Supplementary

S1 Explanation of boundary condition in theoretical model

 $\frac{dP}{dr}|_{r\,=\,0}=0$

For the first BC, we choose the pressure reference at r=R by assuming P(r = R) = 0.

We find the 2nd BC based on the following: For small r, $-\pi r^2 (K_0 + K_1) = (F(r) - F(0))$

$$-\pi r (K_0 + K_1) = \frac{(F(r) - F(0))}{r}$$

When $r \rightarrow 0$,

$$0 = \frac{dF}{dr}|_{r=0}$$

From (4),

$$0 = \frac{dF}{dr}|_{r=0} = -\frac{2\pi h\gamma S}{\eta} \lim_{r \to 0} \frac{d}{dr} \left(r\frac{dP}{dr} \right) = -\frac{2\pi h\gamma S dP}{\eta}|_{r=0}$$

This shows the 2nd BC should be $\frac{dP}{dr}|_{r=0} = 0$

S2. Enrichment setup:



Figure S1. Schematic for the setup during airflow enrichment. The filter cutout for solute enrichment or movement is overlayed a water-saturated replenishing pad with roughly 5mm of contact (red dashed line). This is placed on a hollowed, 3D-printed platform raised 25mm from the workbench's surface.

S3. Operation of the POC device for multistep delivery



Figure S2 (a.) Demonstration of how the slide lock for opening specific valves. **(b.)** Pictures of the case to hold the sections when operating the device. Red arrows show pins that are fitted into the air inlet complex during operation.

Fig. S2a. shows the interior of the central air inlet complex (removing the nozzle adapter) to illustrate how the slide valve works to activate specific valves. Initially, the slide is lined up the center position to activate positions X_1 and X_3 . The center position has marker on the top of the slide to properly align the slide with the starting position. When the user wants to activate the next set of nozzles, the slide is simply pulled to the right to open position X_2 and X_3 while position X_1 becomes closed. The slide has "wings" on the edges to prevent the user from pulling the slide out of the assembled complex. **Fig. S2b.** shows the 3D-printed case which holds all the sections together during operation. It contains a view port to monitor the reaction and pins in its design which connect directly to the central inlet complex.

S4. Paper-based enrichment of RT-LAMP products

Table S1. Primer/probe sets, and positive controls used in RT-qPCR assay

| | N1 primer/ probe mix | |
|--------------------|-------------------------------------|--|
| Primer/probe sets | N2 primer/probe mix - | |
| | RNase P (RP) primer/probe mix | |
| Positive controls: | SARS-CoV-2 viral RNA | |
| | Human specimen- RPP30 plasmid (HSC) | |

Table S2. Concentrations of each component in master mix of RT-qPCR assay

| Volume in 20 µE ixit |
|----------------------|
|----------------------|

| iTaq universal probes reaction mix (2x) | 10 µL |
|---|--------|
| iScript RT | 0.5 μL |
| Primer/probe set | 1.5 μL |
| Nuclease-free H 2 O | 6 μL |

Table S3. The steps and thermocycler conditions for RT-qPCR experiment

| Step Temp Time | Temp | Time |
|---------------------------------|-----------------|--------|
| 1- Reverse Transcription | 50°C | 15 min |
| 2- Enzyme activation | 95°C | 2 min |
| 3 | 95°C | 10 sec |
| 4 | 55°C | 30 sec |
| 5 | Read plate *FAM | |
| Repeat steps 3-5 for 45 cycles. | | |

Table S4. The sequence of primers against Nucleocapsid (N-1) and ORFla-1 genes

| Nucleocapsid (N-1) |
|--|
| N-1-F3 - TGGCTACTACCGAAGAGCT |
| N-1-B3 - TGCAGCATTGTTAGCAGGAT |
| N-1-FIP - TCTGGCCCAGTTCCTAGGTAGTCCAGACGAATTCGTGGTGG |
| N-1-BIP - AGACGGCATCATATGGGTTGCACGGGTGCCAATGTGATCT |
| N-1-LF - GGACTGAGATCTTTCATTTTACCGT |
| N-1-LB - ACTGAGGGAGCCTTGAATACA |
| Orf1a-1 |
| ORF1a-1-F3 - CTGCACCTCATGGTCATGTT |
| ORF1a-1-B3 - AGCTCGTCGCCTAAGTCAA |
| ORF1a-1-FIP - GAGGGACAAGGACACCAAGTGTATGGTTGAGCTGGTAGCAGA |
| ORF1a-1-BIP - CCAGTGGCTTACCGCAAGGTTTTAGATCGGCGCCGTAAC |
| ORF1a-1-LF - CCGTACTGAATGCCTTCGAGT |
| ORF1a-1-LB - TTCGTAAGAACGGTAATAAAGGAGC |

Table S5. Concentrations of each oligonucleotide in the 10x primer mix

| PRIMER | 10X CONCENTRATION (STOCK) | 1X CONCENTRATION (FINAL) |
|--------|---------------------------|--------------------------|
| FIP | 16 μM | 1.6 μΜ |
| BIP | 16 μM | 1.6 μΜ |
| F3 | 2 μΜ | 0.2 μΜ |
| В3 | 2 μΜ | 0.2 μΜ |
| LOOP F | 4 μM | 0.4 μΜ |
| LOOP B | 4 μM | 0.4 μΜ |

| | EXTRACTED RNA TARGET DETECTION | NO- TEMPLATE CONTROL (NTC) |
|--|-----------------------------------|----------------------------------|
| WarmStart Colorimetric LAMP 2X Master Mix | 12.5 μL | 12.5 μL |
| LAMP Primer Mix (10X) | 2.5 μL*2 | 2.5 μL*2 |
| Isolated SARS-CoV-2 RNA | 1 μL | - |
| dH2O | 6.5µL | 7.5 μL |
| Total Volume | 25 μL | 25 μL |

Table S6. Concentrations of each reagents in RT-LAMP assay

In our experiment, the iTaq universal one-step RT-qPCR kit was used for the RT-qPCR process of isolated SARS-CoV-2 RNA. The RT-qPCR assay targeted the nucleocapsid (N) genomic regions by including 2 SARS-CoV-2 specific primer/probe sets (N1, N2) and a human specimen primer/probe set (RNase P). RT-qPCR was used to monitor the amplification of DNA by measuring the fluorescence signals in real time between the thermal cycles. Different concentrations of isolated SARS-CoV-2 RNA (3 samples in each concentration) have been carried out for RT-qPCR reactions untiled the viral cDNA could be detected. The measured Cq values for different concentrations of isolated RNA samples are shown in Figure 1. We were not able to achieve a Cq value for samples with concentration of 3 copies/ µL, which showed limitation of using RT-qPCR to test the existence of isolated SARS-CoV-2 RNA.



Figure S3. RT-qPCR results for isolated SARS-CoV-2 RNA. The Cq value for SARS-CoV-2 RNA with different copy numbers: 192, 96, 48, 24, 12, 6 was measured.



Figure S4. RT-LAMP colorimetric readout system illustration for on-paper RT-LAMP products before and after enrichments.



Figure S5. Image signal analysis process for RT-LAMP colorimetric readout

S5. 3D-printed ELISA Plate



Figure S6. Image of the 3D-printed ELISA plate that was used to secure each 16mm-diameter filter cutout (top row) containing different concentrations of antigen. The holes in the array are 12mm in diameter and double-sided tape was placed over each row prior to cutting out the interior with a blade.