## Supporting Information

# Accurate Electronic Properties and Nonlinear Optical Response of Two-dimensional $\mathrm{MA}_{2} \mathrm{Z}_{4}$ 

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S1 The second-order coefficients for $\mathrm{MA}_{2} \mathrm{Z}_{4}$ family


Figure S1. Besides the large $d_{16}, d_{21}$ and $d_{22}$, other small nonlinear coefficients of second-order susceptibilities of explored $\mathrm{MA}_{2} \mathrm{Z}_{4}$.

S2 Photon energy-dependent imaginary and real components of second-order coefficients for $\mathrm{MA}_{2} \mathrm{Z}_{4}$ family


Figure S2. Photon energy-dependent imaginary and real components of second-order coefficients for $\mathrm{MA}_{2} \mathrm{Z}_{4}$ family.

S3 Polar plot of SHG intensity as a function of azimuthal angle and incident angle

Considering the sample shed by the linearly polarized pump light with an incident angle $\theta$, the electric field of the pump light can be given as follows:

$$
\left[\begin{array}{l}
E_{x}  \tag{S1}\\
E_{y} \\
E_{z}
\end{array}\right]=\left[\begin{array}{c}
E_{0} \cos [\theta] \\
0 \\
E_{0} \sin [\theta]
\end{array}\right]
$$

The relationship between the second order nonlinear susceptibility $\chi_{i j k}^{(2)}$ and the second-order nonlinear coefficient $d_{\mu L}$ can be described as:

$$
d_{\mu L}=\frac{1}{2} \chi_{i j k}^{(2)}=\left[\begin{array}{llllll}
d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\
d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\
d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36}
\end{array}\right] \iota^{*}
$$

MERGEFORMAT (S2)
By the use of rotation operation $T(\phi)$, we can obtain the transformed tensor containing azimuthal angle:

$$
\begin{equation*}
d_{i, j, k}^{(2)}{ }^{\prime}=\sum_{f=1}^{3} T_{i, f} \times \sum_{g=1}^{3} T_{j, g} \times \sum_{h=1}^{3} T_{k, h} \times d_{f, g, h}^{(2)} \tag{*}
\end{equation*}
$$

MERGEFORMAT (S3)

$$
T(\phi)=\left[\begin{array}{ccc}
\cos [\phi] & \sin [\phi] & 0  \tag{S4}\\
-\sin [\phi] & \cos [\phi] & 0 \\
0 & 0 & 1
\end{array}\right]
$$

Here $\phi$ is the azimuthal angle between the mirror plane in the crystal structure and the polarization of the pump beam. $T_{i, f}, T_{j, g}, T_{k, h}$ is a component in $T(\phi)$. Thus the SHG elements can be expressed as:

$$
\left[\begin{array}{c}
P_{x}(2 \omega) \\
P_{y}(2 \omega) \\
P_{z}(2 \omega)
\end{array}\right]=2 \varepsilon_{0} d_{\mu L}^{\prime}(\phi)\left[\begin{array}{c}
E_{x}^{2}(\omega, \theta) \\
E_{y}^{2}(\omega, \theta) \\
E_{z}^{2}(\omega, \theta) \\
2 E_{y}(\omega, \theta) E_{z}(\omega, \theta) \\
2 E_{z}(\omega, \theta) E_{x}(\omega, \theta) \\
2 E_{x}(\omega, \theta) E_{y}(\omega, \theta)
\end{array}\right] \text { ।*MERGEFORMAT }
$$

(S5)
where $\varepsilon_{0}$ represents the permittivity of the space.
With the calculated nonzero second-order nonlinear coefficients and their relationship, the SHG polarization components from $\mathrm{MA}_{2} Z_{4}$ can be expressed as:

$$
\begin{aligned}
P_{M A_{2} Z_{4}}= & {\left[\begin{array}{l}
P_{x} \\
P_{y} \\
P_{z}
\end{array}\right]=2 \varepsilon_{0}\left[\begin{array}{c}
-d_{22} E_{0}^{2} \cos ^{2}[\theta] \sin [3 \phi] \\
-d_{22} E_{0}^{2} \cos ^{2}[\theta] \cos [3 \phi] \\
0
\end{array}\right] \backslash^{*} } \\
& \text { MERGEFORMAT (S6) }
\end{aligned}
$$

Thus, the two polarization components (parallel and perpendicular) of SHG intensity as a function of azimuthal and incident angle can be described as:

$$
\begin{aligned}
& I_{/ /} \propto\left[-P_{x}\left(d_{\mu L}, \phi\right) \cos [\theta]+P_{z}\left(d_{\mu L}, \phi\right) \sin [\theta]\right]^{2} \backslash^{*} \\
& \text { MERGEFORMAT (S7) } \\
& I_{\perp} \propto P_{y}^{2}\left(d_{\mu L}, \phi\right) \quad \quad{ }^{*} \text { MERGEFORMAT (S8) }
\end{aligned}
$$

