

## A Quantum-Mechanical investigation of Oxygen Vacancy and Copper Doping in the Orthorhombic $\text{CaSnO}_3$ Perovskite

Jefferson Maul, S. Casassa, and Alessandro Erba  
*Dipartimento di Chimica, Università di Torino, via Giuria 5, 10125, Torino, Italy\**

I. M. G. Santos  
*Núcleo de Pesquisa e Extensão: Laboratório de Combustíveis e Materiais,  
Universidade Federal da Paraíba, CEP 58051-900, João Pessoa, PB, Brazil*

J. R. Sambrano  
*Computational Simulation Group, CDMF-UNESP, São Paulo State University, CEP 17033-360, Bauru, SP, Brazil*

Animation of the vibrational modes of  $\text{CaSnO}_3$  for the pristine and oxygen vacancy cases, as calculated at PBE level, can be found in the following links respectively:

[http://www.crystal.unito.it/vibs/CaSnO3/index.html?name=CaSnO3\\_Pristine\\_PBE.xyz](http://www.crystal.unito.it/vibs/CaSnO3/index.html?name=CaSnO3_Pristine_PBE.xyz)

[http://www.crystal.unito.it/vibs/CaSnO3/index.html?name=CaSnO3\\_Oxygen\\_Vacancy\\_PBE.xyz](http://www.crystal.unito.it/vibs/CaSnO3/index.html?name=CaSnO3_Oxygen_Vacancy_PBE.xyz)

TABLE I: Bond lengths of oxygen atoms toward Sn in octahedral site and its defective counterpart (XX represent vacant atom), in both position  $O_{ap}$  and  $O_{eq}$  and calculation level (PBE and PBE0). The bond lengths are given in Å and the volume ( $D_{VOL}$ ) is given in % tanking as reference the bulk volume.

Level	Position	Method	Config.	$D_{VOL}$	$O_{ap}1/XX$	$O_{ap}2$	$O_{eq}1$	$O_{eq}2$	$O_{eq}3/XX$	$O_{eq}4$
PBE	<i>Bulk</i>	closed-shell	ref		2.102	2.102	2.095	2.095	2.098	2.098
PBE0	<i>Bulk</i>	closed-shell	ref		2.070	2.070	2.066	2.066	2.069	2.069
		closed-shell	-0.04	-	2.155	2.121	2.112	2.112	2.127	2.124
		AR	singlet	-0.04	-	2.155	2.121	2.112	2.127	2.124
	$O_{ap}$	triplet	0.35	-	2.079	2.123	2.126	2.134	2.133	2.133
		closed-shell	-0.03	2.145	2.150	2.122	2.111	2.126	2.123	2.123
		GH	singlet	-0.03	2.134	2.150	2.122	2.111	2.126	2.123
PBE		triplet	0.36	2.029	2.076	2.122	2.125	2.132	2.131	2.131
		closed-shell	-0.02	2.129	2.128	2.122	2.130	-	2.168	2.168
		AR	singlet	-0.02	2.129	2.128	2.122	2.130	-	2.168
	$O_{eq}$	triplet	0.37	2.123	2.140	2.124	2.122	-	2.079	2.079
		closed-shell	0.00	2.127	2.127	2.120	2.127	1.744	2.154	2.154
		GH	singlet	0.00	2.128	2.127	2.121	2.128	1.898	2.155
		triplet	0.38	2.121	2.136	2.121	2.120	2.159	2.075	2.075
		closed-shell	-0.10	-	2.137	2.090	2.079	2.093	2.091	2.091
		AR	singlet	-0.10	-	2.137	2.090	2.079	2.093	2.091
	$O_{ap}$	triplet	0.28	-	2.053	2.102	2.111	2.121	2.112	2.112
		closed-shell	-0.09	2.201	2.134	2.091	2.080	2.093	2.092	2.092
		GH	singlet	-0.12	2.665	2.136	2.088	2.078	2.092	2.090
		triplet	0.28	2.181	2.050	2.102	2.111	2.119	2.111	2.111

