# Magnetic Field and Dilution Effects on the Slow Relaxation of $\left\{\mathrm{Er}_{3}\right\}$ Triangles Based Arsenotungstate Single-Molecule 

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Fig. S8. At 2.2 K , the real part (left) and imaginary part (right) of compound $\mathbf{1}$ ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.23469 ; \alpha_{2}=0.21989 ; \tau_{1}=1.48248 \times 10^{-4} \mathrm{~s} ; \tau_{2}=0.16258 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=10.16743 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=4.97757 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=4.31230 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.
Fig. S9. At 2.4 K , the real part (left) and imaginary part (right) of compound 1 ac signal frequency dependence diagram, solid line represents Debye model Fitting curve, fitting parameters: $\alpha_{1}=0.2878 ; \alpha_{2}=0.22097 ; \tau_{1}=1.00191 \times 10^{-4} \mathrm{~s} ; \tau_{2}=0.10621 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=9.67703 \mathrm{~cm}^{3} \mathrm{~mol}^{-}$ ${ }^{1} ; \Delta \chi_{1}=5.03636 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=3.30812 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.
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Fig. S13. At 3.2 K , the real part (left) and imaginary part (right) of compound $\mathbf{1}$ ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.45949 ; \alpha_{2}=0.23169 ; \tau_{1}=9.37751 \times 10^{-6} \mathrm{~s} ; \tau_{2}=0.0295 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=7.35975 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=5.3805 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=1.32714 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.
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Fig. S23. Calculated orientations of the local main magnetic axes on $\mathrm{Er}^{3+}$ ions of $\left\{\mathrm{Er}_{3}\right\}$ fragment in 1 in the ground KDs.
Fig. S24. Calculated model structures of individual $\mathrm{Er}^{3+}$ fragments in 1; H atoms are omitted for clarify.

## Material and physical measurements

All other chemicals were commercially purchased and used without further purification. The potassium salt of the dilacunary $\mathrm{K}_{14}\left[\mathrm{As}_{2} \mathrm{~W}_{19} \mathrm{O}_{67}\left(\mathrm{H}_{2} \mathrm{O}\right)\right]$ precursor was synthesized according to the reported literature ${ }^{51}$ and confirmed by IR spectrum. IR spectra of all compounds were recorded on a Bruker VERTEX 70 IR spectrometer using KBr pellets in the range of 4000-400 $\mathrm{cm}^{-1}$. (Figure S1) Elemental analyses (C, H) were conducted on a Perkin-Elmer 2400-II CHNS/O analyzer. ICP analyses were performed on a PerkinElmer Optima 2000 ICP-OES spectrometer. (Table S10) Thermogravimetric analysis (TGA) analyses were performed on a NETZSCH STA 449 F5 Jupiter thermal analyzer in flowing $\mathrm{N}_{2}$ with a heating rate of $10^{\circ} \mathrm{C} \mathrm{min}^{-1}$ (Figure S2). EDX measurements were recorded on a JSM-7610F scanning electron microscope with an OXFORD x-act EDS system (Figure S3). Variable-temperature magnetic susceptibility data were obtained on a SQUID magnetometer (Quantum Design, MPMS3) in the temperature range of 2-300 K.

## X-ray Crystallography

Excellent single crystals of the three compounds were stilled in a capillary tube when prepared for data collection at $150(2) \mathrm{K}$ as they weather easily. Indexing and data collection were performed on a Bruker D8 VENTURE PHOTON II diffractometer with Mo K $\alpha$ radiation ( $\lambda=$ $0.71073 \AA$ Å). Direct methods successfully located the tungsten atoms, and successive Fourier syntheses revealed the remaining atoms. Refinements were done by full-matrix least-squares on $F^{2}$ using the SHELXL-2018 program suite for all data. ${ }^{52}$ In the final refinement, almost all the
atoms except few water O atoms were refined anisotropically; the disordered K countercations and few O atoms were refined isotropically. The hydrogen atoms of the malate groups were placed in calculated positions and then refined using a riding model. All H atoms on water molecules were directly included in the molecular formula.

## Computational details

For three-dimensional cluster of compound 1, we extracted a three-core unit including three types of individual $\mathrm{Er}^{3+}$ fragments indicated as Er1, Er2 and Er3. Complete-active-space self-consistent field (CASSCF) calculations on three types of individual $\mathrm{Er}^{3+}$ fragments from complex 1 (see Figure S24 for the calculated model structures) on the basis of single-crystal X-ray determined geometry have been carried out with MOLCAS $8.4^{53}$ program package. Each individual $\mathrm{Er}^{3+}$ fragment was calculated keeping the experimentally determined structure of the corresponding compound while replacing the nearest neighboring $\mathrm{Er}^{3+}$ ions by diamagnetic $\mathrm{Lu}^{3+}$, and the influence of the other surrounding $\mathrm{W}^{6+}$ ions were taken into account by the closed-shell $\mathrm{Ba}^{2+}$ ab initio embedding model potentials (AIMP; Ba.ECP.Pascual.0s.0s.0e-AIMP-BaF2.). ${ }^{54}$

The basis sets for all atoms are atomic natural orbitals from the MOLCAS ANO-RCC library: ANO-RCC-VTZP for $\mathrm{Er}^{3+}$; VTZ for close O; VDZ for distant atoms. The calculations employed the second order Douglas-Kroll-Hess Hamiltonian, where scalar relativistic contractions were taken into account in the basis set and the spin-orbit couplings were handled separately in the restricted active space state interaction (RASSI-SO) procedure. For individual $\mathrm{Er}^{3+}$ fragment, active electrons in 7 active spaces include all $f$ electrons (CAS(11 in 7 for $\left.\mathrm{Er}^{3+}\right)$ ) in the CASSCF calculation. To exclude all the doubts, we calculated all the roots in the active space. We have mixed the maximum number of spin-free state which was possible with our hardware (all from 35 quadruplets, all from 112 doublets). SINGLE_ANISO ${ }^{55}$ program was used to obtain energy levels, $\boldsymbol{g}$ tensors, magnetic axes, et $\boldsymbol{a l}$., based on the above CASSCF/RASSI-SO calculations.

