

# **Polarization Consistent Basis Sets with Projector Augmented Wave Method: A Renovation Brought by PAW into Gaussian Basis Sets**

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## **Electronic Supplementary Information (ESI)**

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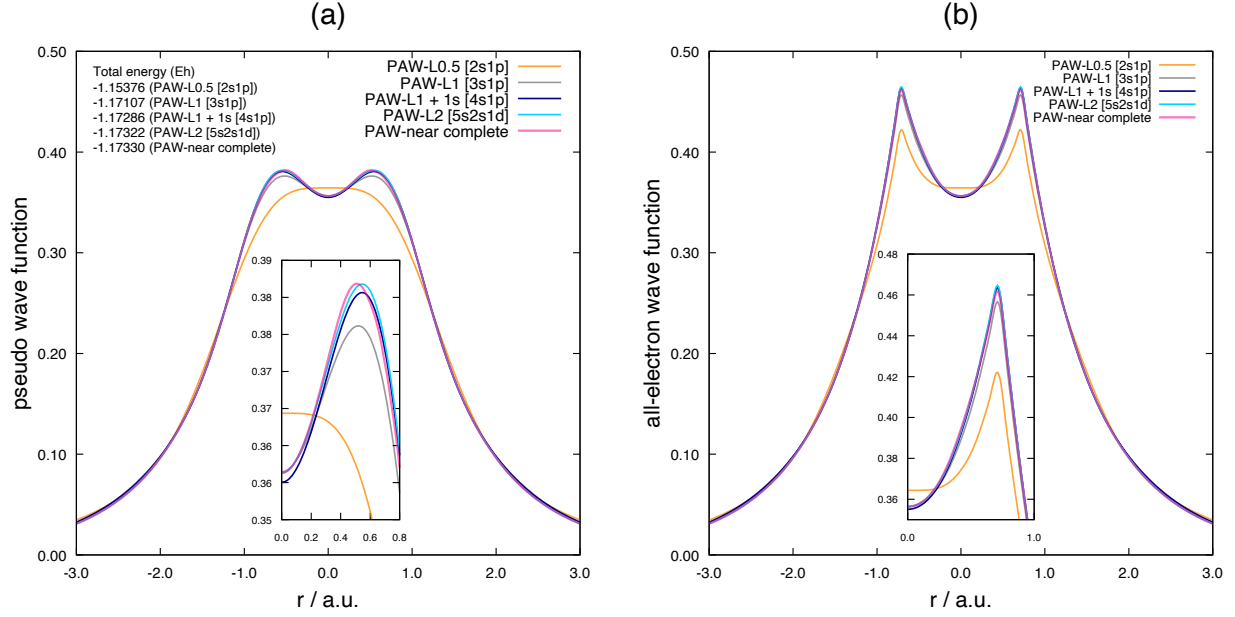


Figure S1: (a) Pseudo wave functions  $\tilde{\Psi}(r)$  obtained with GTF-PAW method using uncontracted PAW- $L_n$  basis for H<sub>2</sub> with  $d(\text{H-H}) = 0.76 \text{ \AA}$ . (b) The corresponding all-electron wave functions  $\Psi(r) = \tilde{\Psi}(r) + \sum_a (\phi^a(r) - \tilde{\phi}^a(r)) \langle p^a | \tilde{\Psi} \rangle$  where  $\phi^a(r)$ ,  $\tilde{\phi}^a(r)$ , and  $p^a(r)$  are partial all-electron waves, partial pseudo waves, and partial projectors, respectively.

