Supporting information

Optical Microring Resonator Based Corrosion Sensing

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Coupled Mode Theory

The transmission characteristic of the waveguide is based on the couple mode theory (CMT). The light propagation through the waveguide and the interaction between the co-propagating light are important to understand the basic principle of different waveguide based devices like micro ring resonator, mach-zender interferometer (MZI) etc.

Two propagating modes in the waveguide can be coupled or interference phenomena can occur due to close proximity. Perturbation theory can be used to analysis the light propagation in the coupled waveguide. Condition is to maintain the same EM-field distribution before and after the coupling. The aim of this section is to give the basic mathematics related to the coupled mode theory for the waveguide structure in the co-directional coupler. The coupling mechanism and power transfer between waveguides for the co-directional coupler are discussed [1].

Consider two waveguides, refractive index n_1 and n_2 with enough distance that each waveguide is effected by other and coupled power to another (Figure S1).

The directional coupler before coupling can support eigen modes, $\mathbf{E_p}$ and $\mathbf{H}_p$ (p=1, 2) which satisfies the following Maxwell's equations [1]:

$$\nabla \times \mathbf{E_p} = -jw\mu_0 \mathbf{H}_p \quad \text{(S.1)}$$

$$\nabla \times \mathbf{H}_p = jw\varepsilon_0 N^2_p \mathbf{E_p} \quad \text{(S.2)}$$

Here, N^2_p = Refractive index distributions of the waveguides.

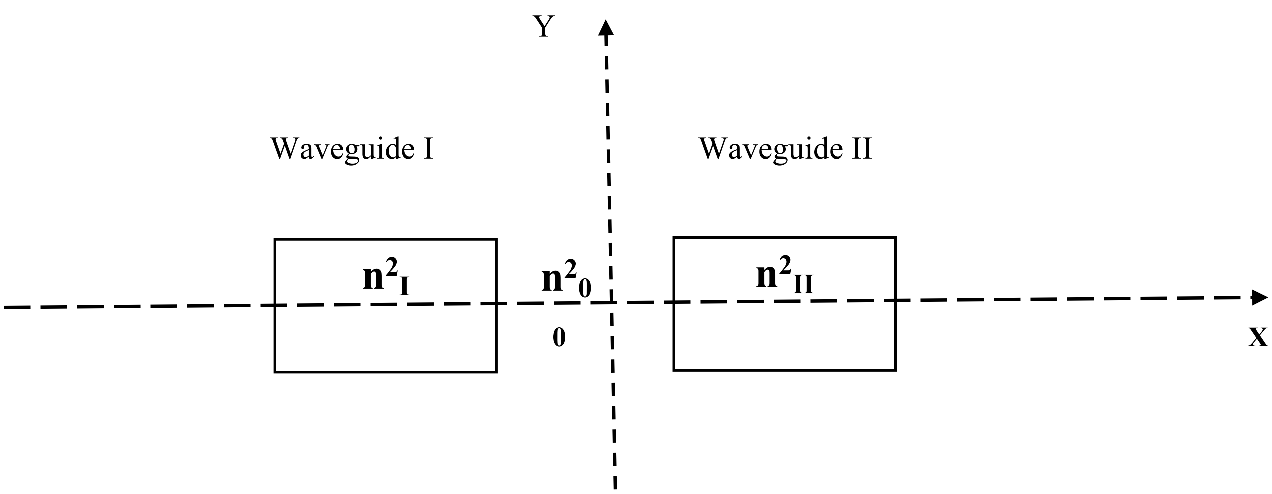

Figure S1: Directional coupler

The coupled EM fields can be considered as sum of eigen modes of the each waveguide:

$$\mathbf{E} = A(z)\,\mathbf{E_1} + B(z)\,\mathbf{E_2} \tag{S.3}$$

$$\mathbf{H} = A(z)\,\mathbf{H_1} + B(z)\,\mathbf{H_2} \tag{S.4}$$

The coupled EM fields also satisfies the Maxwell's equations like before coupling:

$$\nabla \times \mathbf{E} = -jw\mu_0\,\mathbf{H} \tag{S.5}$$

$$\nabla \times \mathbf{H} = jw\varepsilon_0\,N^2_p\,\mathbf{E} \tag{S.6}$$

Now, by replacing (S.3) and (S.4) into (S.5) and (S.6) we will get the equation of (S.7) and (S.8)

$$\frac{dA}{dz} + c_{12}\frac{dB}{dz}\exp\left[-j(\beta_2-\beta_1)z\right] + j\chi_1 A + jk_{12}B\exp\left[-j(\beta_2-\beta_1)z\right] = 0 \tag{S.7}$$

$$\frac{dB}{dz} + c_{21}\frac{dA}{dz}\exp\left[+j(\beta_2-\beta_1)z\right] + j\chi_2 A + jk_{21}A\exp\left[+j(\beta_2-\beta_1)z\right] = 0 \tag{S.8}$$

From those equ. (S.7), (S.8) three important parameter K_{pq}, C_{pq} and χ_p are seen. The properties of the directional coupler can be understand from those parameters.