To fit the exchange interactions between $\mathrm{Er}^{3+}$ ions in compound 1, we took two steps to obtain them. Firstly, we calculated individual $\mathrm{Er}^{3+}$ fragments using CASSCF/RASSI-SO to obtain the corresponding magnetic properties. Then, the exchange interactions between the magnetic centers were considered within the Lines model, ${ }^{56}$ while the account of the dipole-
dipole magnetic couplings were treated exactly. The Lines model is effective and has been successfully used widely in the research field of $d$ and $f$-elements SMMs. ${ }^{57}$

The Ising exchange Hamiltonian for $\mathbf{1}$ is:

$$
\begin{equation*}
\Psi_{\text {exch }}=-\stackrel{\circ}{J}_{1} \$_{E r 1} \$_{E r 2}-\stackrel{\circ}{J}_{2} \$_{E r 1} \$_{E r 3}-\stackrel{\circ}{J}_{3} \$_{E r 2} \$_{E r 3} \tag{1}
\end{equation*}
$$

The 9 total is the parameter of the total magnetic interaction $\left(\mathcal{J}_{\text {total }}=9 /{ }_{\text {dip }}+\mathcal{O}_{\text {exch }}\right)$ between magnetic center ions. The $S / \varepsilon_{r}=1 / 2$ is the ground pseudospin on $\mathrm{Er}^{3+}$ sites. The dipolar magnetic coupling can be calculated exactly, while the exchange coupling constants were fitted through comparison of the computed and measured magnetic susceptibilities using the POLY_ANISO program. ${ }^{57}$

Table S1. Crystallographic data and structure refinements for 1, 2 and Er@2.

|  | 1 | 2 | Er@2 |
| :---: | :---: | :---: | :---: |
| Empirical formula | $\mathrm{C}_{8} \mathrm{H}_{6} \mathrm{As}_{4} \mathrm{Er}_{6} \mathrm{~K}_{18} \mathrm{O}_{210} \mathrm{~W}_{38}$ | $\mathrm{C}_{8} \mathrm{H}_{6} \mathrm{As}_{4} \mathrm{~K}_{16} \mathrm{O}_{208} \mathrm{~W}_{38} \mathrm{Y}_{6}$ | $\begin{aligned} & \mathrm{C}_{8} \mathrm{H}_{6} \mathrm{As}_{4} \mathrm{Er}_{0.3} \mathrm{~K}_{20} \mathrm{O}_{219} \\ & \mathrm{~W}_{38} \mathrm{Y}_{5.7} \end{aligned}$ |
| Formula weight | 12455.47 | 11875.17 | 12231.07 |
| Temperature / K | 150(2) | 150(2) | 150(2) |
| Crystal system | Monoclinic | monoclinic | monoclinic |
| Space group | P2(1)/n | P2(1)/n | P2(1)/n |
| $a[A ̊]$ | 20.6656(19) | 20.6656(19) | 20.520(3) |
| $b[A ̊]$ | 25.056(2) | 25.056(2) | 24.926(4) |
| $c[A ̊]$ | 22.596(2) | 22.596(2) | 22.516(3) |
| $\beta\left[{ }^{\circ}\right]$ | 112.570(4) | 112.570(4) | 112.644(4) |
| $V\left[\AA^{3}\right]$ | 10804.1(17) | 10804.1(17) | 10629(3) |
| $Z$ | 2 | 2 | 2 |
| $\rho_{\text {calcd }}\left[\mathrm{g} \mathrm{cm}^{-3}\right]$ | 3.829 | 3.650 | 3.822 |
| $\mu\left[\mathrm{mm}^{-1}\right]$ | 23.504 | 22.753 | 23.250 |
| F(000) | 10856.0 | 10400.0 | 10745.0 |
|  | $-24 \leq h \leq 21$ | $-24 \leq h \leq 24$ | $-24 \leq h \leq 23$ |
| Index ranges | $-27 \leq k \leq 29$ | $-29 \leq k \leq 29$ | $-29 \leq k \leq 29$ |
|  | $-26 \leq 1 \leq 26$ | $-23 \leq 1 \leq 26$ | $-26 \leq 1 \leq 26$ |
| Reflections collected | 73900 | 122877 | 107923 |
| Independent | 19179 | 19174 | 18904 |
| reflections | $\left[R_{\text {int }}=0.0385\right]$ | $\left[R_{\text {int }}=0.0745\right]$ | $\left[R_{\text {int }}=0.0559\right]$ |
| data/restraints/param eters | 19179/18/ 1257 | 19174/30/1243 | 18904/42/ 1360 |
| Goodness-of-fit on $F^{2}$ | 1.043 | 1.017 | 1.038 |


| $R_{1}, w R_{2}[I>2 \sigma(I)]$ |  | $0.0348,0.0821$ | $0.0394,0.0988$ | $0.0305,0.0713$ |
| :--- | :--- | :--- | :--- | :--- |
| $R_{1}, w R_{2}$ [all data] |  | $0.0412,0.0849$ | $0.0497,0.1045$ | $0.0372,0.0744$ |
| Largest | diff. | $3.71 /-1.96$ | $3.35 /-2.87$ | $3.26 /-1.95$ |
| Peak/hole/e $\AA^{-3}$ |  |  |  |  |