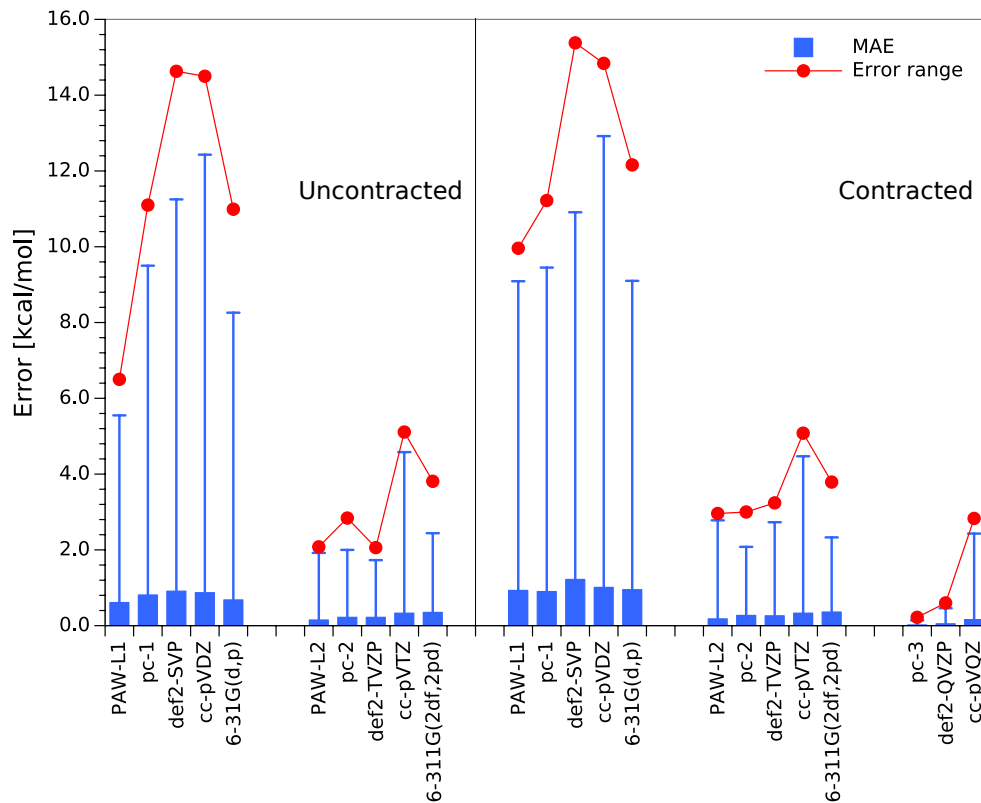


Figure S2: Mean absolute errors (blue bars), maximum absolute errors (vertical blue lines), and error ranges (red circles) of the atomization energies per atom calculated with the PAW- $L_n$  and different all-electron basis sets and the SVWN exchange-correlation functional. The atomization energies calculated with pc-4 are used as references. The atomic energies were calculated as half the energies of the diatomic systems. Energies are in kcal/mol.

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Table S1: Summary of the most important parameters used to construct the PAW atomic dataset.

Element	core electrons	valence electrons	# partial-waves	$r_{\text{paw}}$ (Bohr)
H	-	$1s^1$	3	0.70
Li	-	$1s^2 2s^1$	5	1.60
Be	-	$1s^2 2s^2$	5	1.45
B	$1s^2$	$2s^2 2p^1$	4	1.20
C	$1s^2$	$2s^2 2p^2$	4	1.00
N	$1s^2$	$2s^2 2p^3$	4	1.00
O	$1s^2$	$2s^2 2p^4$	4	1.00
F	$1s^2$	$2s^2 2p^5$	4	1.00
Na	$1s^2$	$2s^2 2p^6 3s^1$	5	1.90
Mg	$1s^2$	$2s^2 2p^6 3s^2$	6	1.90
Al	$1s^2 2s^2 2p^6$	$3s^2 3p^1$	6	1.65
Si	$1s^2 2s^2 2p^6$	$3s^2 3p^2$	6	1.65
P	$1s^2 2s^2 2p^6$	$3s^2 3p^3$	6	1.65
S	$1s^2 2s^2 2p^6$	$3s^2 3p^4$	6	1.65
Cl	$1s^2 2s^2 2p^6$	$3s^2 3p^5$	6	1.65

Table S2: Systems used in the test set.