\*Electronic address: [jmauldea@unito.it](mailto:jmauldea@unito.it)

TABLE II: Atomic net charge ( $q$ ) and spin density ( $\mu$ ) as calculated by both Mulliken ( $q_M$  and  $\mu_M$ ) and Hirshfeld-I ( $q_H$  and  $\mu_H$ ) of SnO<sub>6</sub> octahedral site and vacancy alike at PBE and PBE0 level. Two oxygen position ( $O_{ap}$  and  $O_{eq}$ ) taking as reference the central Sn are displayed, with quantities given in  $|e|$ .

Level	Position	Method	Config.	$q/\mu$	Sn	$O_{ap1}/XX$	$O_{ap2}$	$O_{eq1}$	$O_{eq2}$	$O_{eq3}$	$O_{eq4}$		
PBE	<i>Bulk</i>	PBE	closed-shell	$q_M$	1.516	-1.010	-1.010	-1.014	-1.014	-1.014	-1.014	-1.014	
				$q_H$	2.258	-1.274	-1.274	-1.272	-1.272	-1.272	-1.272	-1.272	-1.272
				$q_M$	0.979	-	-1.015	-0.990	-1.002	-0.995	-0.995	-0.995	-0.995
				$q_H$	1.246	-	-1.173	-1.172	-1.177	-1.176	-1.168	-1.168	-1.168
				$q_M$	0.979	-	-1.015	-0.990	-1.002	-0.995	-0.995	-0.995	-0.995
				$q_H$	1.246	-	-1.173	-1.172	-1.177	-1.176	-1.168	-1.168	-1.168
	$O_{ap}$	AR	triplet	$q_M$	0.991	-	-1.006	-0.975	-0.989	-0.981	-0.979	-0.979	-0.979
				$q_H$	1.350	-	-1.174	-1.183	-1.184	-1.183	-1.177	-1.177	-1.177
				$\mu_M$	0.492	-	0.001	0.073	0.045	0.055	0.070	0.070	0.070
				$\mu_H$	0.367	-	0.005	0.095	0.072	0.082	0.091	0.091	0.091
				$q_M$	1.055	0.174	-1.014	-1.001	-0.998	-0.990	-0.994	-0.994	-0.994
				$q_H$	1.220	0.000	-1.169	-1.166	-1.170	-1.168	-1.162	-1.162	-1.162
	$O_{eq}$	GH	triplet	$q_M$	1.056	0.175	-1.014	-1.001	-0.998	-0.990	-0.995	-0.995	-0.995
				$q_H$	1.227	0.000	-1.170	-1.167	-1.171	-1.169	-1.163	-1.163	-1.163
				$q_M$	1.059	0.139	-1.002	-0.973	-0.992	-0.985	-0.977	-0.977	-0.977
				$q_H$	1.327	0.000	-1.275	-1.177	-1.178	-1.273	-1.172	-1.172	-1.172
				$\mu_M$	0.417	0.126	0.002	0.071	0.045	0.055	0.068	0.068	0.068
				$\mu_H$	0.378	0.000	0.005	0.091	0.067	0.077	0.087	0.087	0.087
$O_{eq}$	AR	triplet	$q_M$	0.987	-0.987	-0.993	-1.001	-0.991	-	-1.018	-1.018	-1.018	
			$q_H$	1.250	-1.170	-1.174	-1.177	-1.168	-	-1.174	-1.174	-1.174	
			$q_M$	0.987	-0.987	-0.993	-1.001	-0.991	-	-1.018	-1.018	-1.018	
			$q_H$	1.250	-1.170	-1.174	-1.177	-1.168	-	-1.174	-1.174	-1.174	