Table S2 BVS values for Er, As, and W atoms in 1.

| Atoms | BVS value | Atoms | BVS value | Atoms | BVS value |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Er1 | 3.08 | W4 | 6.18 | W12 | 6.23 |
| Er2 | 3.19 | W5 | 6.16 | W13 | 6.33 |
| Er3 | 3.20 | W6 | 6.21 | W14 | 6.27 |
| As1 | 2.97 | W7 | 6.10 | W15 | 6.06 |
| As2 | 3.01 | W8 | 5.96 | W16 | 6.16 |
| W1 | 6.09 | W9 | 6.21 | W17 | 5.95 |
| W2 | 6.13 | W10 | 6.27 | W18 | 6.27 |
| W3 | 6.41 | W11 | 6.13 | W19 | 6.24 |

Table S3 Possible geometries of nine coordinated metal centers.

| geometry | point group | polyhedron |
| :--- | :--- | :--- |
| EP-9 | $D_{9 \mathrm{~h}}$ | Enneagon |
| OPY-9 | $C_{8 v}$ | Octagonal pyramid |
| HBPY-9 | $D_{7 \mathrm{~h}}$ | Heptagonal bipyramid |
| JTC-9 | $C_{3 v}$ | Johnson triangular cupola J3 |
| JCCU-9 | $C_{4 v}$ | Capped cube J8 |
| CCU-9 | $C_{4 v}$ | Spherical-relaxed capped cube |
| JCSAPR-9 | $C_{4 v}$ | Capped square antiprism J10 |
| CSAPR-9 | $C_{4 v}$ | Spherical capped square antiprism |
| JTCTPR-9 | $D_{3 \mathrm{~h}}$ | Tricapped trigonal prism J51 |
| TCTPR-9 | $D_{3 \mathrm{~h}}$ | Spherical tricapped trigonal prism |
| JTDIC-9 | $C_{3 v}$ | Tridiminished icosahedron J63 |
| HH-9 | $C_{2 v}$ | Hula-hoop |
| MFF-9 | $C s$ | Muffin |

Table S4 Deviation parameters calculated by SHAPE from each ideal polyhedron for Er1.

|  | Er1 |
| :--- | :--- |
| EP-9 | 37.187 |
| OPY-9 | 21.858 |
| HBPY-9 | 19.340 |
| JTC-9 | 15.716 |
| JCCU-9 | 10.750 |
| CCU-9 | 9.840 |
| JCSAPR-9 | 1.437 |
| CSAPR-9 | 0.640 |
| JTCTPR-9 | 2.170 |
| TCTPR-9 | 1.090 |


| JTDIC-9 | 12.444 |
| :--- | :--- |
| HH-9 | 10.886 |
| MFF-9 | 0.730 |

Table S5 Possible geometries of eight coordinated metal centers.

| geometry | point group | polyhedron |
| :--- | :--- | :--- |
| OP-8 | $D_{8 \mathrm{~h}}$ | Octagon |
| HPY-8 | $C_{7 v}$ | Heptagonal pyramid |
| HBPY-8 | $D_{6 \mathrm{~h}}$ | Hexagonal bipyramid |
| CU-8 | $O_{\mathrm{h}}$ | Cube |
| SAPR-8 | $D_{4 \mathrm{~d}}$ | Square antiprism |
| TDD-8 | $D_{2 \mathrm{~d}}$ | Triangular dodecahedron |
| JGBF-8 | $D_{2 \mathrm{~d}}$ | Johnson gyrobifastigium (J26) |
| JETBPY-8 | $D_{3 \mathrm{~h}}$ | Johnson elongated triangular bipyramid |
|  |  | (J14) |
| JBTPR-8 | $C_{2 \mathrm{v}}$ | Biaugmented trigonal prism (J50) |
| BTPR-8 | $C_{2 \mathrm{v}}$ | Biaugmented trigonal prism |
| JSD-8 | $D_{2 \mathrm{~d}}$ | Snub diphenoid(J84) |
| TT-8 | $T_{\mathrm{d}}$ | Triakis tetrahedron |
| ETBPY-8 | $D_{3 \mathrm{~h}}$ | Elongated trigonal bipyramid |

Table S6 Deviation parameters calculated by SHAPE from each ideal polyhedron for Er2 and Er3.

|  | Er2 | Er3 |
| :--- | :--- | :--- |
| OP-8 | 27.278 | 28.549 |
| HPY-8 | 23.199 | 22.767 |
| HBPY-8 | 15.383 | 16.065 |
| CU-8 | 9.424 | 9.903 |
| SAPR-8 | 0.648 | 0.742 |
| TDD-8 | 2.773 | 1.395 |
| JGBF-8 | 13.806 | 14.903 |
| JETBPY-8 | 25.459 | 27.647 |
| JBTPR-8 | 3.195 | 1.438 |


| BTPR-8 | 2.504 | 1.148 |
| :--- | :--- | :--- |
| JSD-8 | 5.545 | 3.674 |
| TT-8 | 10.219 | 10.732 |
| ETBPY-8 | 21.226 | 23.559 |

Table S7 Selected bond lengths (Å) of 1.