The mode coupling co-efficient, K_{pq} (p,q=1,2) of a directional coupler indicate the possibility of polarization of conversion from the waveguide-I to the waveguide-II. The mode coupling co-efficient, K_{pq} can be

$$K_{pq} = \frac{\omega\varepsilon_0 \int\limits_{+\alpha}^{-\alpha}\int\limits_{+\alpha}^{-\alpha}(N^2 - N_2^2)E_1^* \cdot E_2\,dxdy}{\int\limits_{-\alpha}^{+\alpha}\int\limits_{-\alpha}^{+\alpha} u_z \cdot (E_1^* \times H_1 + E_1 \times H_1^*)\,dxdy} \tag{S.9}$$

This indicate the overlap of the fields. If field of the waveguide-I is orthogonal to the waveguide-II, $K_{12} = 0$ means no power coupling between the modes. In the case of small width waveguide, $K_{12} \neq 0$ means some value of the overlap fields are found due to the absence of pure or 100% TE or TM modes. Modes are known as quasi TE or TM (also say TE like and TM like modes). Therefore, some power exist even in the longitudinal fields. This is only true for small waveguide. Based on this concepts small dimension waveguide are used for the passive polarization devices [1].

Another important parameter butt coupling coefficient, C_{12} is defined as

$$C_{pq} = \frac{\int_{-\alpha}^{+\alpha}\int_{-\alpha}^{+\alpha} u_z \cdot (E_p^* \times H_q + E_p \times H_p^*) dxdy}{\int_{-\alpha}^{+\alpha}\int_{-\alpha}^{+\alpha} u_z \cdot (E_p^* \times H_p + E_p \times H_p^*) dxdy} \qquad \text{(S.10)}$$

This indicate the excitation efficiency of waveguide-I due to the field of waveguide-II. This may happen, when the eigen modes (E_1H_1) of waveguide-I is in the cladding region of waveguide-II and it's eigen modes (E_2 H_2) make excitation to waveguide-I.

Another, important parameter is χ_p, indicate the electric field distribution coupled to the adjacent waveguide. Generally, χ_p is η times smaller than k_{pq} and cannot be neglected when the waveguides are very close to each other. The expression for χ_p is

$$\chi_p = \frac{\omega\varepsilon_0 \int_{+\alpha}^{-\alpha}\int_{+\alpha}^{-\alpha} (N^2 - N_q^2) E_p^* \cdot E_q dxdy}{\int_{-\alpha}^{+\alpha}\int_{-\alpha}^{+\alpha} u_z \cdot (E_p^* \times H_p + E_p \times H_p^*) dxdy} \qquad \text{(S.11)}$$

The optical power carried by the waveguide due to the eigen mode is given by

$$P_p = \frac{1}{2}\iint_{-\infty}^{\infty} (E_p \times H_p^*) u_z \, dx \, dy \qquad \text{(S.12)}$$

The difference of propagation constants between the waveguides, $\delta = (\beta_2 - \beta_1)/2$

For the co-directional coupler $\beta_1, \beta_2 > 0$ but in the contra-directional coupler $\beta_1 > 0$ and $\beta_2 < 0$.

For the lossless waveguide, the power remain constant. Therefore, $\frac{\partial P}{\partial z} = 0$. One important relation between the coupling coefficients and propagation constants differences is

$$k_{21} = k^*_{12} + 2\delta C^*_{12}$$

For the symmetrical waveguide (same dimension), having same mode coupling coefficient ($k_{12} = k^*_{21}$) and butt coupling is zero ($C_{12} = 0$). Therefore, the propagation constant difference of the waveguide is also zero $\delta = (\beta_2 - \beta_1)/2 = 0$. Those are also true if the waveguides are separated from each other at sufficient distances. With asymmetrical waveguide (different WG dimension) all those parameters have non-zero values and cannot be negligible. [1]

Ring Resonator and Spectral Characteristics

Ring resonator is an important element of optical switch, modulator, filter and sensing applications. A ring resonator consist with the bus waveguide and a loop waveguide. The loop can be different types such as circle, ellipse, triangular or racetrack etc. [2]. Figure S2 shows the simple all pass ring resonator.

