# Driving the electrocatalytic activity by interface electronic structure control in metalloprotein hybrid catalyst for efficient hydrogen evolution 

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Supplementary Figure 1 Geometry optimized atomic configurations. (a) The configuration of LYS unit with atoms named in the figure itself. (b) Its counterpart configuration incorporated with silver ions. The red dashed circles show the active hydrogen absorption sites both at carbon and nitrogen atoms. The rectangular red dashed circle shows the interface between silver and protein active site.

## (a)


(b)


Supplementary Figure 2 Geometry optimized atomic configurations. (a) The configuration of LYS unit with atoms named in the figure itself. The rectangular red and green dashed lines show the configuration of retinal chromophore (rc) and LYS unit respectively. The violet dashed circle shows the attachment of rc with LYS unit. (b) Its counterpart configuration incorporated with silver ions. The red dashed circles show the active hydrogen absorption sites both at carbon and nitrogen atoms. The rectangular red dashed circle shows the interface between silver and protein active site.

Absolute Co-ordinates of atoms

| Atoms | x | y | z |
| :---: | ---: | ---: | ---: |
| N | 0.07002 | 0.03774 | -0.00423 |
| C | 1.53710 | 0.00253 | -0.01319 |
| C | 2.06619 | -0.02067 | 1.40813 |
| O | 1.29546 | -0.07594 | 2.35443 |
| C | 2.06907 | -1.21920 | -0.80163 |
| C | 1.69820 | -1.18321 | -2.29278 |
| H | 1.91002 | 0.93845 | -0.48367 |
| H | 1.68521 | -2.15604 | -0.33974 |
| H | 3.17829 | -1.23363 | -0.73821 |
| H | 0.59534 | -1.19943 | -2.41854 |
| C | 2.28861 | -2.39022 | -3.03250 |
| H | -0.31773 | -0.87099 | 0.34171 |
| H | 2.09975 | -0.24734 | -2.73999 |
| H | 1.87066 | -3.32527 | -2.59856 |
| H | 3.39332 | -2.39951 | -2.90439 |
| C | 1.96205 | -2.33395 | -4.53119 |
| N | 2.51497 | -3.48879 | -5.23416 |
| H | 2.38749 | -1.40603 | -4.97230 |
| H | 0.86029 | -2.31650 | -4.67811 |
| H | 2.10846 | -4.36805 | -4.83957 |
| H | 3.55381 | -3.51075 | -5.11311 |
| H | -0.30516 | 0.23680 | -0.95798 |
| O | 3.39461 | 0.04530 | 1.64579 |
| H | 4.03630 | 0.12866 | 0.92674 |

Optimized value of Energy: $\quad 16.4418533 \mathrm{~kJ} / \mathrm{mol}$
Supplementary Table 1 The list of atoms and their absolute coordinates of lysine (LYS) unit of seventh helix (gh) obtained from geometry optimization. This coordinates show the stable structure of active sites with minimum energy configuration with value $16.44185 \mathrm{~kJ} / \mathrm{mol}$.

|  | Absolute Co-ordinates of atoms |  |  |
| :---: | :---: | :---: | :---: |
| Atoms | x | y | Z |
| N | 0.33089 | 0.22737 | -0.21404 |
| C | 1.76498 | -0.05226 | -0.07112 |
| C | 2.14752 | -0.12754 | 1.40102 |
| O | 1.81636 | 0.86104 | 2.04134 |
| C | 2.12247 | -1.38004 | -0.78107 |
| C | 1.86550 | -1.34326 | -2.29064 |
| H | 2.34225 | 0.77936 | -0.53573 |
| H | 1.54933 | -2.21795 | -0.32700 |
| H | 3.20726 | -1.57887 | -0.64054 |
| C | 2.30740 | -2.64873 | -2.95677 |
| H | 1.72328 | -3.49424 | -2.53087 |
| H | 3.38745 | -2.81850 | -2.75839 |
| C | 2.08716 | -2.58677 | -4.47140 |
| N | 2.49079 | -3.83365 | -5.11399 |
| H | 1.93700 | -4.62917 | -4.72088 |
| H | 3.50717 | -4.00127 | -4.93531 |
| O | 3.44119 | -0.48748 | 1.57354 |
| H | 3.97281 | -0.14636 | 2.30772 |
| Ag | 0.97877 | -1.74768 | 2.09750 |
| Ag | 2.97529 | 0.28935 | -3.11383 |
| H | 0.78341 | -1.20033 | -2.48649 |
| Ag | -0.05066 | 0.89067 | -2.13112 |
| H | 0.06319 | 1.05257 | 0.36871 |
| Ag | 3.24784 | -0.99260 | -5.30018 |
| H | 1.01451 | -2.39717 | -4.69373 |