| Bond | Length | Bond | Length | Bond | Length |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Er1-O28 | $2.568(8)$ | Er2-O27 | $2.279(8)$ | Er3-O30 | $2.265(8)$ |
| Er1-O32 | $2.353(8)$ | Er2-O32 | $2.301(8)$ | Er3-O33 | $2.303(7)$ |
| Er1-O33 | $2.340(8)$ | Er2-062 | $2.319(8)$ | Er3-068 | $2.308(8)$ |
| Er1-O63 | $2.605(8)$ | Er2-O63 | $2.484(8)$ | Er3-072 | $2.342(9)$ |
| Er1-O64 | $2.308(8)$ | Er2-O1W | $2.383(7)$ | Er3-O1W | $2.456(7)$ |
| Er1-O1W | $2.328(7)$ | Er2-O4W | $2.352(8)$ | Er3-O7W | $2.393(8)$ |
| Er1-O2W | $2.377(8)$ | Er2-O5W | $2.409(9)$ | Er3-O8W | $2.393(8)$ |
| Er1-O3W | $2.400(8)$ | Er2-O6W | $2.330(8)$ | Er3-O9W | $2.381(8)$ |
| Er1-O8W | $2.506(7)$ |  |  |  |  |

Table S8 Selected bond angles $\left({ }^{\circ}\right)$ of 1.

| Bond | Angel | Bond | Angel |
| :--- | :--- | :--- | :--- |
| O28-Er1-O63 | $124.6(2)$ | O48-Er3-O56 | $75.330(15)$ |
| O32-Er1-O28 | $63.2(3)$ | O30-Er3-O56 | $145.274(22)$ |
| O32-Er1-O63 | $69.6(3)$ | O74-Er3-O56 | $121.051(17)$ |
| O32-Er1-O2W | $134.1(3)$ | O8W-Er3-O56 | $79.343(15)$ |
| O32-Er1-O3W | $82.9(3)$ | O7W-Er3-O56 | $74.347(14)$ |
| O32-Er1-O8W | $137.2(3)$ | O6W-Er3-O56 | $74.599(14)$ |
| O33-Er1-O28 | $62.6(3)$ | O30-Er3-O48 | $139.355(18)$ |
| O33-Er1-O32 | $82.9(3)$ | O74-Er3-O48 | $72.278(13)$ |
| O33-Er1-O63 | $137.7(3)$ | O8W-Er3-O48 | $85.101(13)$ |
| O33-Er1-O2W | $84.7(3)$ | O7W-Er3-O48 | $117.036(15)$ |
| O33-Er1-O3W | $136.1(3)$ |  |  |
| O33-Er1-O8W | $67.4(3)$ |  |  |
| O64-Er1-O28 | $139.1(3)$ |  |  |
| O64-Er1-O32 | $139.5(3)$ |  |  |
| O64-Er1-O33 | $134.9(3)$ |  |  |
| O64-Er1-O63 | $71.1(3)$ |  |  |
| O64-Er1-O1W | $102.8(3)$ |  |  |
| O64-Er1-O2W | $73.9(3)$ |  |  |
| O64-Er1-O3W | $76.7(3)$ |  |  |
| O64-Er1-O8W | $68.8(3)$ |  |  |
| O1W-Er1-O28 | $117.9(3)$ |  |  |
| O1W-Er1-O32 | $72.1(3)$ |  |  |
| O1W-Er1-O33 | $71.5(3)$ |  |  |


| O1W-Er1-063 | 69.7(2) |  |  |
| :---: | :---: | :---: | :---: |
| O1W-Er1-O2W | 142.7(3) |  |  |
| O1W-Er1-O3W | 140.5(3) |  |  |
| O1W-Er1-O8W | 69.7(2) |  |  |
| O2W-Er1-028 | 71.9(3) |  |  |
| O2W-Er1-063 | 137.4(3) | O6W-Er2-O4W | 141.2(3) |
| O2W-Er1-O3W | 76.0(3) | O6W-Er2-05W | 72.7(4) |
| O2W-Er1-O8W | 74.9(3) | O30-Er3-O33 | 77.7(3) |
| O3W-Er1-028 | 73.9(3) | O30-Er3-068 | 141.9(3) |
| O3W-Er1-063 | 73.1(3) | O30-Er3-O72 | 78.9(3) |
| O3W-Er1-O8W | 139.7(3) | O30-Er3-01W | 119.8(3) |
| O8W-Er1-028 | 121.2(2) | O30-Er3-07W | 83.6(3) |
| O3W-Er1-063 | 112.8(2) | O30-Er3-08W | 140.4(3) |
| O27-Er2-O32 | 75.7(3) | O30-Er3-09W | 71.3(3) |
| O27-Er2-062 | 143.7(3) | O33-Er3-068 | 135.0(3) |
| O27-Er2-O63 | 140.4(3) | O33-Er3-072 | 81.0(3) |
| O27-Er2-O1W | 120.2(3) | O33-Er3-O1W | 69.9(3) |
| O27-Er2-O4W | 74.2(3) | O33-Er3-07W | 149.9(3) |
| O27-Er2-O5W | 73.6(3) | O33-Er3-O8W | 69.9(3) |
| O27-Er2-O6W | 79.4(3) | O33-Er3-09W | 115.7(3) |
| O32-Er2-062 | 140.5(3) | O68-Er3-072 | 119.2(3) |
| O32-Er2-O63 | 72.6(3) | O68-Er3-01W | 70.1(2) |
| O32-Er2-O1W | 72.1(3) | O68-Er3-07W | 72.4(3) |
| O32-Er2-O4W | 115.1(3) | O68-Er3-08W | 77.4(3) |
| O32-Er2-O5W | 144.5(3) | O68-Er3-09W | 75.5(3) |
| O32-Er2-O6W | 84.5(3) | 072-Er3-O1W | 139.3(3) |
| O62-Er2-063 | 71.9(3) | 072-Er3-07W | 72.4(3) |
| O62-Er2-O1W | 80.3(3) | 072-Er3-O8W | 74.2(3) |
| O62-Er2-O4W | 83.7(3) | 072-Er3-09W | 140.7(3) |
| O62-Er2-O5W | 72.5(3) | O7W-Er3-O1W | 140.3(3) |
| O62-Er2-06W | 102.4(3) | O7W-Er3-08W | 114.3(3) |
| O1W-Er2-063 | 71.1(3) | O8W-Er3-01W | 69.5(3) |
| O1W-Er2-05W | 140.3(3) | 09W-Er3-01W | 79.1(3) |
| O4W-Er2-063 | 141.6(3) | O8W-Er3-07W | 79.3(3) |
| O4W-Er2-O1W | 76.0(3) | O8W-Er3-O8W | 143.9(3) |
| O4W-Er2-O5W | 72.8(3) | Er1-O42-Er2 | 102.460(15) |
| O5W-Er2-063 | 124.1(3) | Er1-O48-Er3 | 100.564(14) |
| O6W-Er2-063 | 74.6(3) | Er1-O74-Er3 | 98.012(13) |
| O6W-Er2-O1W | 142.8(3) |  |  |