			$q_M$	1.025	-0.970	-0.977	-0.985	-0.978	-	-1.010	-1.010	-1.010	
			$q_H$	1.413	-1.180	-1.186	-1.187	-1.181	-	-1.180	-1.180	-1.180	
$O_{eq}$	GH	triplet	$\mu_M$	0.454	0.067	0.053	0.048	0.061	-	0.000	0.000	0.000	
			$\mu_H$	0.338	0.087	0.075	0.072	0.084	-	0.004	0.004	0.004	
			$q_M$	1.093	-0.990	-0.992	-0.994	-0.996	0.176	-1.016	-1.016	-1.016	
			$q_H$	1.242	-1.166	-1.170	-1.171	-1.163	0.000	-1.171	-1.171	-1.171	
			$q_M$	1.088	-0.990	-0.993	-0.995	-0.997	0.180	-1.017	-1.017	-1.017	
			$q_H$	1.237	-1.166	-1.170	-1.171	-1.163	0.000	-1.170	-1.170	-1.170	
<i>Bulk</i>	PBE0	closed-shell	$q_M$	1.123	-0.975	-0.980	-0.983	-0.986	0.167	-1.008	-1.008	-1.008	
			$q_H$	1.400	-1.176	-1.181	-1.272	-1.176	0.000	-1.178	-1.178	-1.178	
			$\mu_M$	0.370	0.065	0.052	0.046	0.061	0.138	0.001	0.001	0.001	
			$\mu_H$	0.345	0.081	0.069	0.066	0.079	0.000	0.004	0.004	0.004	
			$q_M$	1.762	-1.115	-1.115	-1.115	-1.115	-1.115	-1.115	-1.115	-1.115	
			$q_H$	2.409	-1.355	-1.355	-1.351	-1.351	-1.351	-1.351	-1.351	-1.351	
$O_{eq}$	AR	triplet	$q_M$	1.163	-	-1.120	-1.090	-1.103	-1.097	-1.097	-1.097		
			$q_H$	1.479	-	-1.276	-1.276	-1.277	-1.281	-1.273	-1.273		
			$q_M$	1.162	-	-1.120	-1.090	-1.104	-1.097	-1.097	-1.097		
			$q_H$	1.477	-	-1.276	-1.277	-1.283	-1.280	-1.273	-1.273		
			$q_M$	1.128	-	-1.112	-1.075	-1.092	-1.087	-1.082	-1.082		
			$q_H$	1.489	-	-1.262	-1.285	-1.284	-1.284	-1.281	-1.281		
$O_{eq}$	GH	triplet	$\mu_M$	0.653	0.000	-0.001	0.077	0.043	0.054	0.073	0.073		
			$\mu_H$	0.482	-	0.001	0.110	0.077	0.087	0.103	0.103		
			$q_M$	1.244	0.184	-1.120	-1.099	-1.101	-1.101	-1.096	-1.096		
			$q_H$	1.452	0.000	-1.273	-1.272	-1.276	-1.273	-1.267	-1.267		
			$q_M$	1.226	0.118	-1.120	-1.101	-1.103	-1.100	-1.098	-1.098		
			$q_H$	1.501	0.000	-1.279	-1.278	-1.283	-1.280	-1.274	-1.274		
$O_{eq}$	GH	triplet	$q_M$	1.218	0.184	-1.111	-1.084	-1.089	-1.091	-1.081	-1.081		
			$q_H$	1.462	0.000	-1.258	-1.279	-1.276	-1.276	-1.275	-1.275		
			$\mu_M$	0.562	0.167	-0.001	0.079	0.043	0.054	0.072	0.072		
			$\mu_H$	0.495	0.000	0.001	0.108	0.071	0.082	0.098	0.098		

