

Electronic Supplementary Information for:  
**Understanding the structural diversity of freestanding  $\text{Al}_2\text{O}_3$   
ultrathin films through a DFTB-aided genetic algorithm**

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# 1 Genetic operators in the 2D-GA

The different genetic operators used in the 2D-GA are listed in Table 1. These can be seen as the 2D equivalents of 3D operators used in crystal structure optimization.<sup>1,2,3</sup>

Table 1

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Operator	Probability	Description
Cut-and-splice pairing	50%	Cuts two parent structures along a plane perpendicular to a randomly chosen cell vector and combines the different halves into a new structure. The new <b>a</b> and <b>b</b> vectors are chosen as random linear combinations of the parent vectors and are then rescaled so that the initial surface area corresponds to the average of the best 20% structures in the population.
Strain mutation	15%	Scales the surface unit cell vectors and atomic positions according to strain components drawn from a Gaussian distribution with $\sigma = 0.7$ .
Shear mutation	10%	Applies a shear strain perpendicular to the <b>a</b> or <b>b</b> vectors (chosen at random), which hence only affects the atomic <i>c</i> coordinates.
Rattle mutation	10%	Randomly displaces 80% of the atomic coordinates with amplitudes uniformly selected between 0 and 2.5 Å.
Soft mutation	15%	Moves the atoms along the vibrational mode with the lowest frequency, as identified by a simple pairwise interaction model. <sup>4</sup> If a structure has already been soft mutated, the next vibrational mode is chosen.

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## 2 Bulk energetics with the matsci DFTB parameters

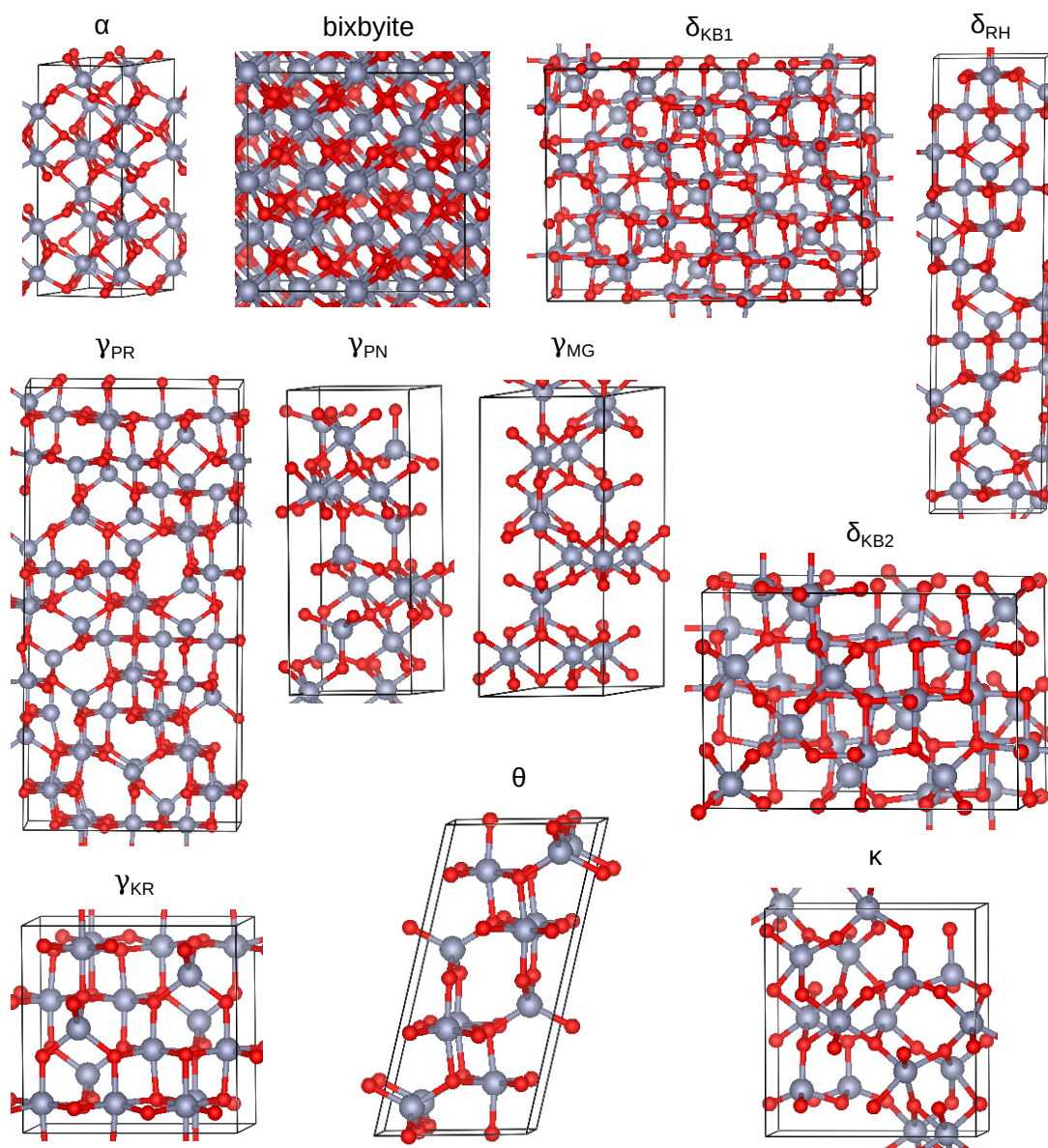
Table 2 shows the relative energies (per formula unit) for various bulk  $\text{Al}_2\text{O}_3$  polytypes calculated with the present DFT setup and with the `matsci[5]` DFTB parameter set using  $\ell$ -dependent Hubbard parameters. Bulk  $\text{Al}_2\text{O}_3$  polymorphism was not taken into account in the `matsci` parametrization procedure, leading to a poor description of the energetical ordering.