Table S9 Relaxation fitting parameters from least-squares fitting of $\chi(\omega)$ data of $\mathbf{1}$.

| $T / K$ | $\alpha_{1}$ | $\alpha_{2}$ | $\Delta \chi_{1} / \mathrm{cm}^{3}$ <br> $\mathrm{~mol}^{-1}$ | $\Delta \chi_{2} / \mathrm{cm}^{3}$ <br> $\mathrm{~mol}^{-1}$ | $\tau_{1} / \mathrm{s}$ | $\tau_{2} / \mathrm{s}$ | $\chi_{\mathrm{s}, \mathrm{to}} / \mathrm{cm}^{3}$ <br> $\mathrm{~mol}^{-1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 2.0 | 0.16421 | 0.39146 | 4.66775 | 8.76193 | $2.22884 \times 10^{-4}$ | 0.24782 | 11.01469 |
| 2.1 | 0.18020 | 0.38504 | 4.62245 | 7.62994 | $1.81589 \times 10^{-4}$ | 0.19757 | 10.61995 |
| 2.2 | 0.23469 | 0.21989 | 4.97757 | 4.31230 | $1.48248 \times 10^{-4}$ | 0.16258 | 10.16743 |
| 2.4 | 0.2878 | 0.22097 | 5.03636 | 3.30812 | $1.00191 \times 10^{-4}$ | 0.10621 | 9.67703 |
| 2.6 | 0.28777 | 0.27865 | 4.44723 | 3.11013 | $7.99563 \times 10^{-5}$ | 0.08430 | 9.66670 |
| 2.8 | 0.37304 | 0.28301 | 4.05652 | 2.01895 | $3.58979 \times 10^{-5}$ | 0.06086 | 9.14171 |
| 3.0 | 0.42971 | 0.18214 | 6.29960 | 1.61919 | $2.33217 \times 10^{-5}$ | 0.04121 | 7.511220 |
| 3.2 | 0.45949 | 0.23169 | 5.38050 | 1.32714 | $9.37751 \times 10^{-6}$ | 0.0295 | 7.35975 |

Table S10 Calculated and found analyses of As, W, Er and Y in 1, 2 and Er@2.

|  | sample | As (\%) | $\mathbf{W}$ (\%) | $\operatorname{Er}(\%)$ | $\mathbf{Y}(\%)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| calcd | $\mathbf{1}$ | 2.47 | 57.47 | 8.26 | 0 |
|  | $\mathbf{2}$ | 2.56 | 59.78 | 0 | 4.56 |
|  | Er@2 | 2.64 | 61.58 | 0.15 | 1.49 |
|  | $\mathbf{1}$ | 2.23 | 56.98 | 8.01 | 0 |
|  | $\mathbf{2}$ | 2.50 | 58.87 | 0 | 4.36 |
|  | Er@2 | 2.52 | 60.59 | 0.13 | 1.37 |

Table S11 Relaxation fitting parameters from least-squares fitting of $\chi(\omega)$ data of $\operatorname{Er@2}$.

| $T / K$ | $\alpha$ | $\chi_{1} / \mathrm{cm}^{3} \mathrm{~mol}^{-1}$ | $\chi_{2} / \mathrm{cm}^{3} \mathrm{~mol}^{-1}$ |
| :--- | :--- | :--- | :--- |
| 2.0 | 0.56977 | 1.88534 | 0.02301 |
| 2.1 | 0.50005 | 1.39909 | 0.06637 |
| 2.4 | 0.52691 | 1.20601 | 0.08321 |
| 2.8 | 0.52698 | 1.00886 | 0.11625 |
| 3.2 | 0.57032 | 0.91715 | 0.11994 |
| 3.6 | 0.59532 | 0.80576 | 0.12169 |

Table S12 Calculated energy levels $\left(\mathrm{cm}^{-1}\right), \boldsymbol{g}\left(g_{x}, g_{y}, g_{z}\right)$ tensors and predominant $m_{J}$ values of the lowest eight Kramers doublets (KDs) of individual $\mathrm{Er}^{3+}$ fragments for compound $\mathbf{1}$ using CASSCF/RASSI-SO with MOLCAS 8.4.

