Electronic Supplementary Information (ESI) for:

## Curvature-driven positioning of Turing patterns in phase-separating curved membranes

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Suppl. Fig. 1: In a smaller domain, the patterns which result at equilibrium are stripes without internal defects. This happens because the influence of the boundaries extends throughout the whole patterning region, i.e. the transversal size of the domain is not much larger than the characteristic length of the patterns.


Suppl. Fig. 2: Phase separation (A) and corresponding Turing pattern dynamics (B) for a sphere with a single bump. Here the phase separation process has a direct coupling with the surface curvature that favours the aggregation of the A-rich phase in the negative-curvature region. The pattern dynamics of case (B) can be compared to the one shown in column (C) the coupling with binary phase separation dynamics has been replaced by a direct coupling of the inhibitor diffusivity to the surface curvature: in this case no conservation law poses any lower limit to the area of the pattern-allowing region, therefore the collar can now be made narrow enough to select only the striped patterning mode, typical of narrow domains.



Suppl. Fig. 3: Here we show that, by tuning properly the strength $c$ of the curvature coupling, one can select the number of the disconnected domains resulting by the arrested coarsening of the binary mixture phases on the surface. In figure (A) a critical value of the coupling strength $c^{*}$ is used, giving rise to an incomplete separation in which two domains are connected through a stripe in the "prohibited" region. In figure (B) it is shown that a value bigger than $c^{*}$ gives rise to a full separation of the domains in which the B-rich phase is excluded from the connected low-curvature region.

Suppl. Movie 1: This movie shows the dynamics corresponding to Fig. 5A and Fig. 5B in the main text, in logarithmic timescale.

