Flow Patterning Capability of Localized Natural Convection

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Supplementary Information

Effect of pressure work and viscous dissipation.

In general, the equation of energy should be expressed as

$$\rho\hat{C}_p\frac{DT}{Dt} = k\nabla^2T + \beta T\frac{Dp}{Dt} + \mu\Phi_v \quad \text{(S1)}$$

where $\frac{D}{Dt}$ [t^{-1}] is the substantial time derivative, ∇ [L^{-1}] is the del operator, ρ [M/L^3] is the density of the fluid, T [T] is the absolute temperature in the system, $\hat{C}_p$ [$L^2/T\ t^2$] is the heat capacity at constant pressure per unit mass of the fluid, k [ML/Tt^3] is the thermal conductivity of the fluid, β [T^{-1}] is the thermal expansion coefficient of the fluid and Φ_v [t^{-2}] is the viscous dissipation function. Dimension above is given in term of mass (M), length (L), time (t), and temperature (T).

We nodimensionalized the equation and obtained the nondimensional parameters, which can indicate the relative importance of each term in the equation. The nondimensional equation is shown as follows [Y. Jaluria, *Pergamon Press Oxford*, 1980].

$$\frac{\partial\theta}{\partial\tau}+\vec{\mathbf{v}}^*\cdot\nabla^*\theta= \frac{1}{Pr\sqrt{Gr}}\nabla^{*2}\theta^* + (\beta T)Ec[\frac{\partial p^*}{\partial\tau}+\vec{\mathbf{v}}^*\cdot\nabla^*p^*] + Ec\frac{1}{\sqrt{Gr}}\Phi_v^* \quad \text{(S2)}$$

The dimensionless variables and parameters introduced in these equations are

$$\tau=\frac{t}{d/v_c};\qquad \theta^*=\frac{T-T_0}{T_H-T_0};\qquad \mathbf{v}^*=\frac{\mathbf{v}}{v_c}=\frac{\mathbf{v}}{\sqrt{g\beta d(T_H-T_0)}};$$

$$\nabla^*=\frac{\nabla}{1/d};\qquad p^*=\frac{p}{\rho v_c^2};\qquad \Phi_v^2=\frac{\Phi_v}{v_c/d^2};$$

$$Ec=\frac{v_c^2}{C_p(T_H-T_0)}=\frac{g\beta d}{C_p};\qquad Gr=\frac{g\beta d^3(T_H-T_0)}{\nu^2};$$

where Ec is Eckert number, which expresses the relationship between the kinetic energy and enthalpy of the fluid.

From the above equation, Ec number (= $\frac{g\beta d}{C_p}$) determines the relative importance of pressure work and viscous dissipation terms in the energy balance equation. If Ec is much less than one, the term Φ_v^* can be neglected. Similarly, if (βT)Ec is much less than one, the term $[\frac{\partial p^*}{\partial\tau}+\vec{\mathbf{v}}^*\cdot\nabla^*p^*]$ can be neglected.

Ec number of a commonly used liquid is usually small. If water is used as an example, at 15°C and 1 atm, its specific heat capacity is 4185.5 J/kg·K, and its thermal expansion coefficient is 1.51 × 10^{-4} 1/K [The Engineering Toolbox: http://www.engineeringtoolbox.com/water-thermal-properties-d_162.html]. If the liquid layer in the system is 1 cm thick, the Ec number of the system is about 3.5×10^{-9}, showing that both pressure work and viscous dissipation terms in the energy balance can be negligible.

Constant density assumption based on Boussinesq approximation.

In the natural convection system, the density varies in space because of the temperature variation in space. When there is density variation in space, the equation of continuity at steady state can be described as

$$\vec{v}\cdot\nabla\rho + \rho\nabla\cdot\vec{v} = 0$$

The relative size of the two terms in the equation can be approximated as

$$\frac{v_i \frac{\partial\rho}{\partial x_i}}{\rho\frac{\partial v_i}{\partial x_i}} \sim \frac{\Delta\rho/\rho}{\Delta v_i/v_i}$$

When the natural convection effect is obvious, $\Delta v_i/v_i$ should not be small. Therefore, when $\Delta\rho/\rho \ll 1$, the size of the density variation term is much smaller than the size of the velocity variation term, and the density variation term can be neglected in the equation of continuity.