Optimized value of Energy: $57.2011111 \mathrm{~kJ} / \mathrm{mol}$
Supplementary Table 2 The list of atoms and their absolute coordinates of lysine (LYS) unit of seventh helix (gh) incorporated with silver ions obtained from geometry optimization. This coordinates show the stable structure of active sites with minimum energy configuration with value $57.2011 \mathrm{~kJ} / \mathrm{mol}$. This shows the excess energy of $40.7593 \mathrm{~kJ} / \mathrm{mol}$. indicating that the presence of silver makes the active site to stable at higher energy value.

Absolute Co-ordinates of atoms

| Atoms | x | y | Z |
| :---: | :---: | :---: | :---: |
| C | -3.57329 | 1.45665 | 1.06811 |
| C | -5.10838 | 1.25519 | 1.27565 |
| C | -5.89929 | 1.77317 | 0.10477 |
| H | -5.47012 | 1.81515 | 2.16963 |
| H | -5.33720 | 0.18138 | 1.45821 |
| C | -5.70736 | 3.27841 | 0.05349 |
| H | -6.98174 | 1.55220 | 0.22710 |
| H | -5.56243 | 1.29698 | -0.83833 |
| C | -4.25579 | 3.72509 | 0.08692 |
| C | -4.07303 | 5.16402 | -0.37930 |
| H | -4.42616 | 5.25338 | -1.42801 |
| H | -3.07819 | 5.62472 | -0.33765 |
| H | -4.70942 | 5.81636 | 0.25532 |
| C | -3.22736 | 2.88270 | 0.50151 |
| C | -1.77986 | 3.34996 | 0.34399 |
| C | -0.66245 | 2.64033 | 0.61898 |
| H | -1.60500 | 4.31692 | -0.06828 |
| C | 0.72609 | 3.14574 | 0.35070 |
| H | -0.72490 | 1.64820 | 1.00589 |
| C | 0.91781 | 4.50581 | -0.30057 |
| H | 0.40650 | 4.51500 | -1.28591 |
| H | 1.95753 | 4.81490 | -0.48467 |
| H | 0.47227 | 5.28777 | 0.34955 |
| C | 1.75789 | 2.32157 | 0.67531 |
| C | 3.20386 | 2.58368 | 0.41749 |
| C | 4.18656 | 1.67309 | 0.67803 |
| C | 5.64241 | 2.01331 | 0.44286 |
| C | 6.00730 | 3.38607 | -0.10336 |
| H | 5.52242 | 3.52733 | -1.09206 |
| H | 7.07707 | 3.58583 | -0.25732 |
| H | 5.64630 | 4.16704 | 0.59838 |
| C | 6.58183 | 1.06632 | 0.72855 |
| C | 8.07771 | 1.19074 | 0.56801 |
| H | 6.25389 | 0.10627 | 1.11412 |
| H | 8.45886 | 2.14657 | 0.17899 |
| H | 8.42509 | 0.39489 | -0.12303 |
| H | 8.55369 | 1.02688 | 1.55698 |
| H | -6.23731 | 3.78119 | 0.89198 |
| H | -6.18464 | 3.61210 | -0.89395 |
| C | -3.06723 | 0.36593 | 0.08454 |
| H | -3.33748 | -0.64705 | 0.45258 |