Table 2

		$\Delta E$ (eV/f.u.)	
	Model	optB86b -vdW	DFTB <code>matsci[5]</code>
$\alpha$		0	0
$\theta$		0.15	-0.56
$\kappa$		0.15	-0.31
$\gamma$	$\gamma_{\text{MG}}^6$	0.30	-0.52
	$\gamma_{\text{PN}}^7$	0.30	-0.52
	$\gamma_{\text{PR}}^{8,9}$	0.42	0.16
	$\gamma_{\text{KR}}^{10}$	0.23	-0.02
$\delta$	$\delta_{\text{RH}}^{11}$	0.43	-0.23
	$\delta_{\text{KB1}}^{12}$	0.14	-0.50
	$\delta_{\text{KB2}}^{12}$	0.14	-0.50
bixbyite		0.17	-0.58

### 3 Structures of known bulk polymorphs

Figure 1: Structures of the known bulk polymorphs listed in Table 1 of the main text and in Table 2 of the ESI. The atomic coordinates and cell vectors are provided in the ZIP archive of the ESI.



## 4 Parity diagrams for the thin film training set

Figures 2 and 3 show the parity diagrams for the `bulk-opt` (blue) and `film-opt` (orange) DFTB total energies for two series of  $\text{Al}_2\text{O}_3$  thin films, in comparison with DFT. The thin film structures have been obtained by 2D-GA searches in the thickness intervals of 3.5-5 Å (Figure 2) and 6.5-8 Å (Figure 3) using the `bulk-opt` parametrization. The `film-opt` parameters have been refined so as to minimize the deviation from parity for these data sets.

Figure 2: 3.5-5 Å thickness interval.

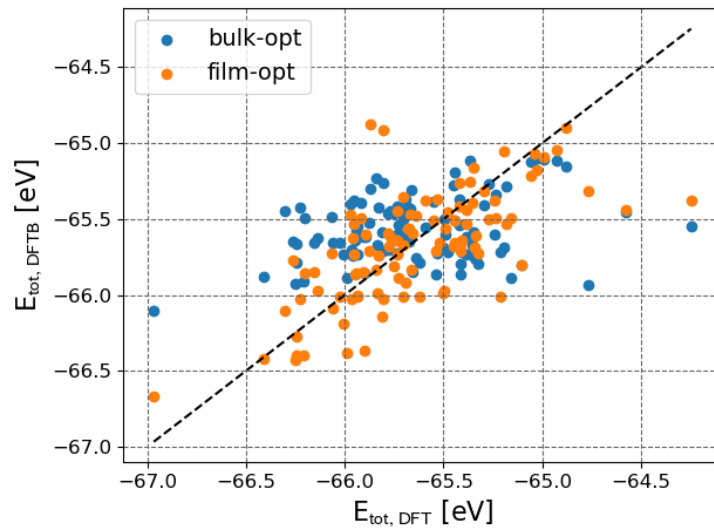
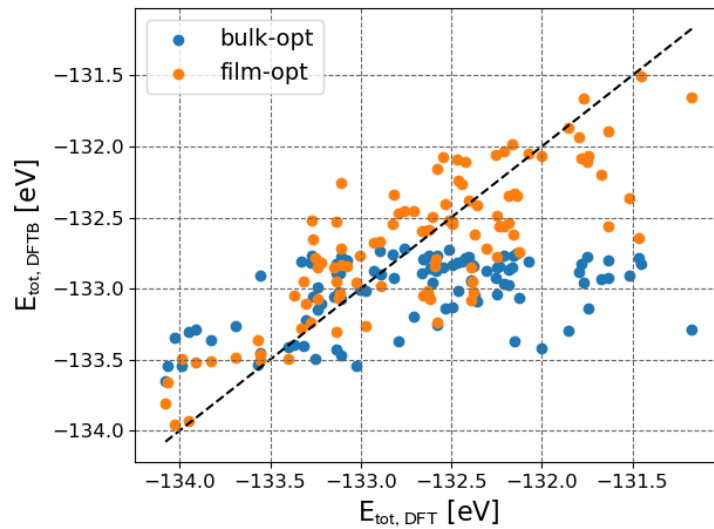


