

**Supplementary Document To: Optical crystallographic Study of Piezoelectric $K_xNa_{1-x}NbO_3$
($x = 0.4, 0.5$ & 0.6) Single Crystals using Linear Birefringence**

1. Crystal Growth, Compositional and Orientation Determination -

Figure 1(a) shows the optical image of as grown crystals ($x = 0.5$) embedded in the re-solidified flux. By successive boiling and treating by ultrasonic waves in water, crystals were separated from flux. Crystal with a cubic geometry were found to have maximum dimensions of about 6 mm X 6 mm X 2 mm. Figure 1(b) shows one of the crystal ($x = 0.5$) having planer dimensions of about 5 mm X 5 mm. After compositional homogenization by annealing at 1000°C, energy dispersive X-ray analysis (EDX) was performed to confirm the composition of crystals. Figure 2 shows the atomic mole % of Na and K ions in the crystals of three batches ($x = 0.4, 0.5$ and 0.6). For optical birefringence study, crystals with higher transparency were chosen from each batch of $x = 0.4, 0.5$ and 0.6 respectively.

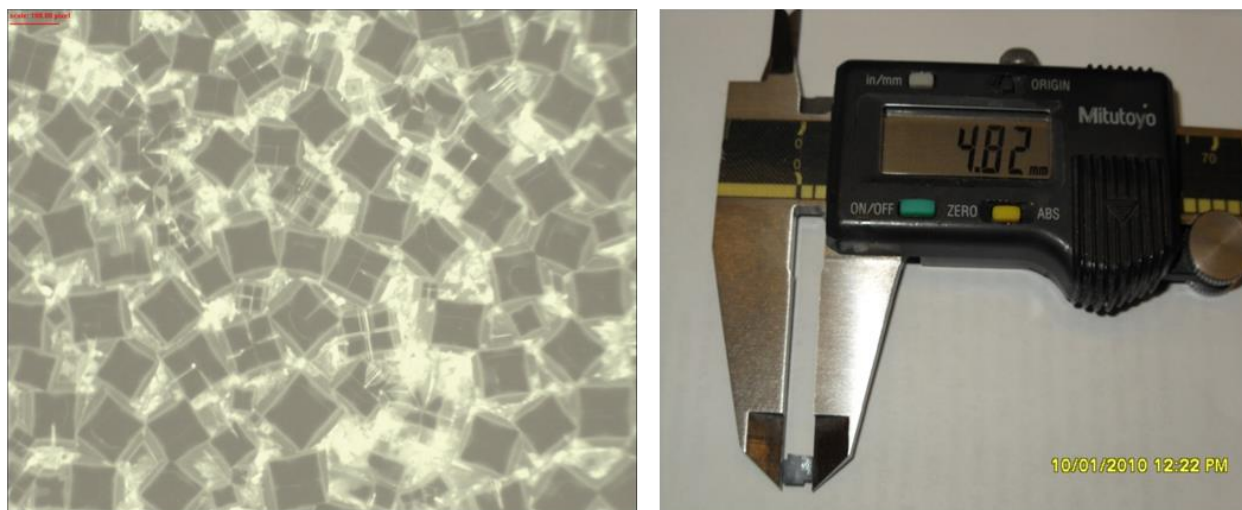


Figure 1 Optical images showing (a) as grown crystals in KF – NaF matrix (b) One of the large size crystals.