| KDs | Er1 |  |  | Er2 |  |  | Er3 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{E} / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ | $\mathrm{E} / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ | $\mathrm{E} / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ |
| 1 | 0.0 | $\begin{gathered} 0.395 \\ 1.191 \\ 16.115 \end{gathered}$ | $\pm 15 / 2$ | 0.0 | $\begin{gathered} 0.043 \\ 0.062 \\ 17.421 \end{gathered}$ | $\pm 15 / 2$ | 0.0 | $\begin{gathered} 0.086 \\ 0.148 \\ 17.068 \end{gathered}$ | $\pm 15 / 2$ |
| 2 | 80.3 | $\begin{gathered} 0.897 \\ 2.471 \\ 10.879 \end{gathered}$ | $\pm 5 / 2$ | 116.6 | $\begin{gathered} 0.874 \\ 1.813 \\ 13.345 \end{gathered}$ | $\pm 13 / 2$ | 138.8 | $\begin{gathered} 1.561 \\ 2.726 \\ 12.392 \end{gathered}$ | $\pm 13 / 2$ |
| 3 | 117.1 | $\begin{array}{r} 0.922 \\ 1.901 \\ 10.162 \end{array}$ | $\pm 1 / 2$ | 161.3 | $\begin{gathered} 0.015 \\ 1.963 \\ 12.012 \end{gathered}$ | $\pm 11 / 2$ | 207.2 | $\begin{aligned} & 1.093 \\ & 2.856 \\ & 9.755 \end{aligned}$ | $\pm 9 / 2$ |
| 4 | 162.5 | $\begin{aligned} & 1.469 \\ & 4.315 \\ & 7.609 \end{aligned}$ | $\pm 11 / 2$ | 220.2 | $\begin{gathered} 0.416 \\ 2.881 \\ 11.691 \end{gathered}$ | $\pm 5 / 2$ | 248.6 | $\begin{aligned} & 7.431 \\ & 6.609 \\ & 2.442 \end{aligned}$ | $\pm 5 / 2$ |
| 5 | 232.3 | $\begin{aligned} & 3.956 \\ & 4.597 \\ & 7.758 \end{aligned}$ | $\pm 9 / 2$ | 270.8 | $\begin{aligned} & 2.705 \\ & 5.406 \\ & 8.505 \end{aligned}$ | $\pm 9 / 2$ | 301.5 | $\begin{aligned} & 0.252 \\ & 3.524 \\ & 9.834 \end{aligned}$ | $\pm 11 / 2$ |
| 6 | 265.2 | $\begin{array}{r} \hline 0.450 \\ 1.191 \\ 14.765 \end{array}$ | $\pm 13 / 2$ | 327.5 | $\begin{gathered} 1.181 \\ 1.984 \\ 10.632 \end{gathered}$ | $\pm 7 / 2$ | 346.5 | $\begin{aligned} & 1.805 \\ & 3.368 \\ & 9.362 \end{aligned}$ | $\pm 7 / 2$ |
| 7 | 309.6 | $\begin{gathered} 0.483 \\ 0.890 \\ 13.740 \end{gathered}$ | $\pm 7 / 2$ | 422.2 | $\begin{gathered} 0.174 \\ 0.247 \\ 13.294 \end{gathered}$ | $\pm 3 / 2$ | 472.2 | $\begin{gathered} 0.491 \\ 0.618 \\ 13.148 \end{gathered}$ | $\pm 3 / 2$ |
| 8 | 463.1 | $\begin{gathered} 0.044 \\ 0.084 \\ 17.138 \end{gathered}$ | $\pm 3 / 2$ | 529.0 | $\begin{gathered} 0.027 \\ 0.051 \\ 16.770 \end{gathered}$ | $\pm 1 / 2$ | 605.9 | $\begin{gathered} 0.000 \\ 0.056 \\ 16.523 \end{gathered}$ | $\pm 1 / 2$ |

Table S13. Wave functions with definite projection of the total moment $\mid m_{J}>$ for the lowest one or two KDs of individual $\mathrm{Er}^{3+}$ fragments from compound 1.

| Er1 | $E / \mathrm{cm}^{-1}$ | wave functions |
| :---: | :---: | :---: |
|  | 0.0 | $70.8 \%\| \pm 15 / 2>+21.7 \%\| \pm 11 / 2>$ |
| Er2 | 80.3 | $18.8 \%\| \pm 7 / 2>+18.6 \%\| \pm 9 / 2>+16.8 \%\| \pm 5 / 2>+15.2 \%\| \pm 3 / 2>+15.0 \%\| \pm 1 / 2>+11.1 \%\| \pm 13 / 2>$ |
|  | 0.0 | $91.4 \%\| \pm 15 / 2>+5.1 \%\| \pm 11 / 2>$ |
|  | 116.6 | $62.1 \%\| \pm 13 / 2>+16.0 \%\| \pm 7 / 2>+8.6 \%\| \pm 5 / 2>+8.3 \%\| \pm 9 / 2>$ |


| Er3 | 0.0 | $86.7 \%\| \pm 15 / 2>+10.8 \%\| \pm 11 / 2>$ |
| :---: | :---: | :---: |
|  | 138.8 | $57.7 \%\| \pm 13 / 2>+10.7 \%\| \pm 9 / 2>+9.3 \%\| \pm 7 / 2>+7.3 \%\| \pm 5 / 2>+5.4 \%\| \pm 1 / 2>+4.9 \%\| \pm 3 / 2>$ |

Table S14. Exchange energies $E\left(\mathrm{~cm}^{-1}\right)$, the energy difference between each exchange doublets $\Delta_{t}\left(\mathrm{~cm}^{-1}\right)$ and the main values of the $g_{z}$ for the lowest four exchange doublets of $\left\{\mathrm{Er}_{3}\right\}$ fragment in 1.

|  | 1 |  |  |
| :---: | :---: | :---: | :---: |
|  | $E$ | $\Delta_{t}$ | $g_{z}$ |
| 1 | 0.0 | $0.3 \times 10^{-11}$ | 29.318 |
| 2 | 0.3 | $0.1 \times 10^{-11}$ | 21.703 |
| 3 | 2.5 | $0.1 \times 10^{-11}$ | 22.833 |
| 4 | 4.9 | $0.1 \times 10^{-11}$ | 34.080 |



Fig. S1. IR spectra of 1, 2 and Er@2, respectively.