The density variation can be expressed as a function of temperature:

$$\rho(T) = \rho_0 - \rho\beta(T-T_0) - \frac{\rho\beta^2}{2!}(T-T_0)^2 - \ldots$$

where β is the thermal volumetric expansion and defined as $\beta = -\frac{1}{\rho}(\frac{\partial\rho}{\partial T})$.

The normalized density difference can be estimated as

$$\Delta\rho/\rho \sim (\rho(T) - \rho_0)/\rho = -\beta(T-T_0) - \frac{\beta^2}{2!}(T-T_0)^2 - \ldots$$

Therefore, the density variation term in the equation of continuity can be neglected compared to the velocity variation term when $\beta(T-T_0) \ll 1$.

System configuration and boundary conditions used in our 2D model to study the effect of dimensionless groups on flow behavior and temperature distribution.

Figure S1 shows the system configuration in our 2D COMSOL simulation model. The values of system height, system width and heating wire width are set as 1, 9 and 0.05 respectively. The governing equation to describe flow behavior and temperature distribution are shown in Eqs. 4-6 in the main text.

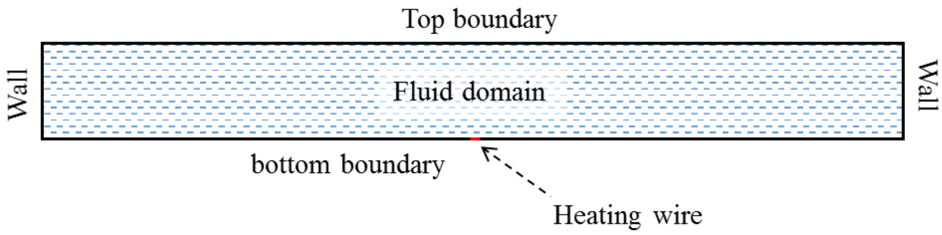

Figure S1 System configuration in our 2D COMSOL simulation model.

The system is initially at rest and the initial conditions are set as follows.

at $\tau = 0,$ $\vec{\mathbf{v}}^* = 0$

$\tau = 0,$ $\theta^* = 0$

We have different boundary conditions at the top plate, the heating wire and the rest of the boundary. The upper boundary of liquid is sealed and continuously cooled to a constant temperature (T_0). In addition, no-slip boundary condition is applied at the upper surface. The heating wire is set at a constant temperature of T_H throughout the entire process. The rest of boundaries are assumed to be no-slip and thermal insolated.

At the entire upper surface, $\vec{\mathbf{v}}^* = 0$

$\theta^* = 0$

At the heating wire, $\vec{\mathbf{v}}^* = 0$

$\theta^* = 1$

At the rest of boundaries, $\vec{\mathbf{v}}^* = 0$

$\vec{\mathbf{n}} \cdot \nabla^* \theta^* = 0$

where $\vec{\mathbf{n}}$ is the normal vector of the boundary.

System configuration of 3D model for obtaining resolution.

Figure S2 shows the configuration of the angle-shaped wire system. The height to radius ratio of the system is set as 1:10, and all the boundary conditions and initial conditions are the same as our 2D simulation model.

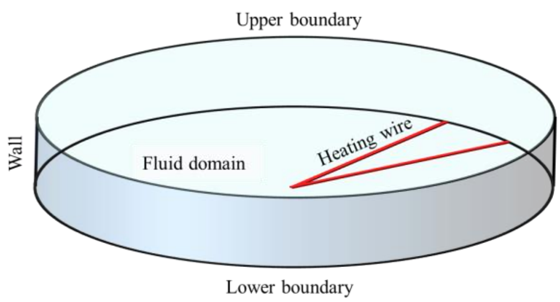

Figure S2 System configuration in our 3D COMSOL simulation model.

Resolution in the systems with different heating wire angles.

