

**Figure S1.** The shear stress-strain curves obtained from 2D diluted triangular networks made up of linear elastic fibers. The coordination number z is about 3.1 in all networks but different value of  $\bar{\kappa}$  are considered. The critical strain  $\gamma_0$ , shown by red circles, denotes the onset of nonlinearity in the overall mechanical response. The top right inset shows the differential shear stiffness K versus the applied shear strain and the top left inset represents the variation of K' =  $d(\log(K))/d(\log(\gamma))$ ; the maximum point of this plot, denoted by the star symbol, is the inflection point of the log(K) versus log( $\gamma$ ) curve and gives an estimate for critical strain  $\gamma_c$ .

The total bending energy H<sub>b</sub> and total stretching energy H<sub>s</sub> in random fiber networks are, respectively, given as  $H_b = \sum_{fibers} \frac{\kappa}{2} \int_{fiber} \left| \frac{d\vec{t}}{ds} \right| ds$  and  $H_b = \sum_{fibers} \frac{\mu}{2} \int_{fiber} \left( \frac{dl}{ds} \right)^2 ds$  where  $\frac{dl}{ds}$  is the change in length and  $\left| \frac{d\vec{t}}{ds} \right|$  is the curvature of a fiber. The relative contributions of bending and stretching energy in networks composed of linear elastic fibers are shown in Figure S2 where H = H<sub>s</sub> + H<sub>b</sub>.



**Figure S2.** The relative contributions of bending and stretching energy in networks composed of linear elastic fibers is plotted as a function of the applied shear strain. The bending energy is dominant for strains less than critical shear strain  $\gamma_c$ . When  $\gamma > \gamma_c$ , the stretching energy becomes important.



**Figure S3.** The variation of the differential shear modulus of random fiber networks versus the shear stress for linear elastic fibers with different bending rigidity  $\bar{\kappa} = 0.00001 - 0.1$ . The shear modulus and stress are plotted in units of  $\mu/l$ . Three different regions are observed. Initially, the network stiffness is independent of the stress. With increasing the stress, the stiffness increases as  $K \propto \sigma^{\alpha}$  where  $\alpha$  varies from 0.6 to 1.5 as  $\bar{\kappa}$  decreases from 10<sup>-1</sup> to 10<sup>-5</sup> (shown in the inset). With further increase of the stress, the stiffness becomes independent of stress (see the red dashed lines); this is a limitation of linear elastic models and does not agrees with experimental measurements where the network stiffness increases until failure.