| H | -1.97633 | 0.36111 | -0.06440 |
| :--- | ---: | ---: | ---: |
| H | -3.50930 | 0.49546 | -0.92344 |
| C | -2.98987 | 1.22658 | 2.49233 |
| H | -3.38207 | 1.99456 | 3.19366 |
| H | -1.89966 | 1.25655 | 2.59647 |
| H | -3.29245 | 0.22663 | 2.87428 |
| N | 1.03319 | -0.42672 | -0.63088 |
| C | 2.49653 | -0.46148 | -0.72117 |
| C | 3.07321 | -0.52157 | 0.68457 |
| O | 2.81194 | -1.48822 | 1.38405 |
| C | 2.98675 | -1.67831 | -1.53963 |
| C | 2.46209 | -1.68140 | -2.98399 |
| H | 2.84373 | 0.45913 | -1.23407 |
| H | 2.66622 | -2.61882 | -1.03855 |
| H | 4.09904 | -1.65770 | -1.56501 |
| H | 1.35304 | -1.73499 | -2.98055 |
| C | 2.99326 | -2.89139 | -3.76017 |
| H | 0.65715 | -1.33979 | -0.28494 |
| H | 2.77836 | -0.74508 | -3.49403 |
| H | 2.68092 | -3.82472 | -3.24175 |
| H | 4.10417 | -2.85335 | -3.79060 |
| C | 2.44332 | -2.90808 | -5.19324 |
| N | 2.93644 | -4.06648 | -5.93331 |
| H | 2.74920 | -1.98124 | -5.72553 |
| H | 1.33207 | -2.94493 | -5.16862 |
| H | 2.64238 | -4.94516 | -5.44772 |
| H | 3.98090 | -4.03604 | -5.97826 |
| H | 0.60437 | -0.19121 | -1.55342 |
| O | 3.89901 | 0.43969 | 1.19245 |
| H | 1.50467 | 1.38390 | 1.14902 |
| H | 3.48713 | 3.53587 | 0.01178 |

Optimized value of Energy: $120.5158133 \mathrm{~kJ} / \mathrm{mol}$
Supplementary Table 3 The list of atoms and their absolute coordinates of lysine (LYS) unit of seventh helix (gh) attached with retinal chromophore (rc). This coordinates show the stable structure of active sites with minimum energy configuration with value $120.5158 \mathrm{~kJ} / \mathrm{mol}$.

| Atoms | Absolute Co-ordinates of atoms |  |  |
| :---: | :---: | :---: | :---: |
|  | x | y | z |
| C | -3.77852 | 1.39111 | 1.25265 |
| C | -5.32145 | 1.53171 | 1.11709 |
| C | -5.70046 | 2.10463 | -0.23219 |
| H | -5.71734 | 2.22772 | 1.89188 |
| H | -5.81068 | 0.54521 | 1.27538 |
| C | -5.14686 | 3.51934 | -0.33485 |
| H | -6.80334 | 2.13974 | -0.35984 |
| H | -5.29035 | 1.47459 | -1.04587 |
| C | -3.69086 | 3.64875 | 0.06518 |
| C | -3.04514 | 4.96554 | -0.31421 |
| H | -3.05619 | 5.08250 | -1.41774 |
| H | -2.01026 | 5.10485 | 0.04546 |
| H | -3.63680 | 5.79354 | 0.12976 |
| C | -3.03504 | 2.66204 | 0.77188 |
| C | -1.57053 | 2.75868 | 1.06942 |
| C | -0.68944 | 2.97726 | 0.07508 |
| C | 0.78791 | 3.00841 | 0.27041 |
| C | 1.35357 | 2.98875 | 1.67289 |
| H | 0.88961 | 3.81352 | 2.25299 |
| H | 2.43593 | 3.19707 | 1.72649 |
| H | 1.14251 | 2.01624 | 2.15989 |
| C | 1.56701 | 3.09701 | -0.83719 |
| C | 3.05440 | 3.06069 | -0.90815 |
| C | 3.89690 | 2.27737 | -0.17992 |
| C | 5.37358 | 2.45204 | -0.35877 |
| C | 5.83276 | 3.84817 | 0.04065 |
| C | 6.18111 | 1.48641 | 0.14019 |
| C | 7.65953 | 1.43522 | -0.11035 |
| H | 8.00897 | 2.22668 | -0.79533 |
| H | 7.90011 | 0.46421 | -0.58910 |
| H | 8.19983 | 1.50230 | 0.85650 |
| C | -3.26235 | 0.19777 | 0.41167 |
| H | -3.77540 | -0.74379 | 0.70322 |
| H | -2.17085 | 0.05137 | 0.56306 |
| H | -3.41892 | 0.35845 | -0.67431 |
| C | -3.50008 | 1.07857 | 2.73780 |
| H | -3.78427 | 1.93689 | 3.38350 |
| H | -2.43637 | 0.82828 | 2.90497 |
| H | -4.07733 | 0.18571 | 3.06351 |
| N | 0.73664 | -0.27755 | -0.97340 |
| C | 2.19466 | -0.14510 | -0.95019 |