Figure S3 shows how the flow pattern and resolution change when the angle of the heating wire varies. We found that the arc lengths where the pattern is just distinguishable in the systems with different heating wire angles varying from 15° to 25° are almost the same. We chose the angle at around 20° because we anticipated that the error of arc length measurement becomes larger when the angle is too small and that the interaction between the two wires from the supplementary angle side can become significant when the angle is too large.

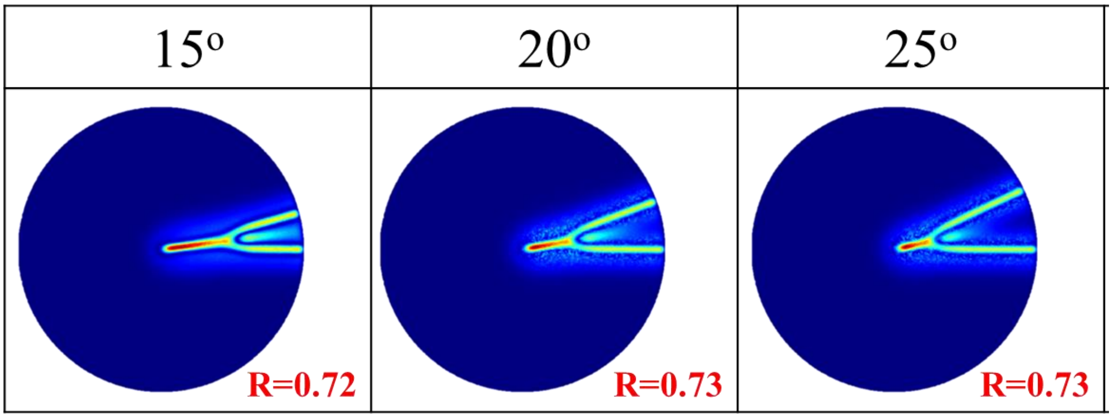

Figure S3 Modeling results of flow pattern and resolution under different angle of heating wire. ($1/\sqrt{Gr}=0.1$ and $1/Pr\sqrt{Gr}=0.001$) for all results. Colors blue to red represent the slow to fast normalized velocity.

Obtaining resolution from our modeling results.

Figure S4(a) shows a typical flow pattern in an angle-shaped system. To obtain a more consistent and quantitative way to obtain the resolution from a velocity magnitude map, we plotted the velocity magnitude profile along the center line of the two wires and used the characteristic property of the velocity magnitude profile to find the radius for the distinguishable arc length, as shown in Figure S4(b). Note that there is a descending flow region with large velocity magnitude close to the location where the two ascending flows induced by the heating wires can be just distinguishable; therefore, there is a clear minimum velocity magnitude appearing in the profile. We used the minimum velocity magnitude to find the radius where the pattern is just distinguishable and obtained the arc length by multiplying the radius by the degree of the angle. The arc length is used as Δx_m to calculate the corresponding resolution. For example, in Figure S4(b), the minimum velocity magnitude occurs at the radius = 3.94. Therefore, Δx_m (3.94× [20× (π/180)]) = 1.375 and the resolution R=1/1.375=0.73.