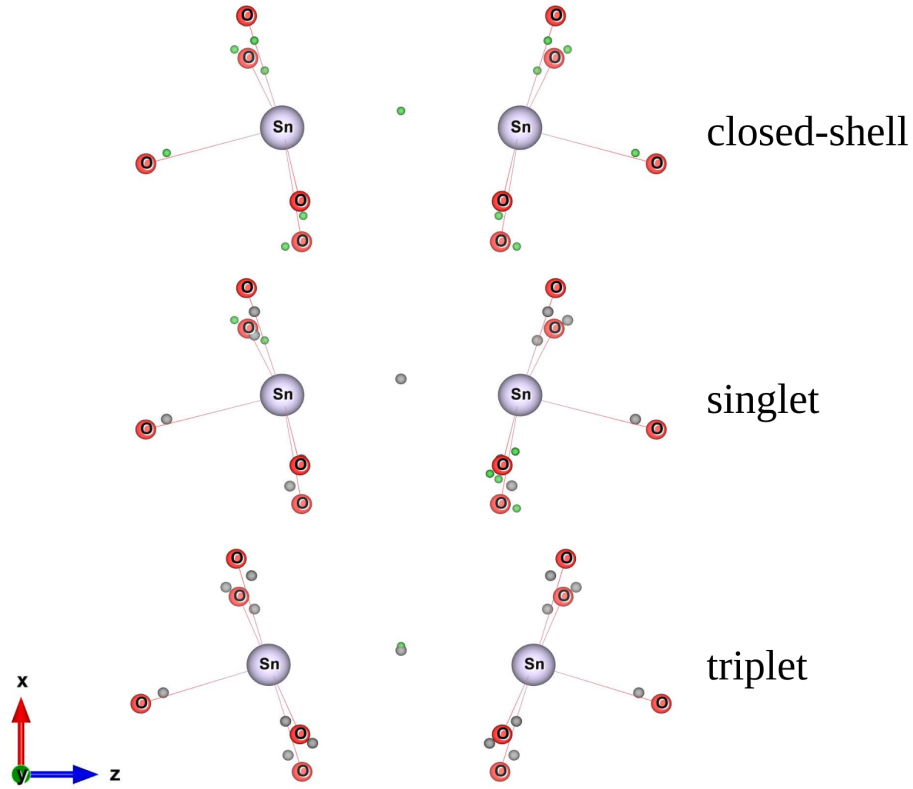


FIG. 1: Wannier centroid's representation of vacancy containing site. Oxygen and Tin atoms are labeled , and the spheres between them represents the centroid of the Wannier localization. The smaller one are characterized as alpha electrons and the other the beta ones.

TABLE IV: Bond lengths of oxygen atoms toward Sn and Cu in octahedral site and its defective counterpart (XX represent vacant atom), at  $O_{ap}$  as calculated at PBE0 level and GH method. The bond lengths are given in Å and the volume ( $D_{VOL}$ ) is given in % tanking as reference the bulk volume.

Atom	Neighbor	$D_{VOL}$	$O_{ap}1/XX$	$O_{ap}2$	$O_{eq}1$	$O_{eq}2$	$O_{eq}3$	$O_{eq}4$
	bulk	ref	2.070	2.070	2.066	2.066	2.069	2.069
	no vacancy	-0.37	2.137	2.066	2.052	2.058	2.061	2.051
Sn	first neighbor	-0.15	1.778	1.969	2.033	2.026	2.038	2.026
	second neighbor	0.04	1.838	1.976	1.961	2.018	2.040	2.066
	third neighbor	0.09	2.153	1.978	2.052	2.065	2.009	1.968
	no vacancy	-0.37	1.945	1.945	2.071	2.072	1.959	1.960
Cu	first neighbor	-0.15	2.551	2.088	2.012	2.038	2.011	2.040
	second neighbor	0.04	2.034	2.043	2.048	2.069	2.253	2.170
	third neighbor	0.09	2.037	2.047	2.165	2.296	2.048	2.092
	no vacancy	-0.37	1.945	1.945	2.071	2.072	1.959	1.960

