

**Supplemental Material (ESI) to:
Electron correlation effects in boron clusters B_n^Q (for $Q = -1, 0, 1$ and $n \leq 13$) based on
quantum Monte Carlo simulations**

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	1.790837	-0.000230	0.000001		1.790276	-0.002948	0.000001		1.766187	-0.002610	0.000001	
	1.116455	1.399947	0.000001		1.107500	1.433577	0.000000		1.094917	1.411738	0.000000	
	-0.398251	1.745673	-0.000001		-0.397137	1.784331	0.000000		-0.392792	1.759181	0.000000	
	-1.613102	0.777172	0.000000		-1.609608	0.802032	0.000000		-1.588973	0.788675	0.000000	
	-1.613244	-0.777080	0.000001		-1.608715	-0.797752	0.000000		-1.587978	-0.784859	0.000000	
	-0.398479	-1.745759	0.000000		-0.392333	-1.783776	0.000000		-0.388641	-1.758529	0.000000	
	1.116279	-1.400230	0.000000		1.110628	-1.435968	0.000001		1.097860	-1.414104	0.000001	
9	1.758925	-0.006585	-0.011055	0.7687	1.711153	0.017080	-0.050012	0.0	0.000000	0.000000	0.000000	0.0
	1.096470	1.368471	-0.018387		1.179405	1.500707	-0.104265		1.974961	-0.000120	0.000000	
	-0.391442	1.708061	-0.017047		-0.388959	1.825657	-0.102707		1.397626	1.397642	0.000000	
	-1.585024	0.757346	-0.008323		-1.503348	0.725233	-0.019758		-0.000121	1.975014	0.000000	
	-1.584373	-0.769247	0.000955		-1.634745	-0.830061	0.080100		-1.397729	1.397750	0.000000	
	-0.391418	-1.720401	0.004883		-0.436950	-1.884748	0.092402		-1.974961	0.000120	0.000000	
	1.096663	-1.381562	-0.000655		1.072702	-1.403148	0.056460		-1.397626	-1.397642	0.000000	
	0.000082	-0.006999	-0.869754		0.121712	0.323764	-0.754077		0.000121	-1.975014	0.000000	
	0.001197	-0.003175	0.843664		-0.119889	-0.328575	0.726138		1.397729	-1.397750	0.000000	
10	-0.176565	0.309350	-0.388888	0.0	-0.165067	0.290476	-0.220815	0.7601	-0.157257	0.277747	-0.204822	0.7848
	1.444187	0.303946	-0.023773		1.483873	0.271838	-0.071579		1.480572	0.315396	-0.080312	
	0.771862	1.771037	-0.074924		0.772005	1.771128	-0.074729		0.791786	1.783252	-0.074568	
	-0.961372	1.588482	0.237131		-0.972682	1.607449	0.067365		-0.980551	1.620304	0.053255	
	-1.909534	0.127144	-0.076098		-1.909855	0.126968	-0.076713		-1.929654	0.114830	-0.076476	
	-0.907336	-1.137810	-0.025716		-0.896675	-1.187593	-0.073397		-0.933988	-1.164875	-0.082216	
	0.669971	-1.071693	-0.118483		0.685348	-1.096825	-0.112990		0.668062	-1.068644	-0.106977	
	-2.582203	1.594261	-0.127180		-2.621813	1.626287	-0.078445		-2.618427	1.582690	-0.070351	
	-1.807774	2.969785	-0.031832		-1.823200	2.994917	-0.036451		-1.805917	2.966730	-0.043463	
	-0.230467	3.035917	-0.124617		-0.241165	3.085775	-0.076629		-0.203858	3.062990	-0.068451	
11	-4.603442	2.858928	0.036733	0.7662	-4.608203	2.843901	-0.049275	0.0	-4.588063	2.846622	0.036919	0.0
	-3.046900	3.084275	-0.012827		-3.041527	3.038619	-0.014371		-3.058621	3.178288	-0.010752	
	-1.483175	3.256969	-0.064711		-1.475336	3.237529	0.020659		-1.495253	3.241062	-0.065507	
	-3.693128	1.387436	-0.027037		-3.665275	1.376172	0.159381		-3.720749	1.351631	-0.028889	
	-1.999016	1.606233	-0.087368		-2.025596	1.595429	-0.274471		-1.964630	1.577092	-0.087337	
	-5.381734	1.521559	0.034496		-5.416829	1.527415	0.061683		-5.376551	1.527392	0.035028	
	-0.397449	2.155848	-0.130466		-0.365914	2.171184	-0.156730		-0.404447	2.159880	-0.130084	
	-1.840636	-0.280315	-0.135468		-1.840278	-0.268536	-0.114585		-1.830702	-0.262796	-0.135349	
	-3.382508	-0.479800	-0.083846		-3.383399	-0.463825	-0.104127		-3.394951	-0.462901	-0.083286	
	-0.588006	0.619134	-0.159556		-0.575691	0.646535	-0.131876		-0.568458	0.598591	-0.160632	
	-4.817376	0.080492	-0.018908		-4.835324	0.106334	-0.045247		-4.830946	0.055897	-0.019070	
12	-0.047676	-0.136640	0.285850	0.0	-0.084868	-0.139677	0.307817	0.7865	-0.078839	-0.119561	0.276395	0.7584
	1.709626	-0.147956	-0.155339		1.755054	-0.172999	-0.211314		1.822320	-0.209844	-0.152078	
	0.762470	1.335743	0.280204		0.746115	1.367627	0.302406		0.732248	1.351395	0.270960	
	-0.917251	1.300926	0.283819		-0.924202	1.305755	0.342624		-0.932586	1.309931	0.190796	
	-1.806126	-0.217853	-0.149880		-1.777674	-0.182318	-0.122759		-1.789388	-0.207572	-0.152303	
	-0.799285	-1.477689	-0.277759		-0.792330	-1.485193	-0.278849		-0.786332	-1.466685	-0.256758	
	0.753732	-1.447160	-0.279952		0.757704	-1.429478	-0.284547		0.788810	-1.419328	-0.276314	
	2.297442	1.353281	-0.289380		2.284934	1.340744	-0.293862		2.292964	1.309169	-0.285515	
	-2.452468	1.259525	-0.282448		-2.459609	1.253064	-0.327090		-2.496208	1.289908	-0.250231	
	1.494823	2.683168	-0.291886		1.504418	2.681068	-0.293122		1.491973	2.666244	-0.270841	
	-0.107696	2.861984	-0.160439		-0.122914	2.819589	-0.133380		-0.107910	2.842784	-0.163174	
	-1.702192	2.619500	-0.286790		-1.701230	2.628644	-0.331926		-1.751653	2.640386	-0.254937	
13	-0.070977	0.060071	-0.106027	0.7596	-0.040725	0.042369	-0.106103	0.0	0.008139	0.025827	-0.244109	0.0
	1.642714	0.104586	-0.113159		1.666290	0.080420	-0.112473		1.561473	0.041935	0.222637	
	0.624701	1.724877	-0.116636		0.615828	1.749102	-0.115824		0.799828	1.592341	-0.051693	
	-0.956861	1.453156	-0.109114		-0.943949	1.454777	-0.108918		-0.816566	1.574932	-0.049870	
	-1.720297	0.137203	-0.100598		-1.715307	0.125166	-0.100446		-1.544179	0.008732	0.225970	
	-1.170328	-1.370622	-0.095241		-1.196439	-1.375017	-0.095915		-0.783211	-1.541100	-0.053277	
	0.739141	-1.450091	-0.102616		0.772480	-1.458740	-0.102623		0.833204	-1.523735	-0.054888	
	2.447427	-2.293801	-0.105676		2.404986	-2.293308	-0.104935		-2.437206	1.376093	-0.046176	
	3.222073	-0.958059	-0.113774		3.201453	-0.921576	-0.113831		2.424304	1.428163	-0.051545	
	2.196099	1.714717	-0.120581		2.202047	1.736096	-0.121738		2.453880	-1.324831	-0.054316	