EDX Analysis

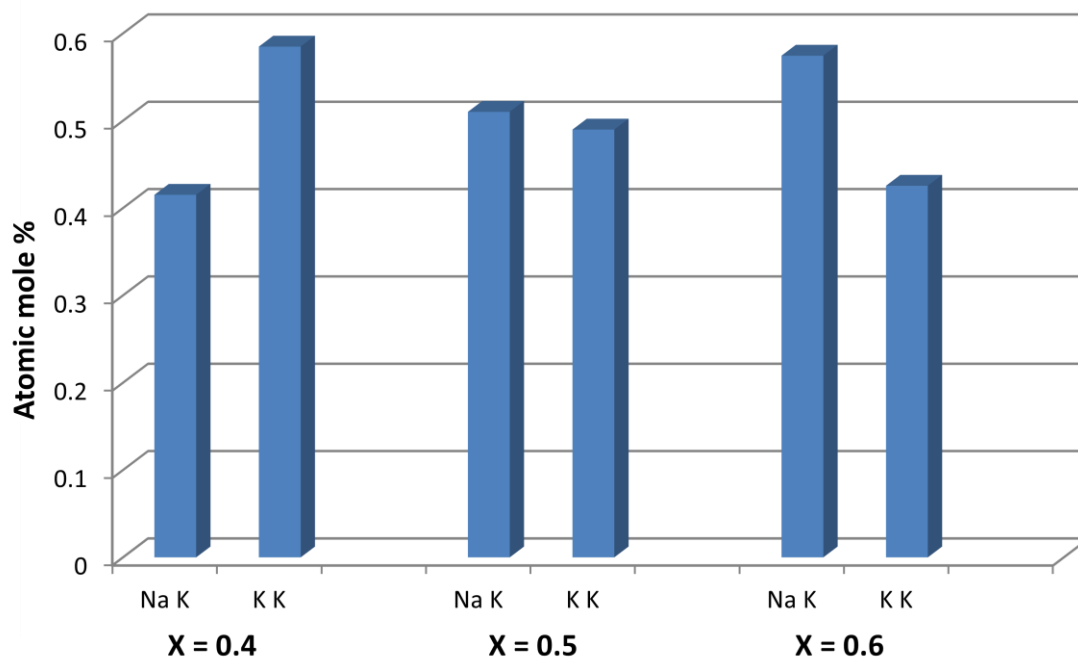


Figure 2 Energy dispersive X-ray (EDX) analysis performed on crystals intended to have composition $x = 0.4, 0.5$ and 0.6 .

Figure 3 shows x-ray diffraction pattern obtained for one of the major faces of a crystal ($x = 0.5$), confirming these crystals to be $\{100\}_{pc}$ oriented. This observation was further confirmed by electron backscattered diffraction analysis (EBSD) performed on two opposite major faces of these crystals.

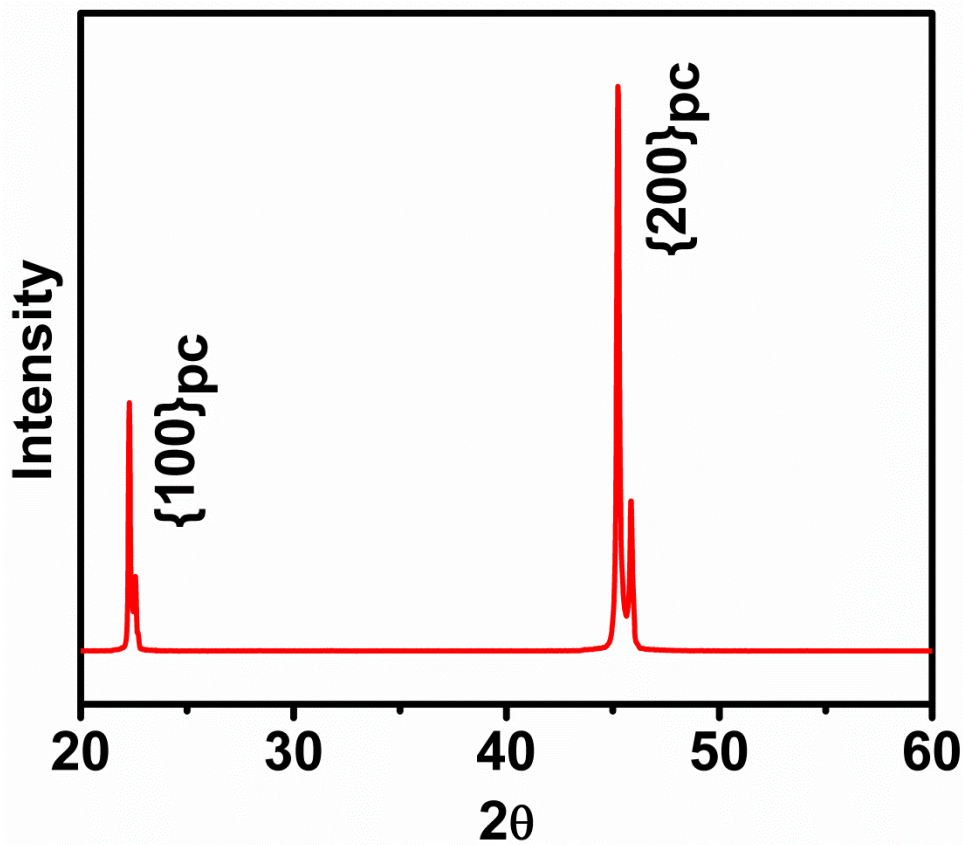


Figure 3 x-ray diffraction pattern showing the presence of only {100} family peaks.