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atomization energies and vertical ionization energies (G2-97' dataset)<sup>a</sup>

1,3-butadiene, 2-butyne, 2-methyl-1-propene, acetaldehyde, acetamide, acetic acid, acetone, acetonitrile, acetylene, acetyl fluoride, acrylonitrile, allene, ammonia, aziridine, B<sub>2</sub>, diborane(4), diborane(6), Be<sub>2</sub>, BeC<sub>2</sub>, Be(CH<sub>3</sub>)<sub>2</sub>, BeF, BeH<sub>2</sub>, benzene, BeO, BeOH, BeH, BF, BH, BH<sub>3</sub>, bicyclobutane, BN, BO, boranamine, borazine, boric acid, B<sub>2</sub>O<sub>3</sub>, C<sub>2</sub>, C<sub>2</sub>H<sub>3</sub><sup>·</sup>, C<sub>2</sub>H<sub>5</sub><sup>·</sup>, CO<sub>2</sub>, carbonic difluoride, CO, CF<sub>4</sub>, CCH<sup>·</sup>, CH<sup>·</sup>, CH<sub>3</sub>CH<sub>2</sub>O<sup>·</sup>, CH<sub>3</sub>CO<sup>·</sup>, CH<sub>3</sub>O<sup>·</sup>, CN<sup>·</sup>, cyanogen, cyclobutane, cyclobutene, cyclopropane, cyclopropene, OF<sub>2</sub>, difluoro methane, dimethylamine, dimethyl ether, ethane, ethanedial, ethanol, ethylamine, ethylene, ethylene oxide, F<sub>2</sub>, fluoro ethene, formaldehyde, formic acid, furan, H<sub>2</sub>, H<sub>2</sub>COH<sup>·</sup>, HCO<sup>·</sup>, hydrazine, HCN, HF, H<sub>2</sub>O<sub>2</sub>, isobutane, isopropanol, isopropyl radical, ketene, Li<sub>2</sub>, Li<sub>2</sub>C<sub>2</sub>, Li<sub>2</sub>NH, Li<sub>2</sub>O, Li<sub>2</sub>O<sub>2</sub>, LiCH<sub>3</sub>, LiNH<sub>2</sub>, LiOH, LiF, LiH, methane, methoxy ethane, methyl radical, methyl alcohol, methylamine, methylene singlet, methylene triplet, methylenecyclopropane, methyl formate, methyl nitrite, N<sub>2</sub>, NH<sup>·</sup>, NH<sub>2</sub><sup>·</sup>, nitric oxide, nitrogen trifluoride, nitro methane, nitrous oxide, NO<sub>2</sub>, O<sub>2</sub>, OH<sup>·</sup>, ozone, propane, propene, propyne, pyridine, pyrrole, spiropentane, t-butyl radical, tetrafluoro ethylene, trans butane, trifluoro acetonitrile, trifluoro borane, trifluoro methane, trimethylamine, trimethylborane, water, 