	-0.382659	-2.730342	-0.092628		-0.398200	-2.745875	-0.092840		-2.407691	-1.376860	-0.049236
	3.243771	0.600412	-0.120123		3.258668	0.622980	-0.120781		-3.167683	-0.008512	0.079678
	1.105039	-3.085837	-0.096897		1.092710	-3.110122	-0.096641		3.184600	0.059548	0.072816
13*	-0.021155	-0.395748	-0.086616	0.7631	-	-	-	-	-	-	-
	2.160032	-0.083440	-0.091845		-	-	-	-	-	-	-
	0.794264	1.173661	-0.113431		-	-	-	-	-	-	-
	-0.901752	1.133977	-0.110613		-	-	-	-	-	-	-
	-2.186719	-0.190549	-0.085558		-	-	-	-	-	-	-
	-1.405651	-1.529881	-0.068528		-	-	-	-	-	-	-
	1.439485	-1.453046	-0.072512		-	-	-	-	-	-	-
	2.419752	1.481260	-0.115627		-	-	-	-	-	-	-
	-2.538473	1.358398	-0.109316		-	-	-	-	-	-	-
	1.493396	2.691797	-0.136223		-	-	-	-	-	-	-
	-0.093111	2.786330	-0.138998		-	-	-	-	-	-	-
	-1.676631	2.613676	-0.132465		-	-	-	-	-	-	-
	0.031014	-2.077526	-0.062278		-	-	-	-	-	-	-
13**	0.008638	0.025391	0.240509	0.7610	-	-	-	-	-	-	-
	1.588487	0.042191	0.334870		-	-	-	-	-	-	-
	0.804803	1.620738	0.085255		-	-	-	-	-	-	-
	-0.822078	1.604679	0.085754		-	-	-	-	-	-	-
	-1.571002	0.008210	0.338067		-	-	-	-	-	-	-
	-0.787822	-1.570248	0.082657		-	-	-	-	-	-	-
	0.838773	-1.553317	0.081262		-	-	-	-	-	-	-
	-2.383679	1.386949	-0.193350		-	-	-	-	-	-	-
	2.370297	1.437869	-0.198903		-	-	-	-	-	-	-
	2.399851	-1.335173	-0.201534		-	-	-	-	-	-	-
	-2.354092	-1.386328	-0.196233		-	-	-	-	-	-	-
	-3.118994	-0.007841	-0.252789		-	-	-	-	-	-	-
	3.135709	0.059412	-0.259574		-	-	-	-	-	-	-