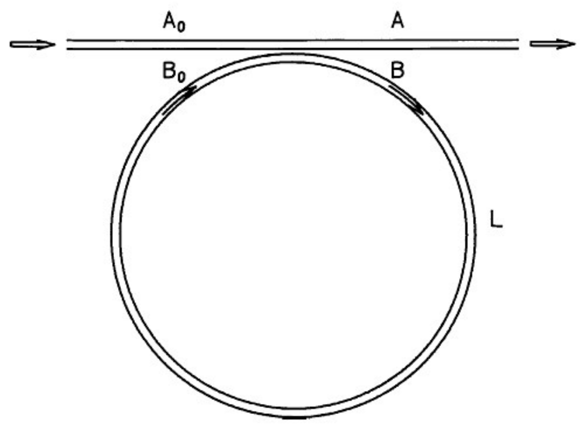

Figure S2: All pass ring resonator

The amplitude transmittance of the ring resonator,

$$\frac{A}{A_0} = (1-\gamma)^{1/2}\left[\frac{\cos(\kappa l) - (1-\gamma)^{1/2}\exp(-\frac{\rho}{2}L - j\beta L)}{1-(1-\gamma)^{1/2}\cos(kl)\exp(-\frac{\rho}{2}L - j\beta L)}\right] \quad (S.13)$$

Let us consider,

$$a = (1-\gamma)^{1/2}\exp(-\frac{\rho}{2}L)$$

$$t = \cos(kl)$$

$$\phi = \beta L$$

Then, Transmission intensity,

$$T(\phi) = \left|\frac{A}{A_0}\right|^2 = (1-\gamma)\left[\frac{a^2 - 2at\cos(\phi) + t^2}{1-2at\cos(\phi) + (at)^2}\right] \quad (S.14)$$

Here, a = single round trip amplitude transmission,

t= transmission co-efficient and

ϕ= single round trip phase shift

In the lossless case, $t^2 + a^2 = 1$. Now, the maximum and minimum transmission are

$$T_{max} = (1-\gamma)\frac{(a+t)^2}{(1+at)^2} \tag{S.15}$$

$$T_{min} = (1-\gamma)\frac{(a-t)^2}{(1-at)^2}$$

The resonance wavelength of the of the resonator is defined as

$$\lambda_{res} = \frac{n_{eff} L}{m} \quad m= 1, 2, 3.... \tag{S.16}$$

The critical coupling is define as the resonance condition in which the transmission at the output port is zero. That means T_{min} drops to zero, $a = t$ or $1 - a^2 = k^2$, which indicate the transmitted power equals to the loss in the ring.

The full width at half maximum (FWHM) is the 3dB resonance width is defined as

$$\text{FWHM} = \frac{(1-at)\lambda_{res}^2}{\pi n_g L\sqrt{at}} \tag{S.17}$$

Here, n_g = group index of waveguide and function of effective index and wavelength is defined as

$$n_g = n_{eff} - \lambda\frac{dn_{eff}}{d\lambda} \tag{S.18}$$

The on-off extinction ratio is defined as the ratio of T_{max} and T_{min}:

$$\text{ER} = \frac{T_{max}}{T_{min}} = \frac{(a+t)^2(1-at)^2}{(a-t)^2(1+at)^2} \tag{S.19}$$

For the critical coupling, $T_{min} = 0$, so ER value is large. For our simulation

$$\text{ER} = \frac{(a+t)^2(1-at)^2}{(a-t)^2(1+at)^2} = \frac{(0.95+0.75)^2(1-0.95*0.75)^2}{(0.95-0.75)^2(1+0.95*0.75)^2} = 2.036349832$$

The wavelength range between two resonances is known as free spectral range (FSR):

$$\text{FSR} = \frac{\lambda^2}{n_g L} \quad \text{(S.20)}$$

For 486x220 nm^2 SOI waveguide group index is found $n_g \approx 4.55$.

The finesses is defined as the ratio of FSR and resonance width:

$$\text{Finesse} = \frac{FSR}{FWFM} \quad \text{(S.21)}$$

The finesses indicate the sharpness of the resonance relative to their spacing. The Q-factor is measured as the sharpness relative to the central frequency:

$$\text{Q-factor} = \frac{\lambda_{res}}{FWHM} \quad \text{(S.22)}$$

It can be also defined as

$$\text{Q-factor} = \frac{\pi n_g L\sqrt{at}}{\lambda_{res}(1-at)} \quad \text{(S.23)}$$

The physical meaning of the Q-factor indicate the number of roundtrip by the light through the ring waveguide before it loss to initial values. More specifically, at Q-value energy lost 1/e of the initial energy. To define the Q-factor, ring is excited to some energy level and consider the loss of energy with time. Therefore, to make high Q-factor reduction of loss due to coupler is important. This section is an adaption of [1, 3].

References

1. Katsunari Okamoto, Fundamentals of Optical Waveguides, Second Edition, chapter 2, 4, Elsevier, 2006.
2. Rohit Grover, “Indium Phosphide Based Optical Micro Ring resonators” PhD thesis, pages:1-50, Graduate School of the University of Maryland, Collage Park, 2003.
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