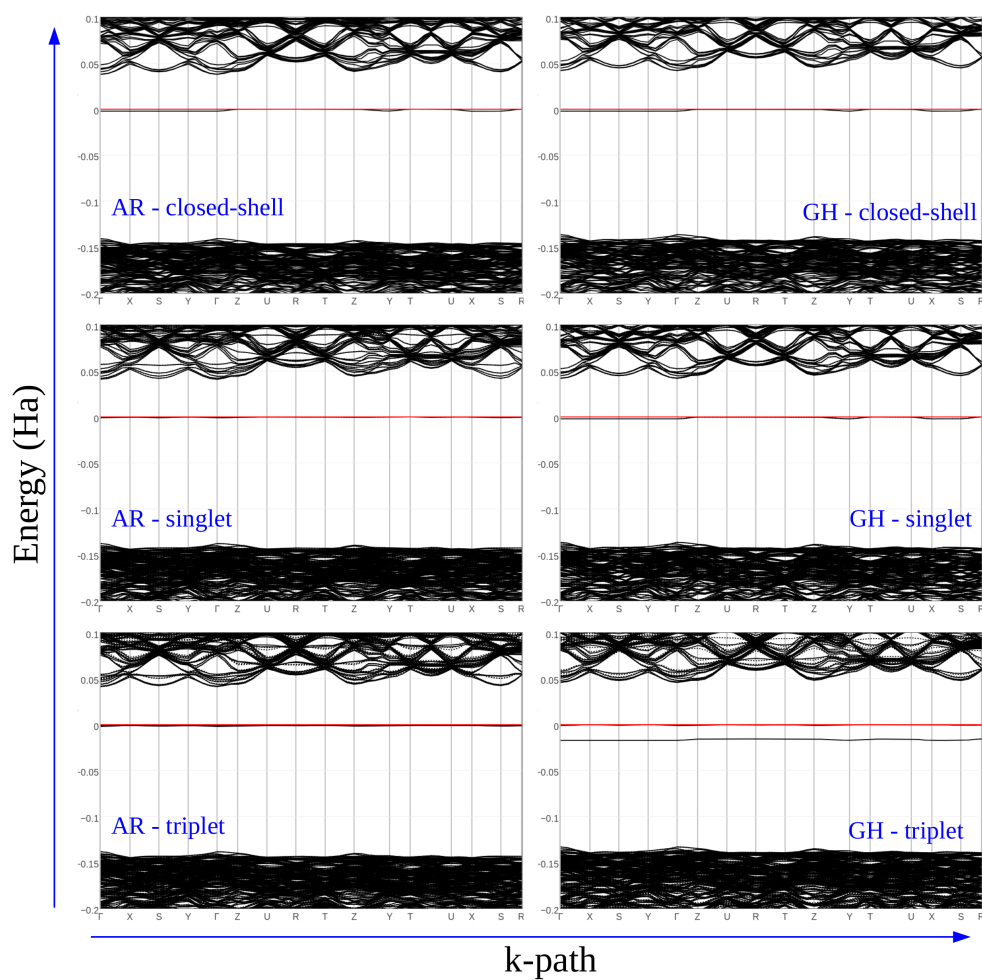


FIG. 2: Band structure of  $\text{CaTiO}_3$  at PBE0 level for the three electronic configuration closed-shell, singlet and triplet. The methods **AR** and **GH** are placed into the left and right columns, respectively. Fermi energy are placed as a straight red line.