S2. THE CBS EXTRAPOLATION SCHEMES

We use the Gaussian program[1] to carry out the HF calculations with subsequent extrapolation to the CBS limit. To do this, we employed a hierarchically designed correlated basis set, aug-cc-pVXZ, where X is a cardinal number, the highest angular momentum included in the basis set. For the HF/CBS energy extrapolation we use five schemes[3] which will denoted as CBS $_n$ where $n = 1, 2, \dots, 5$. The first one, CBS $_1$, is an exponential scheme[4],

$$Y(X) = Y_{CBS} + A \exp(-BX) \quad (\text{S1})$$

where Y_{CBS} is the energy in the CBS limit, $Y(X)$ the energy with a given basis set. Y_{CBS} , A and B are fitting parameters. The second one, CBS $_2$, is a power extrapolation scheme[5]

$$Y(X) = Y_{CBS} + AX^{-B} \quad (\text{S2})$$

the third one, CBS $_3$, is a mixed exponential scheme[5]

$$Y(X) = Y_{CBS} + A \exp(-(X-1)) + B \exp(-(X-1)^2), \quad (\text{S3})$$

the fourth one[6, 7], CBS $_4$, is given by

$$Y(X) = Y_{CBS} + \frac{A}{(X+1/2)^B}, \quad (\text{S4})$$

the last one[6, 7] CBS $_5$, is

$$Y(X) = Y_{CBS} + \frac{A}{(X+1/2)^4} + \frac{B}{(X+1/2)^6}. \quad (\text{S5})$$

The CBS extrapolations of the HF energies of B_n^Q (for $Q = -1, 0, 1$ and $n \leq 13$) clusters according to the above schemes Eqs. (S1)-(S5) are shown in Table S2. It can be seen that the five adopted extrapolation schemes work well for clusters with up to five atoms, whereas, for larger clusters, we found good fitting results with the schemes CBS₃ and CBS₅.

TABLE S2: The HF energies of the B_n^Q (for $Q = -1, 0, 1$ and $n \leq 13$) clusters at different basis set calculated using Gaussian program[1] and the CBS extrapolation of the HF energies using the methods denoted as CBS_{*n*} where $n = 1, 2, \dots, 5$.