| C | 2.65562 | 0.25062 | 0.44476 |
| :---: | :---: | :---: | :---: |
| O | 2.34634 | -0.46806 | 1.38314 |
| C | 2.84929 | -1.48347 | -1.33310 |
| C | 2.44989 | -1.96210 | -2.73270 |
| H | 2.50510 | 0.60767 | -1.69883 |
| H | 1.35737 | -2.15810 | -2.75865 |
| C | 3.18369 | -3.25389 | -3.10014 |
| H | 2.70227 | -1.17454 | -3.47556 |
| H | 2.94117 | -4.04187 | -2.35343 |
| H | 4.28037 | -3.07136 | -3.08570 |
| C | 2.76552 | -3.73841 | -4.49593 |
| N | 3.46038 | -4.97087 | -4.85718 |
| H | 2.99918 | -2.95746 | -5.25210 |
| H | 1.66927 | -3.92472 | -4.51600 |
| H | 3.24004 | -5.71701 | -4.15803 |
| H | 4.49269 | -4.80297 | -4.86250 |
| O | 3.45359 | 1.33222 | 0.69758 |
| H | 3.49067 | 3.69626 | -1.67232 |
| Ag | -0.91247 | 2.40835 | 3.06021 |
| Ag | 0.64628 | 3.36483 | -2.71951 |
| Ag | 4.74098 | 0.76883 | -3.29171 |
| Ag | 6.04387 | 3.04731 | -2.27794 |
| Ag | 7.87062 | 4.37727 | -0.36663 |
| H | 5.67597 | 3.99212 | 1.13215 |
| H | 5.24473 | 4.62643 | -0.48827 |
| Ag | 5.40251 | -0.13167 | 1.26555 |
| Ag | 0.07709 | -0.21015 | -2.92808 |
| H | 0.28721 | 0.53790 | -0.50845 |
| Ag | -5.40115 | 4.12496 | -2.37014 |
| H | -5.73077 | 4.21778 | 0.30362 |
| Ag | 4.97102 | -1.21596 | -1.28689 |
| H | 2.57751 | -2.26760 | -0.59115 |
| Ag | -1.49124 | 3.15282 | -1.87739 |
|  |  |  |  |

Optimized value of Energy: $281.7356627 \mathrm{~kJ} / \mathrm{mol}$
Supplementary Table 4 The list of atoms and their absolute coordinates of lysine (LYS) unit of seventh helix (gh) attached with retinal chromophore and incorporated with silver ions obtained from geometry optimization. This coordinates show the stable structure of active sites with minimum energy configuration with value $281.7356 \mathrm{~kJ} / \mathrm{mol}$. This shows the excess energy of $161.2198 \mathrm{~kJ} / \mathrm{mol}$. indicating that the presence of silver makes the active site to stable at higher energy value.

| Energy <br> Components <br> (Eh) | chu | chuag | fh | fhag | gh | ghag | ghrc | ghrcag |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Kinetic | 353.4 | 577.7 | 572.1 | 675.1 | 317.9 | 572.8 | 418.3 | 591.3 |
| Pseudopotential <br> (local) | -3518.3 | -4766.0 | -4213.8 | -5734.6 | -3101.5 | -4266.6 | -4302.3 | -4651.9 |
| Pseudopotential <br> (non-local) | 61.9 | -179.4 | 76.4 | -281.1 | 58.5 | -213.1 | 74.2 | -144.9 |
| Hartree | 1572.5 | 2220.7 | 2091.7 | 2706.1 | 1379.3 | 1988.7 | 1928.1 | 2141.9 |
| Exchange- | -140.9 | -182.2 | -157.0 | -198.3 | -127.4 | -172.6 | -173.6 | -201.3 |
| correlation |  |  |  |  |  |  |  |  |
| Ewald | 1469.9 | 1562.9 | 19227.9 | 1917.0 | 1031.0 | 1336.1 | 1519.4 | 1486.3 |

Supplementary Table 5 The energy components of protein samples obtained after optimization to get stable structure. The geometry optimization is performed following the various energy terms to achieve stable configurations.