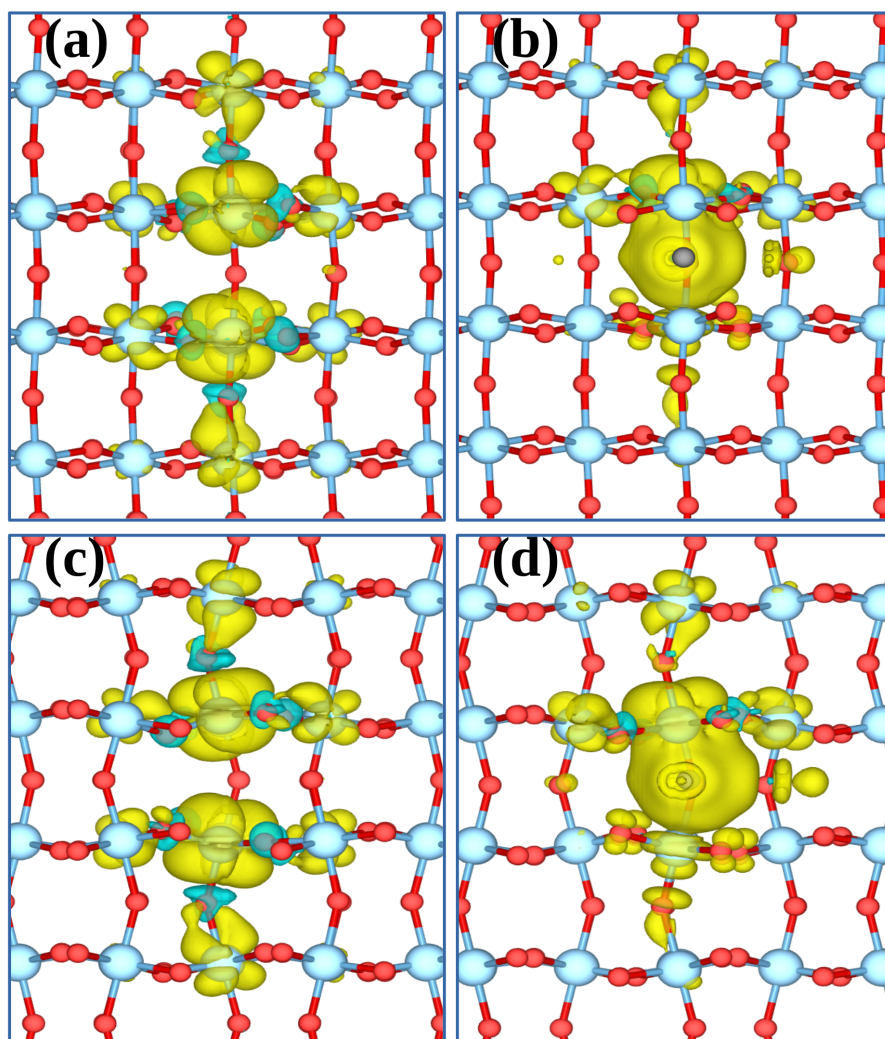


FIG. 3: Spin density 3D maps (cutoff  $0.001 e/a_0^3$ ) for the  $\text{CaTiO}_3$  (a) and (c) are two views of the **AR** model and (b) and (d) are two views of the **GH** model. All calcium atoms have been removed for clarity.

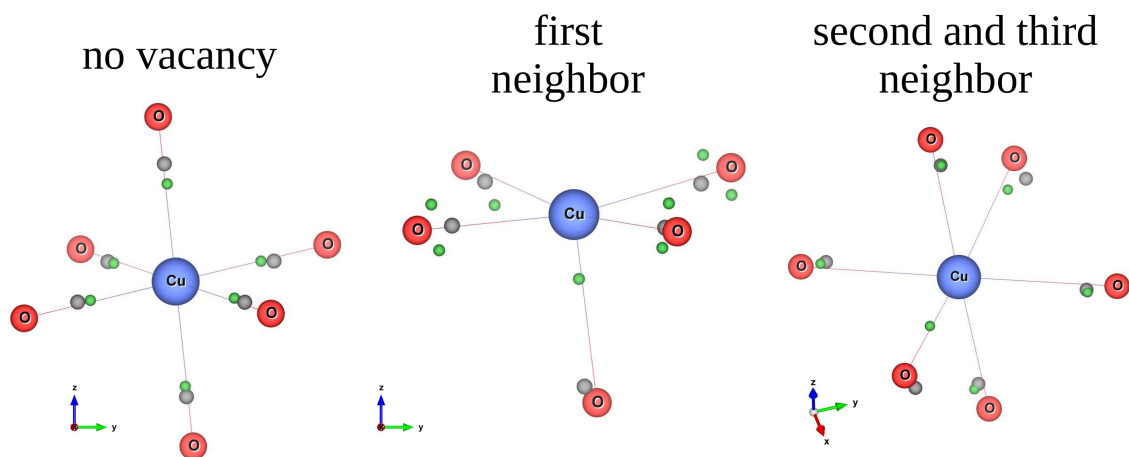


FIG. 4: Wannier centroid's representation of the doping site. Oxygen and Copper atoms are labeled, and the spheres between them represents the centroid of the Wannier localization. The smaller one are characterized as alpha electrons and the other the beta ones.