1-chloropropane, acetyl chloride, Al<sub>2</sub>, Al<sub>2</sub>H<sub>6</sub>, Al<sub>2</sub>O<sub>3</sub>, Al<sub>3</sub>P<sub>3</sub>H<sub>6</sub>, AlCl, AlH, AlH<sub>2</sub>PH<sub>2</sub>, AlH<sub>3</sub>, Al(PH<sub>2</sub>)<sub>3</sub>, AlS, Al(SiH<sub>3</sub>)<sub>3</sub>, AlCl<sub>3</sub>, AlF<sub>3</sub>, BeCl<sub>2</sub>, BeS, Be(SH)<sub>2</sub>, BeSi<sub>2</sub>, methylsulfonylmethane, vinylsilane, CS<sub>2</sub>, CS, CCl<sub>4</sub>, carbonyl sulfide, CF<sub>3</sub>Cl, CH<sub>3</sub>MgCl, CH<sub>3</sub>PH<sub>2</sub>, CH<sub>3</sub>S<sup>·</sup>, ClF, ClO, ClF<sub>3</sub>, chloroethene, chloroform, Cl<sub>2</sub>, sulfuryl chloride, ClN, dimethyl sulfide, dimethyl sulfoxide, disilane, ethanethiol, chloroethane, thioformaldehyde, HCl, H<sub>2</sub>S, hypochlorous acid, Li<sub>2</sub>PH, Li<sub>2</sub>S<sub>2</sub>, Li<sub>2</sub>Si<sub>2</sub>, LiCl, LiPH<sub>2</sub>, LiSH, methanethiol, methylchloride, dichloromethane, methylsilane, Mg<sub>2</sub>, Mg<sub>2</sub>H<sub>4</sub>, MgC<sub>2</sub>, Mg(CH<sub>3</sub>)<sub>2</sub>, MgCl, MgF<sub>2</sub>, MgH, MgH<sub>2</sub>, MgO, MgOH, MgS, MgSH, Na<sub>2</sub>, Na<sub>2</sub>C<sub>2</sub>, Na<sub>2</sub>NH, Na<sub>2</sub>O, Na<sub>2</sub>O<sub>2</sub>, Na<sub>2</sub>PH, Na<sub>2</sub>S<sub>2</sub>, Na<sub>2</sub>Si<sub>2</sub>, NaCH<sub>3</sub>, NaCl, NaNH<sub>2</sub>, NaOH, NaPH<sub>2</sub>, NaSH, nitrosyl chloride, P<sub>2</sub>, P<sub>4</sub>, PCl<sub>3</sub>, PCl<sub>5</sub>, PF<sub>5</sub>, PH, PH<sub>2</sub><sup>·</sup>, PH<sub>3</sub>, PF<sub>3</sub>, PN, PO, phosphoryl chloride, PS, S<sub>2</sub>, S<sub>2</sub>Cl<sub>2</sub>, SCl, SCl<sub>2</sub>, SF<sub>6</sub>, SH<sup>·</sup>, Si<sub>2</sub>, disilyne, disilene, Si<sub>2</sub>H<sub>5</sub><sup>·</sup>, SiCl, SiCl<sub>2</sub>, SiH, silane, SiO, SiF<sub>4</sub>, SiH<sub>3</sub><sup>·</sup>, SiH<sub>2</sub> singlet, SiH<sub>2</sub> triplet, SiN, SiP, SiS, SN, SO<sub>3</sub>, NaF, NaH, SO<sub>2</sub>, SO, tetrachloroethylene, SiCl<sub>4</sub>, thiirane, thiophene, BCl<sub>3</sub>