IR spectra of all compounds have been conducted on a Nicolet 170 SXFT-IR spectrometer in the range of $400-4000 \mathrm{~cm}^{-1}$ with KBr pellets, which are all very similar with only slight shifts in the position of the bands (Fig. S1). In their high wave-number region, a significant widen obvious broad band in the range of $3400-1623 \mathrm{~cm}^{-1}$ which are attributed to the stretching vibration $v(\mathrm{O}-\mathrm{H})$ and bending vibration $\delta(\mathrm{O}-\mathrm{H})$ of free modes of lattice and coordinated water
molecules, respectively. Due to the existence of the trivacant Keggin $\left[\mathrm{B}-\alpha-\mathrm{AsW}_{9} \mathrm{O}_{33}\right]^{9-}$ fragments in the skeletons of the three compounds, 1 shows bands in the region 1000-400 $\mathrm{cm}^{-1}$, which correspond to the fingerprint region. In the low wave-number region, four characteristic peaks are seen, which are respectively explained by the asymmetric vibrations of the terminal $v\left(\mathrm{~W}-\mathrm{O}_{t}\right)$, corner-sharing $v\left(\mathrm{~W}-\mathrm{O}_{\mathrm{b}}\right)$ and $v\left(\mathrm{As}-\mathrm{O}_{\mathrm{a}}\right)$, and edge-sharing $v\left(\mathrm{~W}-\mathrm{O}_{\mathrm{c}}\right)$ bonds. The strong peak at $948 \mathrm{~cm}^{-1}$ is due to the $v\left(W-\mathrm{O}_{\mathrm{t}}\right)$ bond and the intense peak at 864 $\mathrm{cm}^{-1}$ is due to an $v\left(\mathrm{~W}-\mathrm{O}_{\mathrm{b}}\right)$ bond. Two medium intensity peaks at 789 and $718 \mathrm{~cm}^{-1}$ are due to $v\left(\mathrm{As}-\mathrm{O}_{\mathrm{a}}\right)$ and $v\left(\mathrm{~W}-\mathrm{O}_{\mathrm{c}}\right)$ bonds. In addition, in comparison with uncoordinated carboxylate acid (about $1700 \mathrm{~cm}^{-1}$ ), the band at $1630 \mathrm{~cm}^{-1}$ is assigned to $\mathrm{COO}^{-}$stretching vibrations from the carboxylate of the monodentate-coordinated mal carboxylate ligands.


Fig. S2. Thermogravimetric curves of 1, 2 and Er@2.


Fig. S3. EDX spectra of 1 and Er@2 showing the presence of C, O, As, K, W, Er and Y element in the lattice. The measured ratios of $E r^{3+} / Y^{3+}$ ions are almost consistent with nominal ratios.
(a)

(b)


Fig. S4. Frequency dependence of the in-phase ( $\chi_{\mathrm{M}}{ }^{\prime}$ ) and out-of-phase ( $\chi_{\mathrm{M}}{ }^{\prime \prime}$ ) products under a zero dc field for 1.


Fig. S5. At 2.0 K , sweep field diagram of 1 from 0 to 5000 Oe , and the optimal dc field was determined to be 1500 Oe at $2.0 \mathrm{~K}, 707.2 \mathrm{~Hz}$.


Fig. S6. At 2.0 K , the real part (left) and imaginary part (right) of compound $\mathbf{1}$ ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.16421 ; \alpha_{2}=0.39146 ; \tau_{1}=2.22884 \times 10^{-4} \mathrm{~s} ; \tau_{2}=0.24782 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=11.01469 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=4.66775 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=8.76193 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.


Fig. S7. At 2.1 K , the real part (left) and imaginary part (right) of compound $\mathbf{1}$ ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.18020 ; \alpha_{2}=0.38504 ; \tau_{1}=1.81589 \times 10^{-4} \mathrm{~s} ; \tau_{2}=0.19757 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=10.61995 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=4.62245 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=7.62994 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.


Fig. S8. At 2.2 K , the real part (left) and imaginary part (right) of compound 1 ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.23469 ; \alpha_{2}=0.21989 ; \tau_{1}=1.48248 \times 10^{-4} \mathrm{~s} ; \tau_{2}=0.16258 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=10.16743 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=4.97757 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=4.31230 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.


Fig. S9. At 2.4 K , the real part (left) and imaginary part (right) of complex 1 ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.2878 ; \alpha_{2}=0.22097 ; \tau_{1}=1.00191 \times 10^{-4} \mathrm{~s} ; \tau_{2}=0.10621 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=9.67703 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{1}=$ $5.03636 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=3.30812 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.


Fig. S10. At 2.6 K , the real part (left) and imaginary part (right) of compound $\mathbf{1}$ ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.28777 ; \alpha_{2}=0.27865 ; \tau_{1}=7.99563 \times 10^{-5} \mathrm{~s} ; \tau_{2}=0.08430 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=9.66670 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=4.44723 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=3.11013 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.