TABLE V: Atomic net charge ( $q$ ) and spin density ( $\mu$ ) as calculated by both Mulliken ( $q_M$  and  $\mu_M$ ) and Hirshfeld ( $q_H$  and  $\mu_H$ ) of  $\text{SnO}_6$  and  $\text{CuO}_6$  “octahedral” sites and its defective sites or surrounds. Both quantities are given in  $|e|$ . The Sn atom is always a first neighbor of the vacancy, apart from the NV case in which it is neighbor of the doped site.

Atom	Cu Neighbor	Method	$q/\mu$	Sn	$O_{ap1}/XX$	$O_{ap2}$	$O_{eq1}$	$O_{eq2}$	$O_{eq3}$	$O_{eq4}$
<i>Bulk</i>	-	-	$q_M$	1.762	-1.115	-1.115	-1.115	-1.115	-1.115	-1.115
			$q_H$	2.409	-1.355	-1.355	-1.351	-1.351	-1.351	-1.351
	no vacancy	-	$q_M$	1.737	-0.945	-1.113	-1.116	-1.104	-1.110	-1.110
			$q_H$	2.376	-1.054	-1.353	-1.351	-1.338	-1.344	-1.345
	first neighbor	<b>AR</b>	$q_M$	1.546	-	-1.087	-1.078	-1.073	-1.072	-1.067
			$q_H$	2.275	-	-1.334	-1.325	-1.328	-1.322	-1.318
	first neighbor	<b>GH</b>	$q_M$	1.565	0.080	-1.081	-1.072	-1.071	-1.071	-1.069
			$q_H$	2.265	0.000	-1.334	-1.324	-1.325	-1.319	-1.316
Sn	second neighbor	<b>AR</b>	$q_M$	1.495	-	-1.086	-1.039	-1.082	-1.078	-1.086
			$q_H$	2.230	-	-1.337	-1.246	-1.323	-1.333	-1.338
	second neighbor	<b>GH</b>	$q_M$	1.522	0.072	-1.083	-1.042	-1.078	-1.081	-1.086
			$q_H$	2.221	0.000	-1.337	-1.242	-1.322	-1.331	-1.335
	third neighbor	<b>AR</b>	$q_M$	1.489	-	-1.080	-1.080	-1.094	-1.067	-1.042
			$q_H$	2.230	-	-1.334	-1.347	-1.340	-1.315	-1.248
	third neighbor	<b>GH</b>	$q_M$	1.513	0.075	-1.076	-1.073	-1.094	-1.072	-1.046
			$q_H$	2.223	0.000	-1.334	-1.344	-1.341	-1.312	-1.245
	no vacancy	-	$q_M$	0.889	-0.945	-0.949	-1.007	-1.007	-0.954	-0.953
			$q_H$	0.926	-1.054	-1.061	-1.150	-1.151	-1.066	-1.064
			$\mu_M$	0.469	-0.026	0.012	0.231	0.228	0.034	0.003
			$\mu_H$	0.519	-0.038	0.019	0.207	0.205	0.031	-0.004
	first neighbor	<b>AR</b>	$q_M$	0.689	-	-1.103	-1.068	-1.073	-1.064	-1.068
			$q_H$	0.668	-	-1.275	-1.220	-1.215	-1.211	-1.220
			$\mu_M$	0.712	-	0.000	0.058	0.064	0.068	0.059
			$\mu_H$	0.728	-	-0.002	0.055	0.059	0.062	0.056
	first neighbor	<b>GH</b>	$q_M$	0.720	0.080	-1.099	-1.069	-1.078	-1.068	-1.073
			$q_H$	0.661	0.000	-1.274	-1.218	-1.214	-1.208	-1.218
			$\mu_M$	0.712	-0.003	0.000	0.059	0.064	0.068	0.059
			$\mu_H$	0.727	0.000	-0.002	0.055	0.060	0.063	0.056
Cu	second neighbor	<b>AR</b>	$q_M$	0.798	-1.092	-1.091	-1.093	-1.093	-1.039	-1.106
			$q_H$	0.747	-1.235	-1.235	-1.232	-1.239	-1.246	-1.288
			$\mu_M$	0.758	0.053	0.053	0.051	0.048	0.001	0.000
			$\mu_H$	0.766	0.051	0.052	0.049	0.047	-0.002	-0.002
	second neighbor	<b>GH</b>	$q_M$	0.798	-1.092	-1.091	-1.093	-1.094	-1.042	-1.106
			$q_H$	0.746	-1.234	-1.234	-1.232	-1.239	-1.242	-1.288
			$\mu_M$	0.757	0.053	0.053	0.051	0.048	0.001	0.000
			$\mu_H$	0.766	0.051	0.052	0.049	0.047	-0.002	-0.002
	third neighbor	<b>AR</b>	$q_M$	0.792	-1.092	-1.088	-1.108	-1.042	-1.086	-1.098
			$q_H$	0.742	-1.235	-1.231	-1.283	-1.248	-1.233	-1.243
			$\mu_M$	0.755	0.055	0.055	0.000	0.001	0.054	0.046
			$\mu_H$	0.764	0.053	0.053	-0.002	-0.002	0.052	0.046
	third neighbor	<b>GH</b>	$q_M$	0.792	-1.092	-1.087	-1.108	-1.046	-1.087	-1.098
			$q_H$	0.741	-1.234	-1.228	-1.283	-1.245	-1.233	-1.243
			$\mu_M$	0.755	0.055	0.055	0.000	0.001	0.054	0.046
			$\mu_H$	0.763	0.053	0.053	-0.002	-0.002	0.052	0.046