Figure 3: 6.5-8 Å thickness interval.



## 5 Structure-stability relations in bulk Al<sub>2</sub>O<sub>3</sub>

To illustrate the difficulties in relating the relative stabilities of the bulk Al<sub>2</sub>O<sub>3</sub> polymorphs to differences in local bonding patterns, we analyze the variation in cation coordination as well as a model based on correlations between bond strengths and bond lengths.

### 5.1 Cation coordination

The abundances of four- and six-fold coordinated Al atoms for different known bulk polymorphs are listed in Table 3 together with energy differences calculated with DFT. The lack of correlation between both properties can be appreciated by considering e.g. the most stable  $\alpha$  polymorph where all Al atoms are six-fold coordinated. Assigning a higher stability to this local environment, however, would be contradicted by the relative instability of the bixbyite phase (consisting also exclusively of 6-fold Al) compared to several other phases with significant concentrations of 4-fold Al atoms (i.e.  $\kappa$ ,  $\theta$ ,  $\delta_{\text{KB1}}$  and  $\delta_{\text{KB2}}$ ).

### 5.2 Bond-length-bond-strength correlation

The relative stabilities of different bulk structures may also be assessed in terms of the number and strength of the Al-O bonds. A commonly applied expression for bond strengths using only local geometric information is given by the model of Pauling[13] and Brown and coworkers[14]:

$$E_{\text{bond}}(r) = \exp\left(\frac{r - r_0}{b}\right), \quad (1)$$

with parameters  $b$  and  $r_0$ . Energy differences per formula unit may then be written as:

$$\Delta E_{\text{A-B}} = \left( \sum_{\text{Al-O bonds}} E_{\text{bond}}(r_{\text{Al-O}})/N_{\text{f.u.}} \right)_A - \left( \sum_{\text{Al-O bonds}} E_{\text{bond}}(r_{\text{Al-O}})/N_{\text{f.u.}} \right)_B. \quad (2)$$

The last column in Table 3 shows the performance of such a model where  $r_0$  and  $b$  have been adjusted to reproduce the DFT energy differences w.r.t.  $\alpha$ -Al<sub>2</sub>O<sub>3</sub> via least-squares fitting. The fitted parameter values are  $r_0 = 2.29 \text{ \AA}$  and  $b = 0.35 \text{ \AA}$ . While the model retrieves  $\alpha$ -Al<sub>2</sub>O<sub>3</sub> as the most stable polymorph, it is clear that also this approach is not sufficiently accurate for the present purposes.

Table 3

Model	Al coordination		$\Delta E$ (eV/f.u.)	
	4-fold (%)	6-fold (%)	optB86b -vdW	Bond model
$\alpha$	0	100	0	0
$\delta_{\text{KB1}}^{12}$	38	62	0.14	0.27
$\delta_{\text{KB2}}^{12}$	38	62	0.14	0.29
$\theta$	50	50	0.15	0.51
$\kappa$	25	75	0.15	0.34
bixbyite	0	100	0.17	0.57
$\gamma_{\text{KR}}^{10}$	25	75	0.23	0.19
$\gamma_{\text{MG}}^6$	38	62	0.30	0.40
$\gamma_{\text{PN}}^7$	38	62	0.30	0.41
$\gamma_{\text{PR}}^{8,9}$	33	67	0.42	0.43
$\delta_{\text{RH}}^{11}$	38	62	0.43	0.69

## 6 Vacancy refilling on $\delta_{\text{KB1}}(100)$ and $\delta_{\text{KB2}}(100)$

Figures 4 and 5 show the displacements of low-coordinated Al atoms into surface cation vacancies on  $\delta_{\text{KB1}}(100)$  and  $\delta_{\text{KB2}}(100)$ . The displacements for  $\gamma_{\text{MG}}(001)$  are shown in Figure 7 in the main text.

