

Supplementary Material

- Figure FS1: The spin-orbit splitting ΔE for the $1^3\Sigma^+$ and $1^3\Phi$ parent states and their corresponding daughter states.
- Figure FS2: The spin-orbit splitting ΔE for the $1^5\Sigma^-$ parent states and its corresponding daughter state.
- Figure FS3: Permanent dipole moment curves for singlet, triplet and quintet electronic states of LaNa molecule using spin-free MRCI + Q calculation.
- Figure FS4: Permanent dipole moment curves of the electronic states $\Omega = 0^+, 0^-, 1$ of LaNa. Molecule
- Fig. FS5: potential energy curves of the Ω states $(1)0^+, (1)1, (1)2, (1)0^-, (2)1, (2)0^+$.
- Fig. FS6: The FCF plotting of the considered transitions LaNa $X^1\Sigma^+ - (1)^1\Pi$ and $(1)0^+ - (1)1$ of LaNa molecule.
- Table TS1: Positions of the avoided crossings R_{AC} of Ω states of LaNa molecule.
- Table TS2: The radiative lifetimes τ and vibrational branching ratio of the spin orbit vibrational Transitions of the molecule LaNa.
- Table TS3: Variation of the laser slowing distance (L) in function of the number of the lasers needed (Laser N°), the number of cycles (N) for photon absorption/emission and the total decay channels involved (η) for cooling LaNa molecular beam for the transitions $(1)\Omega=0^+ - (2)\Omega=1$ and $(1)\Omega=0^+ - (2)\Omega=1b$
- Table TS4: The vibrational energy E_v , the rotational constant B_v , the centrifugal distortion constant D_v , and the abscissas of the turning points R_{min} and R_{max} for the vibrational levels of the $(1)0^+$ and $(1)1$ states of LaNa molecule.
- Table TS5: The values of the vibrational energy E_v , the rotational constants B_v , the centrifugal distortion constant D_v and the abscissas of the turning points for the different vibrational levels of the ground state $X^1\Sigma^+$ of LaNa molecule.
- Table TS6: The values of the vibrational energies E_v , the rotational constants B_v , the centrifugal distortion constant D_v and the abscissas of the turning points for the different vibrational levels of states $1^3\Pi$ and $1^3\Delta$ of LaNa molecule.

- Table TS7: The values of the vibrational energies E_v , the rotational constants B_v , the centrifugal distortion constant D_v and the abscissas of the turning points for the different vibrational levels of states $(1)^1\Pi$ and $1^3\Sigma^+$ of LaNa molecule.
- Table TS8: The values of the vibrational energies E_v , the rotational constants B_v , the centrifugal distortion constant D_v and the abscissas of the turning points for the different vibrational levels of state $1^1\Delta$ of LaNa molecule.