

FIG. 1. (Color online) The side views of schematic crystal structures of  $Fe_2I_2$  (a) and  $Fe_2IBr$  (b).

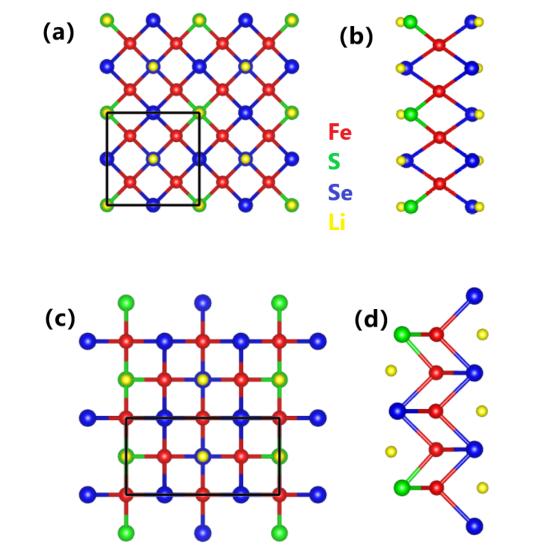


FIG. 2. (Color online) The top and side views of schematic crystal structures of LiFeSe<sub>0.75</sub>S<sub>0.25</sub> with  $\alpha$  (a,b) and  $\beta$  phases (c,d).

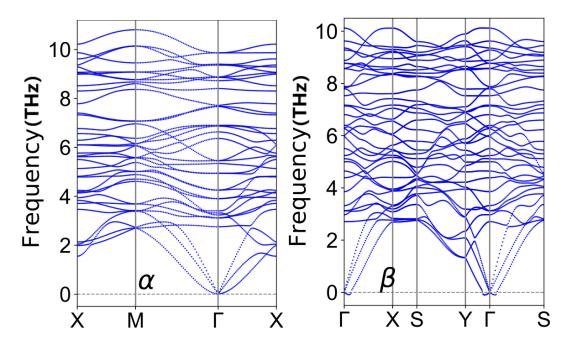


FIG. 3. (Color online)The phonon dispersions of monolayer LiFeSe $_{0.75}$ S $_{0.25}$  with  $\alpha$  and  $\beta$  phases.

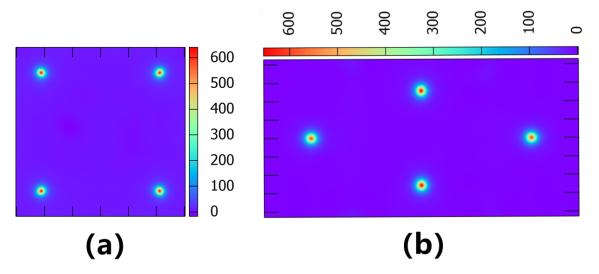


FIG. 4. (Color online) For monolayer LiFeSe<sub>0.75</sub>S<sub>0.25</sub> with  $\alpha$  (a) and  $\beta$  (b) phases, the distribution of Berry curvature contributed by occupied valence bands in the momentum space.

TABLE I. For monolayer LiFeSe<sub>0.75</sub>S<sub>0.25</sub> with  $\alpha$  and  $\beta$  phases, the lattice constants  $a_i$  in Å, the elastic constants  $C_{ij}$  in Nm<sup>-1</sup>, the piezoelectric strain coefficients  $d_{31}$  and  $d_{32}$  in pm/V, MAE in  $\mu eV/\text{Fe}$ , easy axis (EA), normalized exchange parameter J in meV and Curie temperature  $T_C$  in K.

Name	$a_1$	$a_2$	$C_{11}$	$C_{22}$	$C_{12}$	$C_{66}$	$d_{31}$	$d_{32}$	MAE	EA	J	$T_C$
$\alpha$	5.19	5.19	100.80	100.80	17.88	29.04	-0.064	-0.064	176	$\mathbf{c}$	129.54	950
β	7.33	3.67	88.76	89.41	30.11	41.11	-0.564	-0.011	191	c	131.81	966

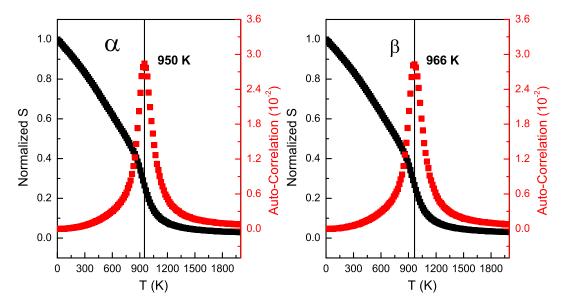


FIG. 5. (Color online) The normalized magnetic moment (S) and auto-correlation of monolayer LiFeSe<sub>0.75</sub>S<sub>0.25</sub> with  $\alpha$  and  $\beta$  phases as a function of temperature.