Lastly, the $I_{/ /}$and $I_{\perp}$ from $\mathrm{MA}_{2} \mathrm{Z}_{4}$ can be expressed as:

$$
\begin{array}{ll}
I_{/ /} \propto d_{22}^{2} \sin ^{2}[3 \phi] \cos ^{4}[\theta] & \quad \text { }{ }^{*} \text { MERGEFORMAT (S9) } \\
I_{\perp} \propto d_{22}^{2} \cos ^{2}[3 \phi] \cos ^{4}[\theta] & \backslash * \text { MERGEFORMAT (S10) }
\end{array}
$$

## S4 Polar plots of SHG intensity from $\mathrm{MA}_{2} \mathrm{Z}_{4}$ family under different

## incident photon energy


$\mathrm{CrSi}_{2} \mathrm{~N}_{4} \quad \mathrm{Max}\left|\mathrm{d}_{22}\right|^{2}=6.8 \times 10^{6} \mathrm{pm}^{2} / V^{2}$
$\mathrm{ZrSi}_{2} \mathrm{~N}_{4} \quad \operatorname{Max}\left|d_{22}\right|^{2}=5.3 \times 10^{4} \mathrm{pm}^{2} / V^{2}$
(b)
$-I_{/} @ 3.26 \mathrm{eV}$
$60^{\circ} 1_{1} @ 3.26 \mathrm{eV}$
${ }_{135^{\circ}}^{120^{\circ}} \underbrace{60^{\circ} 1^{\circ}}_{45^{\circ}}$
$\mathrm{MoSi}_{2} \mathrm{Hf}_{4} \quad$ Max $\left|d_{22}\right|^{2}=1.4 \times 10^{7} \mathrm{pm}^{2} / V^{2}$
(c)
$-l_{\|} @ 1.50 \mathrm{eV}$

$\mathrm{MoSi}_{2} \mathrm{~N}_{4}$
$\operatorname{Max}\left|d_{22}\right|^{2}=7.6 \times 10^{5} \mathrm{pm}^{2} / V^{2}$
(d) $120^{\circ} \underbrace{105^{\circ} 90^{\circ} \quad 75^{\circ}} \begin{array}{r}-I_{/} I_{1} @ 1.43 \mathrm{eV} \\ \hline 1.4 \mathrm{eV}\end{array}$

$\mathrm{MoGe}_{2} \mathrm{~N}_{4} \quad \mathrm{Max}\left|\mathrm{d}_{22}\right|^{2}=9.3 \times 10^{6} \mathrm{pm}^{2} / \mathrm{V}^{2}$
(g) $\quad \begin{array}{r}105^{\circ} 90^{\circ} \quad 75^{\circ} \quad-l_{/} @ 1.17 \mathrm{eV} \\ 60^{\circ} I_{+} @ 1.17 \mathrm{eV}\end{array}$

$\mathrm{WSi}_{2} \mathrm{~N}_{4} \quad \operatorname{Max}\left|d_{22}\right|^{2}=1.9 \times 10^{5} \mathrm{pm}^{2} / V^{2}$

(e) $20^{\circ}{ }^{105^{\circ} 90^{\circ} .75^{\circ}} \quad \begin{aligned} & -l_{/, @ 2.29 \mathrm{eV}}^{60^{\circ} 1_{1} @ 2.29 \mathrm{eV}} .\end{aligned}$
$135^{\circ} \rightarrow 45^{\circ}$
(h) $120^{105^{\circ} 90^{\circ}} 75^{75^{\circ}}{ }_{60^{\circ} 1}^{-1, @ 0.93 \mathrm{eV}}$
$\mathrm{MoSi}_{2} \mathrm{P}_{4} \quad \mathrm{Max}\left|\mathrm{d}_{22}\right|^{2}=3.5 \times 10^{6} \mathrm{pm}^{2} / \mathrm{V}^{2}$

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$-1.1 .99 \mathrm{eV}$
(i) $120{ }^{105^{\circ} 90^{\circ} \quad 75^{\circ}}{ }_{60^{\circ} 1_{1} @ 1.99 \mathrm{eV}}$
(f)

- $l_{/ @ 3.40 \mathrm{eV}}$ $135^{\circ}{ }_{45^{\circ}}^{120}$


$135^{\circ} \rightarrow 45^{\circ}$


Figure S4. Polar plots of the SHG intensity from $\mathrm{MA}_{2} \mathrm{Z}_{4}$ as a function of the crystal's azimuthal angle under different photon energy. Their maximum values are represented by the purple circles in the polar plot and listed on the top.