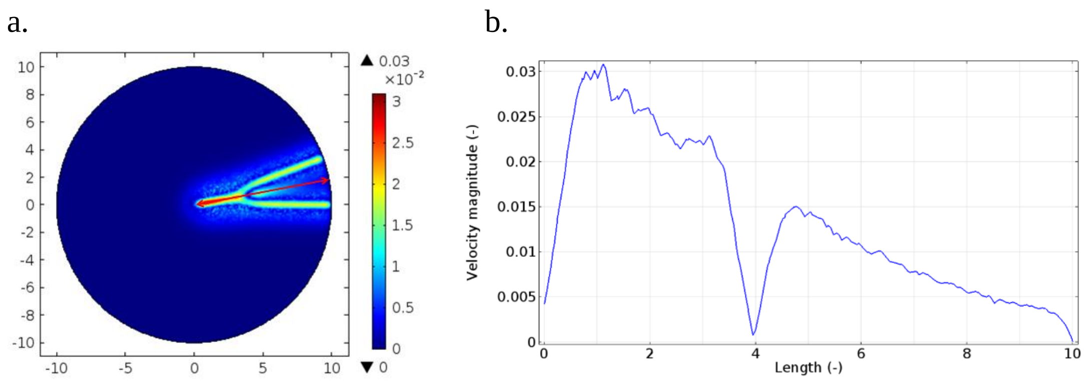

Figure S4 (a) Flow pattern of velocity magnitude in the plane at half of the system height. $1/\sqrt{Gr}$=0.1, $1/Pr\sqrt{Gr}$=0.001 and the heating wire angle is 20°. (b) Velocity magnitude profile along the red line in (a).

Alignment of silver nanowires (AgNWs).

Oriented AgNWs have been shown to be excellent substrates for exciting surface plasma, and for photonic applications. We have used this method to do a preliminary test on aligning AgNWs. As shown in Figure S5, a heating wire was set at the bottom of a well PDMS (Polydimethylsiloxane) well on a cover glass (2.45 mm x 3 mm). The AgNW solution (0.5% AgNWs in IPA) was poured into the well. After 10 min heating, we removed the PDMS well and heating wire, and rinsed the glass by pure IPA and sonicated it in water for about 10 secs. The right image in Figure S5 shows the alignment morphology of AgNWs on the cover glass. The preliminary result shows that the natural convection flow is strong enough to align the nanowires on a substrate.

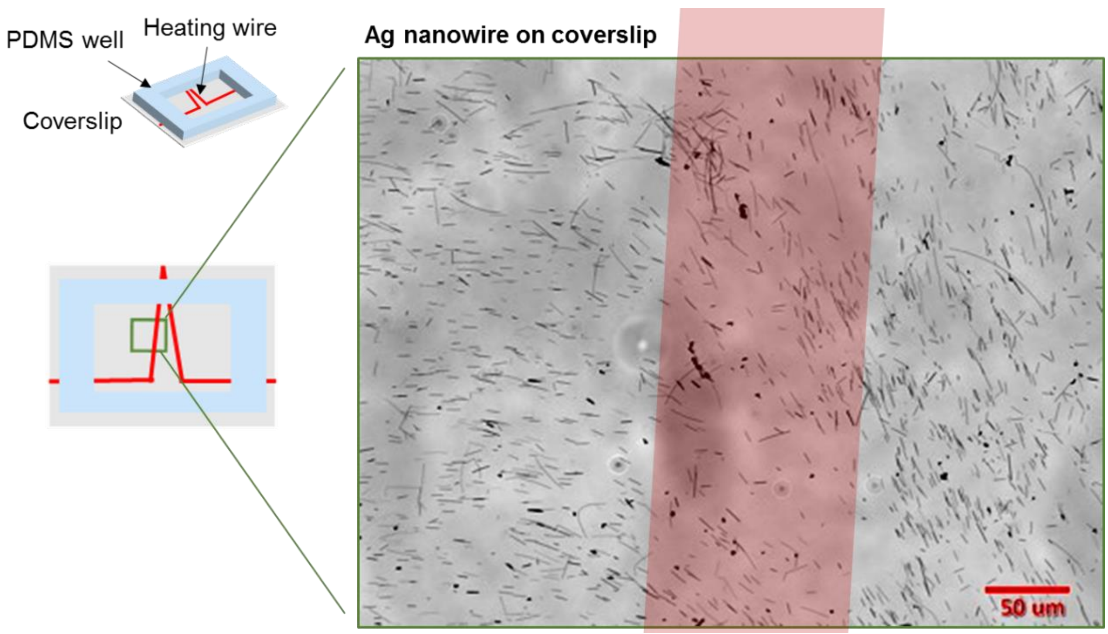

Figure S5 Alignment morphology of AgNWs adsorbed on a cover glass generated by the localized natural convection method. The red region in the morphology image indicates the heating wire location. The oriented directions of the AgNWs on the two sides of the heating wire are different since the flow direction on the right is significantly influenced by the other part of the wire.