## Supplementary Methods

## Computational details

## Linear Scaling density functional theory (LS-DFT)

The theoretical framework is based on linear scaling density functional theory (LS-DFT) and its implementation has been done by using Order N Electronic Total Energy Package (ONETEP) codes [1]. Within ONETEP, the single-particle density matrix, $\rho\left(r, r^{\prime}\right)$, is expressed in a separable form via atom-centred functions (Non-orthogonal Generalized Wannier functions, NWGFs), $\Phi \alpha(\mathrm{r})$, as:

$$
\begin{equation*}
\sum_{\rho\left(\mathrm{r}, \mathrm{r}^{\prime}\right)=\alpha \beta} \phi_{\alpha}(r) K^{\alpha \beta} \phi_{\beta}^{*}\left(r^{\prime}\right) \tag{1}
\end{equation*}
$$

Where, $K^{\alpha \beta}$ is the matrix element of the density kernel, which is non-zero only if $\left.\right|_{\alpha}-\left.r_{\beta}\right|_{<} r_{c}$, where $r_{\alpha}$ and $r_{\beta}$ indicate the coordinates of the centres of the NWGFs, $\phi_{\alpha \text { and }} \phi_{\beta .} r_{c}$ is the realspace cutoff length. The truncation of the density $\operatorname{Kernel}\left(K^{\alpha \beta}\right)$, leading to sparse density matrix $\rho\left(\mathrm{r}, \mathrm{r}^{\prime}\right)$, is justified by the known exponential decay of $\rho\left(\mathrm{r}, \mathrm{r}^{\prime}\right)$ with respect to $\left.\right|^{r_{\alpha}}-r_{\beta} \mid$ for systems with an electronic band gap, which makes any insulating or semiconducting systems amenable to LS-DFT simulation. The NGWFs are centered on the nuclear coordinates and strictly localized within a sphere of radius $\left(r_{\alpha}\right)$.Being non-orthogonal, the NGWFs are characterized by a nondiagonal overlap matrix, $S_{\alpha \beta}$

$$
\begin{equation*}
S_{\alpha \beta=} \int d r \phi_{\alpha}(r) \phi_{\beta}(r) \tag{2}
\end{equation*}
$$

The NGWFs are in turn expanded as a linear combination of coefficients ( ${ }^{C} m \alpha$ ) of localized and orthogonal periodic cardinal sine ( $p \sin c$ ) functions, $D_{m}(r)$, given by:

$$
\begin{equation*}
\phi_{\alpha}(r)=\sum_{m} C_{m \alpha} D_{m}\left(r-r_{m}\right) \tag{3}
\end{equation*}
$$

with $m$ indexing the real-space Cartesian grid inside the spherical localization region of $\phi_{\alpha}$. The psinc functions are formed from discrete sum of plane waves, which makes the set of $D_{m}(r)$ independent from the nuclear coordinates and systematically improvable on increase of
the kinetic energy cutoff. In this approach using ONETEP codes, the total energy of the system under study is minimized self-consistently with respect to $K^{\alpha \beta}$ and $C_{m \alpha \text { in two nested loops. The }}$ inner loop contains the codes to optimize total energy (E) with respect to the density kernel ( $K^{\alpha \beta}$ ) and outer loop does the same task with respect to the Wannier functions ( $\phi_{\alpha}$ and $\phi_{\beta}$ ). As a result, the NGWFs are optimized in situ by iteratively improving the set of coefficients $\left(C_{m \alpha}\right)$ that minimize the total energy under the constraints of idempotency of the density matrix (equation (1)) and conservation of the number of electrons in the simulated system. The NGWFs are represented in terms of the underlying orthogonal psinc basis, $D_{i}(r)$, in ONETEP code.

$$
\begin{equation*}
\phi_{\alpha}(r)=\sum_{i \in L R(\alpha)} C_{i \alpha} D_{i}(r) \tag{4}
\end{equation*}
$$

Where $L R(\alpha)$ is the spherical and atom-centered localization region of NGWF $\left(\phi_{\alpha}\right)$ and $C_{i \alpha}$ is its expansion coefficients in the psinc basis. This approach has been used to lead to near complete convergence of the density functional theory optimization with respect to the basis set even for minimal number of NGWFs employed in the simulations.

