Supplementary Information (ESI) for Soft Matter

Geometric Reconstruction of Biological Orthogonal Plywoods

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The following is the supplementary information regarding the detailed flowchart in the error assessment procedure, comparison of the incision angles fixed (α^M) in the Mayavi visualizations and calculated from the measurements of the herringbone angle (β_m), a mathematical description of the sensitivity of the pitch in terms of the two sources of errors ε_β and ε_L , discussed in Section 3. Table E2 presents and defines all the symbols used in the main article as well as the equation defining it when applicable. This supplementary information should be read in conjunction with section 3.

Table E2. Symbols used in the main article.

Name	Symbol	Equation or defintion
Herringbone periodicity	L	1
Pitch	p_o	1
Incision angle	α	1
Unit vector	$\underline{\delta}_i$	$\underline{\delta}_x$ $\underline{\delta}_y$
Projected unit vector	$\underline{\delta}_i^p$	2
Herringbone angle	β	3
Normal unit vector to incision plane	$\underline{\hat{k}}$	$\cos\theta\sin\phi\,\underline{\hat{\delta}}_x+\sin\theta\sin\phi\,\underline{\hat{\delta}}_y+\cos\phi\,\underline{\hat{\delta}}_z$
Azimuthal angle	θ	4
Polar angle	ϕ	$\phi=\pi-\alpha$
Ratio of projected lengths	R	6
Pitch fixed in Mayavi visualizations	p_o^M	Section 3.2
Incision angle fixed in Mayavi visualizations	α^M	Section 3.2
Exact periodicity value from Mayavi visualizations	L_c^M	Section 3.2
Exact herringbone angle from Mayavi visualizations	β_c^M	Section 3.2
Measured periodicity from in silico herringbone patterns	L_m	Section 3.2
Measured herringbone angle from in silico herringbone patterns	β_m	Section 3.2
Calculated incision angle from in silico herringbone patterns	α_c	Section 3.2
Calculated pitch from measurements of in silico herringbone patterns	p_{oc}	Section 3.2

The flowchart for the error assessment, described in Section 3, is shown in figure E1.