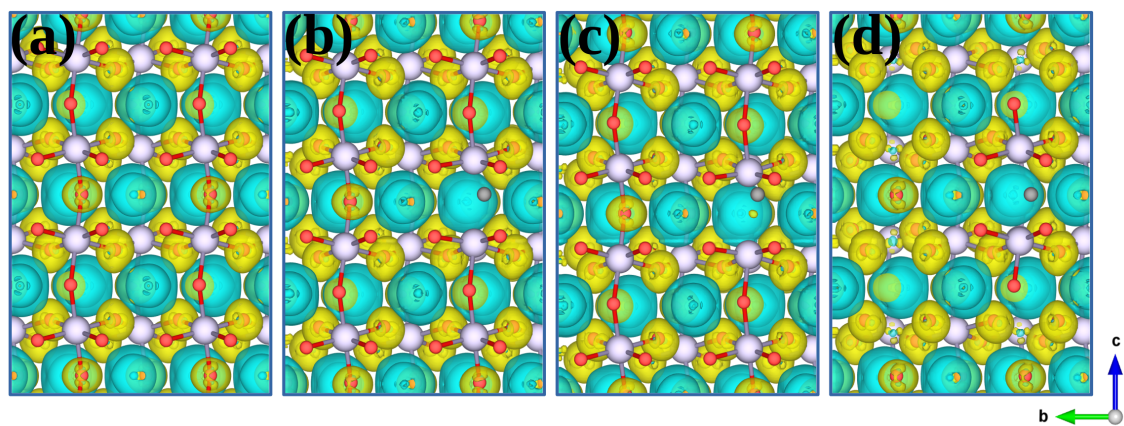


FIG. 5: CaSnO<sub>3</sub> 3D interaction densities (difference between the electron density of the interacting system and the superposition of atomic non-interacting densities) with a cutoff  $0.015 e/a_0^3$  (a) Pristine System (b) Restricted (c) Singlet (d) Triplet. The Calcium atoms are omitted for a better view. The grey small ball presents the "ghost" atom, a.k.a. vacant atom.