The design of catalysts for water electrolysis is based on the first-principles calculation. The hydrogen evolution reaction of the active sites is studied to improve its catalytic ability. In our current work, the linear scaling based ONETEP codes are incorporated with the plane wave scheme, which is based on the density functional theory [2] and the Perdew-Burke-Ernzerhof generalized gradient approximation (PBE-GGA) [3], is used for our calculations. Supercells with lattices larger than $8 \AA$ are used to investigate the catalytic effect and hydrogen-density dependent HER ability. A $2 \times 2 \times 1$ grid for k point sampling, based on the Monkhorst and Pack scheme [4], for geometry optimization of supercells, and an energy cutoff of 800 eV are consistently used in our calculations. Densities of states (DOSs) are calculated based on a k-point sampling of $3 \times 3 \times 1$. Good convergence is obtained with these parameters, and the total energy was converged to $2.0 \times 10-6 \mathrm{eV} /$ atom.

## Ionic forces and Ground state geometry optimization

The forces on ions at ${ }^{R}$ can be calculated by taking the derivative of the total energy with respect to ionic positions following the equation as

$$
\begin{equation*}
F_{\gamma}=-\left(\frac{d E}{d R_{\gamma}}\right)=-\left(\frac{\partial E}{\left.\partial R_{\gamma}\right)} \sum_{-\alpha \beta}\left(\frac{\partial E}{\partial K^{\alpha \beta}}\right) \frac{d K^{\alpha \beta}}{\left(d R_{\gamma}\right.}\right) \quad \sum_{\alpha} \int\left(\frac{\delta E}{\delta \phi_{\alpha}(r)}\right){\left.\frac{d \phi_{\alpha}(r)}{d R_{\gamma}}\right)^{d^{3} r}}^{\text {r }} \tag{5}
\end{equation*}
$$

To achieve total electronic energy minimization with respect to the degree of freedom [5] of the density, the following conditions are satisfied during simulation given as

$$
\begin{equation*}
\frac{\partial E}{\partial K^{\alpha \beta}}=0 \quad \frac{\partial E}{\text { and }} \frac{\partial C_{i \alpha}=0}{} \tag{6}
\end{equation*}
$$

These calculations lie on the Born-Oppenheimer surface for specific ionic calculations. Our calculations converges to Hellmann-Feynman force by vanishing second and third term of equation (5). The components of the total energy (Supplementary Table 5), which will be considered for minimization, consists of ion-ion and ion-electron terms with both local and nonlocal pseudopotential contributions. This non-local contributions to the energy and ionic forces are calculated in linear scaling $\{O(N)\}$ computational effort. This $\{O(N)\}$ has larger pre-factor of force calculation and consumes less memory compared to its square scaling counterpart $\{O$ ( $N^{2}$ ) $\}$. As a result, larger systems like biological systems can be handled smoothly for DFT calculations using this linear scaling methods [6].

## Hydrogen Coverage calculation

The hydrogen adsorption energies were calculated using the $2 \times 2 \times 1$ surface supercell. The total energy from DFT calculations were obtained as $E^{t o t}=\left[E(N R+n H)-E(N R)-\frac{n}{2} E\left(H_{2}\right)\right]$, where $E(N R+n H), E(N R)$ and $E\left(H_{2}\right)$ are the total energies for the protein active sites with n hydrogen atoms adsorbed, pristine sites and the molecular hydrogen in the gaseous phase, respectively. We define H coverage as the ratio of the number of H and C atoms in the interfacial plane of the metalloprotein complex. Thermal corrections were done to calculate adsorption free energy. $\Delta E_{Z P E}$ and $T \Delta S_{H}$ are the differences in the zero point energy and entropy contribution between the ${ }^{H}$ adsorbed state and $H_{2}$ in the gas phase respectively. The Gibbs free energy of hydrogen absorption was calculated as $\Delta G_{H}=\Delta E_{H}+0.22 \mathrm{eV}$.

## Supplementary References

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