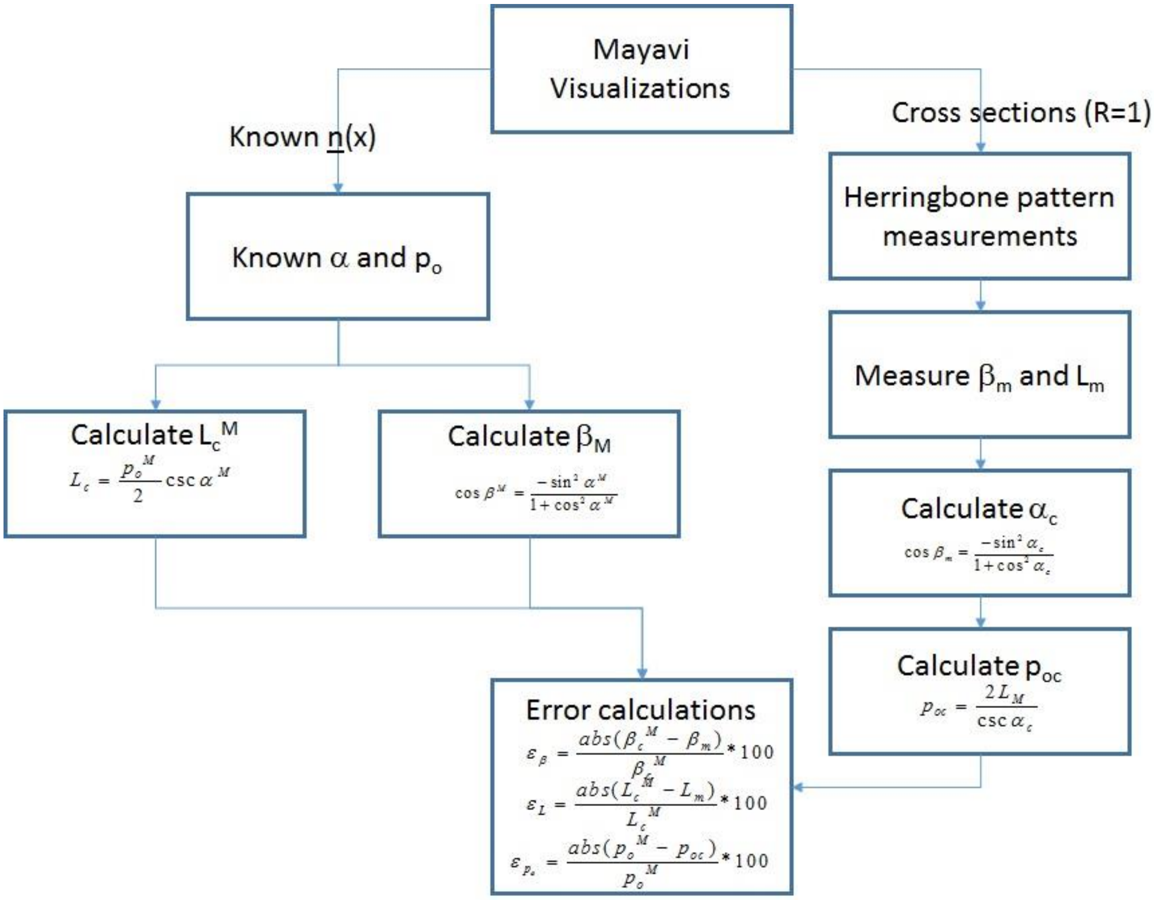

Figure E1. Flowchart detailing the error assessment procedure.

The left stream is known information (α, p_o) or exactly calculated pattern parameters $\left(L_c^M, \beta_M\right)$ since the plywood was pre-specified and then visualized in the Majavi software. The right column is information measured $\left(L_m, \beta_m\right)$ and calculated $\left(p_{oc}, \alpha_c\right)$ on the in-silico 2D patterns found from slicing a box the plywood also in the Majavi environment. The last box indicates the error calculations.

Table E1 shows the negligible errors found for the incision angles found in all cases. Hence this particular quantity will not affect the predictions of the domain size p_o.

Table E1. Incision Angle Errors ε (α)

α^M	α_{calc}	ε (α)
94.04	93.92	0.13%
98.05	98.29	0.25%
112.99	113.29	0.27%
125.26	125.60	0.27%
140.24	139.44	0.57%
151.46	151.52	0.04%
81.95	81.71	0.29%
74.21	73.53	0.91%
54.74	54.40	0.61%

A mathematical description of the pitch error sensitivity is as follows: the error in the calculation of the pitch is a function of errors emerging from β and L:

$$\varepsilon_{p_o} = \varepsilon_{p_o}(\varepsilon_\beta, \varepsilon_L) \qquad \text{(E.1)}$$

Given the results in Table E1, the differential $d\varepsilon_{p_o}$ is:

$$d\varepsilon_{p_o} = \left(\frac{\partial \varepsilon_{p_o}}{\partial \varepsilon_\beta}\right)_L d\varepsilon_\beta + \left(\frac{\partial \varepsilon_{p_o}}{\partial \varepsilon_L}\right)_\beta d\varepsilon_L \qquad \text{(E.2)}$$

The partial derivatives are expanded as follows:

$$\frac{\partial \varepsilon_{p_o}}{\partial \varepsilon_\beta} = \frac{\partial \varepsilon_{p_o}}{\partial p_o}\frac{\partial p_o}{\partial \beta}\frac{\partial \beta}{\partial \varepsilon_\beta}, \frac{\partial \varepsilon_{p_o}}{\partial \varepsilon_L} = \frac{\partial \varepsilon_{p_o}}{\partial p_o}\frac{\partial p_o}{\partial L}\frac{\partial L}{\partial \varepsilon_L} \qquad \text{(E.3)}$$

From the definition of the errors, given these are linear functions of their respective variable, the derivatives involving these terms will have order of magnitude of the unity:

$$\frac{\partial \varepsilon_{p_o}}{\partial p_o} \approx O(1); \frac{\partial \beta}{\partial \varepsilon_\beta} \approx O(1); \frac{\partial \varepsilon_{p_o}}{\partial p_o} \approx o(1); \frac{\partial L}{\partial \varepsilon_L} \approx o(1) \qquad \text{(E.4)}$$

On the other hand the terms with the derivative of the pitch are the ones that determine the order of magnitude of the error. The former leads to a term involving the product of $\sin\beta\cos\beta$ while the latter $\sqrt{\cos\beta}$ being smaller the former.

$$\frac{\partial p_o}{\partial \beta} = f\left(\sin\beta\cos\beta\right); \frac{\partial p_o}{\partial L} = g\left(\sqrt{\cos\beta}\right) \qquad \text{(E.5)}$$

Since $\frac{\partial p_o}{\partial \beta}\frac{\partial \beta}{\partial \varepsilon_\beta}$ and $\frac{\partial p_o}{\partial L}\frac{\partial L}{\partial \varepsilon_L}$ can also be written as: $\frac{\partial p_o}{\partial \varepsilon_\beta}$ and $\frac{\partial p_o}{\partial \varepsilon_L}$, which can be interpreted as the change of the pitch prediction with respect to errors in the measured variables, from the above analysis and (E.5) , it can be inferred that the pitch predictions are more sensitive to errors in L than in β:

$$\left|\frac{\partial p_o}{\partial \varepsilon_\beta}\right| < \left|\frac{\partial p_o}{\partial \varepsilon_L}\right| \tag{E.6}$$

In conclusion, for high accuracy in p_o, the error in L should be minimized by careful measurements and high precision instruments.