Cluster size, n	Basis set	B_n	B_n^+	B_n^-
1	aug-cc-pVDZ	-24.53057377	-24.23501373	-24.51954239
	aug-cc-pVTZ	-24.53217153	-24.23681573	-24.52094707
	aug-cc-pVQZ	-24.53298389	-24.23747329	-24.52178338
	aug-cc-pV5Z	-24.53313019	-24.23754633	-24.52194427
	CBS ₁	-24.5334	-24.2377	-24.5223
	CBS ₂	-24.5339	-24.2379	-24.5230
	CBS ₃	-24.5333	-24.2377	-24.5222
	CBS ₄	-24.5337	-24.2378	-24.5228
	CBS ₅	-24.5334	-24.2378	-24.5223
2	aug-cc-pVDZ	-49.08766994	-48.76340558	-49.16993215
	aug-cc-pVTZ	-49.09264235	-48.76844821	-49.17480280
	aug-cc-pVQZ	-49.09457495	-48.77032570	-49.17678100
	aug-cc-pV5Z	-49.09488696	-48.77062018	-49.17712554
	CBS ₁	-49.0953	-48.7710	-49.1775
	CBS ₂	-49.0960	-48.7716	-49.1784
	CBS ₃	-49.0954	-48.7712	-49.1776
	CBS ₄	-49.0959	-48.7715	-49.1782
	CBS ₅	-49.0956	-48.7713	-49.1778
2*	aug-cc-pVDZ	-49.03890972	-	-
	aug-cc-pVTZ	-49.04375611	-	-
	aug-cc-pVQZ	-49.04566991	-	-
	aug-cc-pV5Z	-49.04597036	-	-
	CBS ₁	-49.0464	-	-
	CBS ₂	-49.0471	-	-
	CBS ₃	-49.0465	-	-
	CBS ₄	-49.0469	-	-
	CBS ₅	-49.0466	-	-
3	aug-cc-pVDZ	-73.77983557	-73.43296012	-73.82014472
	aug-cc-pVTZ	-73.79555343	-73.44556281	-73.83438643
	aug-cc-pVQZ	-73.79968694	-73.44932157	-73.83821354
	aug-cc-pV5Z	-73.80033485	-73.44992715	-73.83882411
	CBS ₁	-73.8008	-73.4504	-73.8392
	CBS ₂	-73.8018	-73.4515	-73.8402
	CBS ₃	-73.8013	-73.4508	-73.8397
	CBS ₄	-73.8016	-73.4512	-73.8400
	CBS ₅	-73.8015	-73.4511	-73.8400
4	aug-cc-pVDZ	-98.43989048	-98.14816045	-98.52748143
	aug-cc-pVTZ	-98.45924494	-98.16649325	-98.54674035
	aug-cc-pVQZ	-98.46423315	-98.17157859	-98.55169773
	aug-cc-pV5Z	-98.46513747	-98.17249174	-98.55261104
	CBS ₁	-98.4656	-98.1731	-98.5531
	CBS ₂	-98.4668	-98.1758	-98.5543
	CBS ₃	-98.4663	-98.1752	-98.5537
	CBS ₄	-98.4666	-98.1749	-98.5541
	CBS ₅	-98.4666	-98.1734	-98.5540
5	aug-cc-pVDZ	-123.15227140	-122.83571838	-123.15377873