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noncovalent binding energies (S22 dataset)<sup>b</sup>

2-pyridoxine–2-aminopyridine, adenine–thymine stack, adenine–thymine Watson-Crick, ammonia dimer, benzene–methane, benzene–ammonia, benzene dimer parallel displaced, benzene dimer T-shaped, benzene–HCN, benzene–water, ethene dimer, ethene–ethyne, formamide dimer, formic acid dimer, indole–benzene stack, indole–benzene T-shape, methane dimer, phenol dimer, pyrazine dimer, uracil dimer H-bonded, uracil dimer stack, water dimer

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<sup>a</sup>The structures of the neutral molecules were optimized. <sup>b</sup>Only single-point calculations.

Table S3: Mean and maximum absolute errors (in kcal/mol) of atomization energies per atom and vertical ionization energies; mean and maximum root-mean-square deviation (RMSD) of atomic positions (in Å), calculated with different basis sets and the SVWN exchange-correlation functional. The values calculated with pc-4 are used as references. MAE = mean absolute error. MaxAE = maximum absolute error.

Basis set	Atomization energies per atom		Atomization energies per atom <sup>b</sup>		Ionization energies <sup>a</sup>		RMSD	
	MAE	MaxAE	MAE	MaxAE	MAE	MaxAE	Mean	Max
Uncontracted basis sets								
PAW-L0.5	1.74	15.22	1.49	16.09	5.13	34.58	0.021	0.276
PAW-L1	1.64	8.37	0.61	5.55	1.49	7.76	0.014	0.307
pc-1	1.94	12.36	0.81	9.50	2.00	11.30	0.015	0.366
pc-1 X2C	2.05	12.72	0.84	9.77	1.91	11.39	0.015	0.366
def2-SVP	1.15	11.13	0.91	11.25	2.37	10.92	0.013	0.283
cc-pVDZ	1.69	13.81	0.87	12.43	2.13	10.94	0.014	0.123
6-31G(d,p)	1.24	9.65	0.68	8.26	2.78	12.22	0.013	0.338
PAW-L2	0.28	2.33	0.15	1.92	0.33	2.54	0.005	0.054
pc-2	0.47	3.02	0.22	2.00	0.51	2.92	0.005	0.147
def2-TZVP	0.32	1.85	0.22	1.73	0.47	3.44	0.006	0.241
cc-pVTZ	0.50	4.82	0.33	4.58	0.80	4.24	0.007	0.082
6-311G(2df,2pd)	0.43	2.40	0.35	2.44	1.21	5.58	0.005	0.086
Contracted basis sets								
PAW-L1	2.38	10.75	0.93	9.09	1.87	8.57	0.019	0.374
pc-1	2.33	12.55	0.90	9.45	2.12	11.02	0.018	0.325
def2-SVP	1.26	9.76	1.22	10.91	2.79	11.61	0.017	0.257
cc-pVDZ	2.15	13.60	1.01	12.92	3.53	13.60	0.018	0.145
6-31G(d,p)	1.44	9.72	0.95	9.10	4.26	15.22	0.015	0.215
PAW-L2	0.39	3.09	0.18	2.78	0.40	2.73	0.005	0.057
pc-2	0.47	3.07	0.27	2.08	0.54	3.07	0.006	0.202
def2-TZVP	0.37	2.90	0.26	2.73	0.54	3.35	0.006	0.242
cc-pVTZ	0.53	4.87	0.33	4.47	0.85	4.06	0.007	0.087
6-311G(2df,2pd)	0.43	2.33	0.36	2.33	1.23	5.64	0.006	0.055
pc-3	0.03	0.18	0.02	0.14	0.08	1.11	0.002	0.033
def2-QZVP	0.12	0.62	0.05	0.46	0.19	1.36	0.003	0.041
cc-pVQZ	0.24	2.57	0.16	2.43	0.35	2.02	0.004	0.041
ECP basis sets								
STRLC	9.37	50.97	3.21	34.21	7.02	34.50	0.064	0.485
SBK	6.19	35.78	1.71	28.50	4.03	17.19	0.041	0.484

<sup>a</sup>Calculations of boric acid, isobutane, and trifluoroacetonitrile with PAW-*Ln* basis sets did not converge due to electronic degeneracy. <sup>b</sup>The atomic energies were calculated as half the energies of the diatomic systems.



Table S4: Mean and maximum absolute errors (in kcal/mol) of atomization energies per atom and vertical ionization energies; mean and maximum root-mean-square deviation of atomic positions (in Å), calculated with different basis sets and the BLYP exchange-correlation functional. The values calculated with pc-4 are used as references. MAE = mean absolute error. MaxAE = maximum absolute error.

