the interfaces of the droplets for their small sizes. On the other hand, for the small sizes of droplets, they can actually act as tracer droplets of the flow fields. Furthermore, we analysis the droplet velocity at the very beginning of droplet formation, which is measured as:

$$u = w_d f \quad (S14)$$

where f is the droplet formation frequency and w_d is the interval of the droplets.

Measured velocities are presented in Fig S2. As comparison, the calculated velocities by the other two methods are also exhibited. The measured values fit well with the calculated ones.

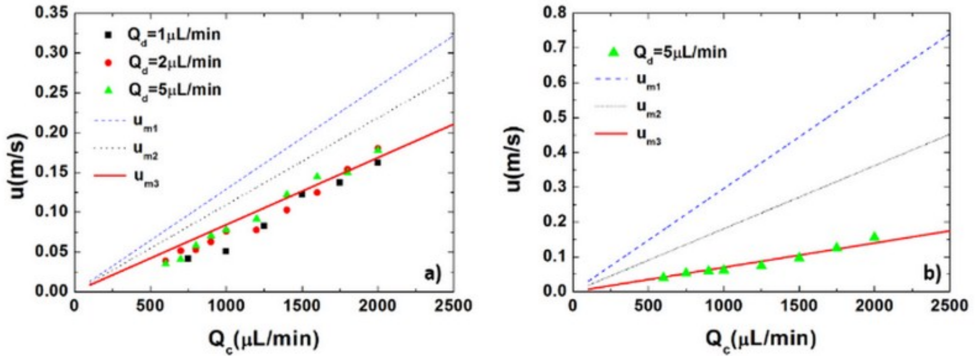

Figure S2 (Color online) Measured velocities and model velocities in step channels: u_{m1} stands for the calculated velocity in our model, u_{m2} is the velocity based on the 3D integration area (in this model, we assume $u_1 = u_2$ and u_1 is calculated by $u_1 = V/(S_1+S_2)$, see eqn S4, eqn S5 and eqn S12) and u_{m3} is the calculated velocity by projected areas, similar to that in literature[34]. Measured velocities are obtained under low disperse phase flow rates to guarantee the small sizes of the droplets. a) Velocities in step channels with extending length of l/W=0.77. b) Velocities in step channels with extending length of l/W=0.90.