2. Birefringence Measurements –

Figure 4 shows the schematic diagram of the Metripol birefringence measurement system.¹

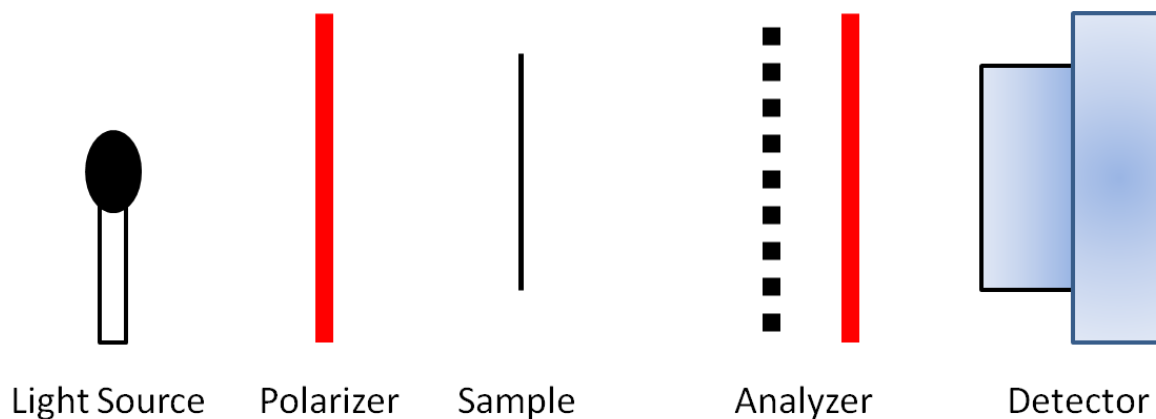


Figure 4 Schematic diagram of Metripol birefringence measurement system.

It consists of a monochromatic light source, a rotating polarizer followed by a sample stage. The transmitted light is allowed to go to the circular analyzer followed by a CCD detector. Intensity measured at each pixel of detector is given by Equation 1.¹

$$I = I_o/2[1 + \text{Sin}(2\alpha - 2\varphi)\text{Sin}\delta]$$

Or

$$I = \frac{1}{2}I_o + \frac{1}{2}I_o \text{Sin}\delta \text{Cos}2\varphi \text{Sin}2\alpha - \frac{1}{2}I_o \text{Sin}\delta \text{Sin}2\varphi \text{Cos}2\alpha \quad (1)$$

Where I_o and φ are the transmittance of the sample and the angle between the minor axis of the indicatrix with an arbitrary coordination axis, respectively. δ is the phase difference introduced by the crystal to the transmitted light ray. To determine the values of all three components I_o , $|\text{Sin}\delta|$, and φ , the intensity of the transmitted light is measured for different values of the rotation angle α which the polarizer makes from a fixed reference.

Least square fitting method is used to determine the three parameters of interest, according to equation (2)¹

$$y(x) = a_o + a_1\text{Sin}x + a_2 \text{Cos}x \quad (2)$$

On comparing it with equation (1), “a” parameters are found to be

$$\begin{aligned} a_o &= \frac{1}{2}I_o \\ a_1 &= \frac{1}{2}I_o \text{Sin}\delta \text{Cos}2\varphi \\ a_2 &= \frac{1}{2}I_o \text{Sin}\delta \text{Sin}2\varphi \end{aligned} \quad (3)$$

If the polarizer is stepped through N steps in the process of collecting data, “a” parameters can be given as in equation (4).

$$a_0 = \sum_{i=1}^N \frac{1}{N} y_i$$

$$a_1 = \sum_{i=1}^N \frac{2}{N} y_i \sin x_i$$

$$a_2 = \sum_{i=1}^N \frac{2}{N} y_i \cos x_i \quad (4)$$

Values for two parameters of interest $|\sin\delta|$ and φ can be determined from values of parameters “a” according to equation (5).¹

$$|\sin\delta| = \frac{\sqrt{(a_1^2 + a_2^2)}}{a_0}$$

$$\varphi = \frac{1}{2} \arcsin\left(-\frac{a_2}{\sqrt{(a_1^2 + a_2^2)}}\right) \quad (5)$$

3. Determination of Orientation Histogram –

To obtain the orientation histograms for three ($x = 0.4, 0.5$ and 0.6) compositions at different temperatures, φ false color images were obtained from an area of about $2.5 \text{ mm} \times 1.9 \text{ mm}$. Each of these images consisted of about 1.39 million (1360×1024) pixels, each representing a value of φ between $0^\circ - 180^\circ$. Each histogram was created using these 1.39 millions φ values with the bin size of 1° .

4. Reference -

1. A. M. Glazer, J. G. Lewis and W. Kaminsky, *Proceedings: Mathematical, Physical and Engineering Sciences*, 1996, **452**, 2751-2765.