## S5 All the locations of prominent peaks and their corresponding $d_{22}$

in $\mathrm{MA}_{2} \mathrm{Z}_{4}$ family

| Compound | $\begin{gathered} d_{22} \\ (\mathrm{pm} / \mathrm{V}) \\ @ \mathrm{E}_{1} \end{gathered}$ | $\begin{gathered} d_{22} \\ (\mathrm{pm} / \mathrm{V}) \\ @ \mathrm{E}_{2} \end{gathered}$ | $\begin{gathered} d_{22} \\ (\mathrm{pm} / \mathrm{V}) \\ @ \mathbf{E}_{3} \end{gathered}$ | $\begin{gathered} d_{22} \\ (\mathrm{pm} / \mathrm{V}) \\ @ \mathrm{E}_{4} \end{gathered}$ | $\begin{gathered} d_{22} \\ (\mathrm{pm} / \mathrm{V}) \\ @ \mathrm{E}_{5} \end{gathered}$ | $\begin{gathered} d_{22} \\ (\mathrm{pm} / \mathrm{V}) \\ @ \mathrm{E}_{6} \end{gathered}$ | $\begin{gathered} d_{22} \\ (\mathrm{pm} / \mathrm{V}) \\ @ \mathrm{E}_{7} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{TiSi}_{2} \mathrm{~N}_{4}$ | 116.26 @ | 120.73@ | 179.10 @ | 210.75 @ | 209.40 @ | 195.28 @ | 161.10 @ |
|  | 1.43 eV | 1.99 eV | 2.84 eV | 3.00 eV | 3.06 eV | 3.17 eV | 3.66 eV |
| $\mathrm{ZrSi}_{2} \mathrm{~N}_{4}$ | 201.71 @ | 207.92 @ | 147.19 @ | 183.18 @ | 229.84 @ | 128.26 @ | 137.93 @ |
|  | 1.37 eV | 1.89 eV | 2.95 eV | 3.08 eV | 3.26 eV | 3.78 eV | 4.10 eV |
| $\mathrm{HfSi}_{2} \mathrm{~N}_{4}$ | 3697.04 @ | 1632.08 @ | 1929.53 @ | 2575.91 @ | 2665.56 @ | 2619.93 @ | 2115.52 @ |
|  | 1.50 eV | 1.85 eV | 2.08 eV | 3.34 eV | 3.71 eV | 4.22 eV | 4.44 eV |
| $\mathrm{CrSi}_{2} \mathrm{~N}_{4}$ | 1012.17 @ | 900.43 @ | 2608.85 | 582.02 @ | 747.84 @ | 414.39 @ | 382.10 @ |
|  | 1.21 eV | 1.33 eV | @ 1.43 eV | 2.70 eV | 2.86 eV | 3.19 eV | 3.97 eV |
| $\mathrm{MoSi} \mathrm{N}_{4}$ | 471.62 @ | 870.82 @ | 737.41 @ | 256.51 @ | 281.23 @ |  |  |
|  | 1.65 eV | 2.29 eV | 3.30 eV | 3.58 eV | 4.30 eV |  |  |
| WSi2 $\mathrm{N}_{4}$ | 239.61 @ | 147.50 @ | 176.72 @ | 184.53 @ | 441.19 @ | 199.41 @ | 129.83 @ |
|  | 1.70 eV | 2.40 eV | 2.48 eV | 2.98 eV | 3.40 eV | 3.68 eV | 4.17 eV |
| MoGe ${ }_{2} \mathrm{~N}_{4}$ | 3042.65 @ | 1294.50 @ | 764.71 @ |  |  |  |  |
|  | 1.17 eV | 2.34 eV | 2.67 eV |  |  |  |  |
| $\mathrm{MoSi}_{2} \mathrm{P}_{4}$ | 1874.79 @ | 1085.45 @ | 1295.03 @ | 1129.80 @ | 1403.39 @ | 1345.30 @ |  |
|  | 0.93 eV | 1.46 eV | 1.72 eV | 2.16 eV | 2.30 eV | 2.90 eV |  |
| $\mathrm{MoSi}_{2} \mathrm{As}_{4}$ | 1052.39 @ | 2143.43 @ | 2464.77 @ | 1017.45 @ | 1163.34 @ | 1674.23 @ | 969.74 @ |
|  | 0.45 eV | 1.55 eV | 1.99 eV | 2.23 eV | 2.73 eV | 2.95 eV | 3.10 eV |

Table S1. All the locations of prominent peaks and their corresponding $d_{22}$ in $M A_{2} Z_{4}$ family.