Fig. S11. At 2.8 K , the real part (left) and imaginary part (right) of compound 1 ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.37304 ; \alpha_{2}=0.28301 ; \tau_{1}=3.58979 \times 10^{-5} \mathrm{~s} ; \tau_{2}=0.06086 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=9.14171 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=4.05652 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=2.01895 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.


Fig. S12. At 3.0 K , the real part (left) and imaginary part (right) of compound 1 ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.42971 ; \alpha_{2}=0.18214 ; \tau_{1}=2.33217 \times 10^{-5} \mathrm{~s} ; \tau_{2}=0.04121 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=7.511220 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=6.29960 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=1.61919 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$.


Fig. S13. At 3.2 K , the real part (left) and imaginary part (right) of compound $\mathbf{1}$ ac signal frequency dependence diagram, solid line represents Debye model fitting curve, fitting parameters: $\alpha_{1}=0.45949 ; \alpha_{2}=0.23169 ; \tau_{1}=9.37751 \times 10^{-6} \mathrm{~s} ; \tau_{2}=0.0295 \mathrm{~s} ; \chi_{\mathrm{s}, \text { tot }}=7.35975 \mathrm{~cm}^{3}$ $\mathrm{mol}^{-1} ; \Delta \chi_{1}=5.3805 \mathrm{~cm}^{3} \mathrm{~mol}^{-1} ; \Delta \chi_{2}=1.32714 \mathrm{~cm}^{3} \mathrm{~mol}^{-1}$. The magnetic susceptibility data were described by the sum of two modified Debye functions:

$$
\begin{aligned}
& \frac{\left(\omega \tau_{1}\right)^{1-\alpha_{1}} \cos \left(\pi \alpha_{1} / 2\right)}{\chi^{\prime \prime}(\omega)=\Delta \chi_{1}{ }^{1+2\left(\omega \tau_{1}\right)^{1-\alpha_{1}} \sin \left(\pi \alpha_{1} / 2\right)+\left(\omega \tau_{1}\right)^{\left(2-2 \alpha_{1}\right)}}+\Delta \chi_{2}} \\
& \frac{\left(\omega \tau_{2}\right)^{1-\alpha_{2}} \cos \left(\pi \alpha_{2} / 2\right)}{1+2\left(\omega \tau_{2}\right)^{1-\alpha_{2}} \sin \left(\pi \alpha_{2} / 2\right)+\left(\omega \tau_{2}\right)^{\left(2-2 \alpha_{2}\right)}} \\
& \Delta \chi_{1}, \tau_{1}, \alpha_{1}, \Delta \chi_{2}, \tau_{2}, \alpha_{2} \\
& \chi^{\prime}(\omega)= \\
& 1+2\left(\omega \tau_{2}\right)^{1-\alpha_{2}} \sin \left(\pi \alpha_{2} / 2\right)+\left(\omega \tau_{2}\right)^{\left(2-2 \alpha_{2}\right)}
\end{aligned}
$$

$\chi_{s, t o t,} \Delta \chi_{1}, \tau_{1}, \alpha_{1}, \Delta \chi_{2}, \tau_{2}, \alpha_{2}$
(a)

(b)


Fig. S14. Plot of $\ln (\tau / s)$ versus $T^{-1}$ for compound 1 under 1500 dc field. Left: FR process, right: SR process.


Fig. S15. Frequency dependence of the in-phase ( $\chi^{\prime}{ }_{M}$ ) and out-of-phase ( $\chi^{\prime \prime}{ }_{M}$ ) products under a zero dc field for Er@2.


Fig. S16. Cole-Cole plot under a zero dc field for Er@2 at the indicated temperatures. The lines are guides to the eyes.


Fig. S17. Plot of $\ln (\tau / \mathrm{s})$ versus $T^{-1}$ of Er@2 under a zero dc field with fitting results.


Fig. S18. Frequency dependence of the in-phase ( $\chi_{M}{ }^{\prime}$ ) and out-of-phase ( $\chi_{M}{ }^{\prime \prime}$ ) products under an optimized 1500 Oe dc field for $\mathbf{E r @ 2}$. The lines are guides to the eyes.


Fig. S19. plot of $\ln (\tau / s)$ versus $T^{-1}$ for compound $\operatorname{Er@2}$ under 1500 dc field.

(c)


Fig. S20. Magnetization blocking barriers of individual $\mathrm{Er}^{3+}$ fragments from compound 1: (a) Er1, (b) Er2 and (c) Er3. The thick black lines represent the KDs as a function of their magnetic moment along the magnetic axis. The green lines correspond to diagonal quantum tunneling of magnetization (QTM); the blue line represent off-diagonal relaxation process. The path shown by the red arrows represents the most probable path for magnetic relaxation in the corresponding compounds. The numbers at each arrow stand for the mean absolute value of the corresponding matrix element of transition magnetic moment.


Fig. S21. Scheme of the $\mathrm{Er}^{3+}-\mathrm{Er}^{3+}$ interactions in compound 1.


Fig. S22. Calculated (red solid line) and experimental (white circle) data of magnetic susceptibility of $\left\{\mathrm{Er}_{3}\right\}$ fragment in $\mathbf{1}$. The intermolecular interaction $z J^{\prime}$ of $\left\{\mathrm{Er}_{3}\right\}$ fragment in $\mathbf{1}$ was fitted to $-0.16 \mathrm{~cm}^{-1}$.


Fig. S23. Calculated orientations of the local main magnetic axes on $\mathrm{Er}^{3+}$ ions of $\left\{\mathrm{Er}_{3}\right\}$ fragment in 1 in the ground KDs.


Fig. S24. Calculated model structures of individual $\mathrm{Er}^{3+}$ fragments in 1; H atoms are omitted for clarify.

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