Figure 4:  $\delta_{\text{KB1}}(100)$

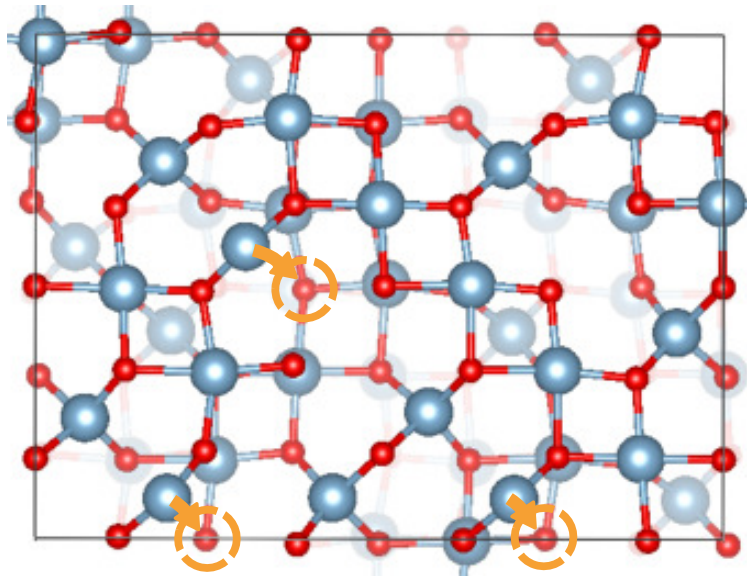
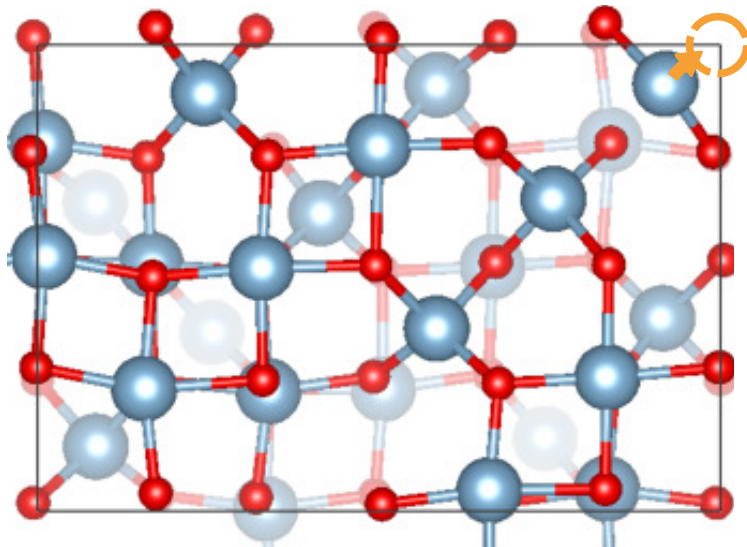


Figure 5:  $\delta_{\text{KB2}}(100)$





## References

- [1] C. W. Glass, A. R. Oganov, and N. Hansen. *Comput. Phys. Commun.* **175**, 11-12 (2006), 713–720.
- [2] D. C. Lonie and E. Zurek. *Comput. Phys. Commun.* **182**, 2 (2011), 372–387.
- [3] M. Van den Bossche, H. Grönbeck, and B. Hammer. *J. Chem. Theory Comput.* **14**, 5 (2018), 2797–2807.
- [4] A. O. Lyakhov, A. R. Oganov, and M. Valle. *Comput. Phys. Commun.* **181**, 9 (2010), 1623–1632.
- [5] J. Frenzel et al. *Z. Anorg. Allg. Chem.* **631**, 6-7 (2005), 1267–1271.
- [6] E. Menéndez-Proupin and G. Gutiérrez. *Phys. Rev. B* **72**, 3 (2005), 035116.
- [7] H. P. Pinto, R. M. Nieminen, and S. D. Elliott. *Phys. Rev. B* **70**, 12 (2004), 125402.
- [8] G. Paglia et al. *Phys. Rev. B* **71**, 22 (2005), 224115.
- [9] G. Paglia et al. *Phys. Rev. B* **68**, 14 (2003), 144110.
- [10] X. Krokidis et al. *J. Phys. Chem. B* **105**, 22 (2001), 5121–5130.
- [11] Y. Repelin and E. Husson. *Mat. Res. Bull.* **25**, 5 (1990), 611–621.
- [12] L. Kovarik et al. *J. Phys. Chem. C* **118**, 31 (2014), 18051–18058.
- [13] L. Pauling. *J. Am. Chem. Soc.* **69**, 3 (1947), 542–553.
- [14] I. D. Brown and R. D. Shannon. *Acta Cryst. A* **29**, 3 (1973), 266–282.