Basis set	Atomization energies per atom		Atomization energies per atom <sup>b</sup>		Ionization energies <sup>a</sup>		RMSD <sup>c</sup>	
	MAE	MaxAE	MAE	MaxAE	MAE	MaxAE	Mean	Max
Uncontracted basis sets								
PAW-L0.5	1.69	14.64	1.54	16.17	6.06	35.84	0.022	0.226
PAW-L1	1.47	7.36	0.53	4.75	1.61	8.68	0.013	0.182
pc-1	1.62	10.54	0.81	8.63	1.79	12.83	0.013	0.110
def2-SVP	0.94	9.10	0.81	10.07	2.85	12.96	0.012	0.132
cc-pVDZ	1.34	11.74	0.84	11.33	2.62	12.82	0.014	0.125
6-31G(d,p)	1.07	7.97	0.62	7.44	3.24	14.03	0.011	0.089
PAW-L2	0.48	1.88	0.19	1.53	0.71	3.18	0.005	0.052
pc-2	0.40	2.74	0.20	1.87	0.48	2.81	0.005	0.037
def2-TZVP	0.26	1.69	0.22	1.69	0.61	3.52	0.005	0.075
cc-pVTZ	0.42	3.99	0.30	4.00	1.03	4.74	0.007	0.053
6-311G(2df,2pd)	0.48	2.83	0.35	2.49	1.59	7.38	0.006	0.044
Contracted basis sets								
PAW-L1	2.23	8.72	0.88	7.97	2.04	9.39	0.016	0.106
pc-1	2.10	10.73	0.87	8.53	1.92	12.43	0.015	0.108
def2-SVP	1.08	8.25	1.14	9.85	3.31	13.54	0.015	0.244
cc-pVDZ	1.88	11.58	0.91	11.70	3.91	15.29	0.018	0.137
6-31G(d,p)	1.22	8.13	0.87	8.20	4.69	16.73	0.013	0.102
PAW-L2	0.57	2.51	0.22	2.09	0.73	3.40	0.005	0.052
pc-2	0.47	2.82	0.23	1.86	0.51	2.88	0.005	0.037
def2-TZVP	0.34	2.59	0.26	2.62	0.68	3.70	0.005	0.087
cc-pVTZ	0.43	4.18	0.30	3.97	1.06	4.70	0.007	0.054
6-311G(2df,2pd)	0.46	2.67	0.34	2.44	1.63	7.55	0.006	0.055
pc-3	0.04	0.19	0.02	0.13	0.07	0.80	0.003	0.040
def2-QZVP	0.11	0.53	0.05	0.41	0.22	2.20	0.003	0.047
cc-pVQZ	0.22	2.21	0.15	2.13	0.40	2.08	0.004	0.041
ECP basis sets								
STRLC	8.43	47.05	3.25	30.91	7.15	34.48	0.053	0.562
SBK	5.36	33.44	1.36	25.52	3.65	16.39	0.027	0.229

<sup>a</sup>Calculations of boric acid with PAW- $L_n$  basis sets did not converge due to electronic degeneracy. <sup>b</sup>The atomic energies were calculated as half the energies of the diatomic systems. <sup>c</sup>The potential energy curve of Mg<sub>2</sub> is too shallow and the RMSD of Mg<sub>2</sub> was not included.

Table S5: Mean and maximum absolute errors (in kcal/mol) of the binding energies of S22 complexes calculated with different contracted basis sets. The values calculated with pc-4 are used as references. MAE = mean absolute error. MaxAE = maximum absolute error.

Basis set	SVWN		BLYP	
	MAE	MaxAE	MAE	MaxAE
PAW-L0.5	6.87	14.08	9.19	21.23
PAW-L1	1.47	3.49	1.39	2.68
pc-1	2.00	4.34	1.87	4.03
def2-SVP	2.34	6.07	2.83	6.73
cc-pVDZ	2.03	5.82	2.65	6.62
6-31G(d,p)	1.97	4.89	2.77	5.81
aug-pc-1	1.81	4.61	1.54	3.90
def2-SVPD	1.86	4.69	1.69	4.12
aug-cc-pVDZ	0.74	1.87	0.68	1.65
6-31++G(d,p)	0.81	1.95	0.66	1.35
PAW-L2	0.17	0.44	0.21	0.40
pc-2	0.34	0.95	0.29	0.64
def2-TZVP	0.46	1.06	0.53	1.18
cc-pVTZ	0.77	1.60	1.01	1.92
6-311G(2df,2pd)	1.63	3.24	1.49	3.82
aug-pc-2	0.27	0.74	0.09	0.30
def2-TZVPD	0.23	0.74	0.15	1.49
aug-cc-pVTZ	0.17	0.44	0.18	0.50
6-311++G(2df,2pd)	0.50	1.46	0.30	0.83
pc-3	0.02	0.05	0.02	0.05
def2-QZVP	0.12	0.33	0.16	0.40
cc-pVQZ	0.36	0.78	0.50	1.04
aug-pc-3	0.02	0.05	0.01	0.04
def2-QZVPD	0.04	0.13	0.05	0.15
aug-cc-pVQZ	0.09	0.31	0.10	0.34
aug-pc-4 <sup>a</sup>	0.00	0.01	0.00	0.01
STRLC	2.18	6.29	2.69	7.17
SBK	3.10	7.08	3.44	7.78

<sup>a</sup>Due to SCF convergence problem related to aug-pc-4 basis set linear dependencies, only 18 and 15 complexes were calculated with SVWN and BLYP, respectively