	aug-cc-pVTZ	-123.17436796	-122.85739648	-123.17493761
	aug-cc-pVQZ	-123.18027427	-122.86318736	-123.18066563
	aug-cc-pV5Z	-123.18138860	-122.86429776	-123.18174708
	CBS ₃	-123.1827	-122.8656	-123.1830
	CBS ₅	-123.1831	-122.8660	-123.1834
6	aug-cc-pVDZ	-147.76319869	-147.48717237	-147.87803539
	aug-cc-pVTZ	-147.79017225	-147.51313297	-147.90445261
	aug-cc-pVQZ	-147.79737620	-147.52031461	-147.91137507
	aug-cc-pV5Z	-147.79878584	-147.52169885	-147.91271401
	CBS ₃	-147.8004	-147.5233	-147.9143
	CBS ₅	-147.8009	-147.5239	-147.9147
6*	aug-cc-pVDZ	-147.80143161	-	-147.83667918
	aug-cc-pVTZ	-147.82888500	-	-147.86437640
	aug-cc-pVQZ	-147.83589621	-	-147.87142761
	aug-cc-pV5Z	-147.83723797	-	-147.87275897
	CBS ₃	-147.8388	-	-147.8743
	CBS ₅	-147.8392	-	-147.8748
7	aug-cc-pVDZ	-172.48112730	-172.22154661	-172.53475769
	aug-cc-pVTZ	-172.51303088	-172.25186286	-172.56792644
	aug-cc-pVQZ	-172.52131038	-172.26004120	-172.57633832
	aug-cc-pV5Z	-172.52291920	-172.26163308	-172.57795736
	CBS ₃	-172.5248	-172.2635	-172.5798
	CBS ₅	-172.5253	-172.2641	-172.5804
7*	aug-cc-pVDZ	-	-	-172.51550737
	aug-cc-pVTZ	-	-	-172.54777036
	aug-cc-pVQZ	-	-	-172.55603704
	aug-cc-pV5Z	-	-	-172.55764341
	CBS ₃	-	-	-172.5595
	CBS ₅	-	-	-172.5600
8	aug-cc-pVDZ	-197.19110703	-196.91019526	-197.24912507
	aug-cc-pVTZ	-197.22774609	-196.94485270	-197.28661156
	aug-cc-pVQZ	-197.23697413	-196.95394358	-197.29590363
	CBS ₃	-197.2419	-196.9589	-197.3009
	CBS ₅	-197.2426	-196.9596	-197.3016
9	aug-cc-pVDZ	-221.82186050	-221.54740028	-221.89635257
	aug-cc-pVTZ	-221.86192843	-221.58643569	-221.93880291
	aug-cc-pVQZ	-221.87225965	-221.59680346	-221.94914155
	CBS ₃	-221.8779	-221.6025	-221.9547
	CBS ₅	-221.8787	-221.6033	-221.9554
10	aug-cc-pVDZ	-246.50683338	-246.22722364	-246.58056522
	aug-cc-pVTZ	-246.55325972	-246.27350908	-246.62857419
	aug-cc-pVQZ	-246.56483211	-246.28505647	-246.64023476
	CBS ₃	-246.5710	-246.2913	-246.6465
	CBS ₅	-246.5719	-246.2921	-246.6473
11	aug-cc-pVDZ	-271.18462827	-270.92479966	-271.27608286
	aug-cc-pVTZ	-271.23631534	-270.97407869	-271.32684132
	aug-cc-pVQZ	-271.24888667	-270.98648913	-271.33932128
	CBS ₃	-271.2556	-270.9932	-271.3460
	CBS ₅	-271.2565	-270.9941	-271.3469
12	aug-cc-pVDZ	-295.87496079	-295.58802694	-295.91991489
	aug-cc-pVTZ	-295.93099327	-295.64367079	-295.97647807
	aug-cc-pVQZ	-295.9448853	-295.65762905	-295.990396
	CBS ₃	-295.9523	-295.6651	-295.9978
	CBS ₅	-295.9534	-295.6662	-295.9989

13	aug-cc-pVDZ	-320.53345755	-320.30108995	-320.59841749
	aug-cc-pVTZ	-320.59407279	-320.35980147	-320.65943769
	aug-cc-pVQZ	-320.60881158	-320.37440282	-320.67434864
	CBS ₃	-320.6167	-320.3822	-320.6823
	CBS ₅	-320.6177	-320.3833	-320.6834
13*	aug-cc-pVDZ	-320.53144825	—	—
	aug-cc-pVTZ	-320.59199321	—	—
	aug-cc-pVQZ	-320.60674255	—	—
	CBS ₃	-320.6146	—	—
	CBS ₅	-320.6157	—	—
13 **	aug-cc-pVDZ	-320.50633686	—	—
	aug-cc-pVTZ	-320.56605607	—	—
	aug-cc-pVQZ	-320.5809227	—	—
	CBS ₃	-320.5889	—	—
	CBS ₅	-320.5900	—	—

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