## Supporting Information

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## EXPERIMENTAL SECTION

Materials and methods. Unless otherwise noted, all manipulations were carried out at room temperature under an atmosphere of argon in a glovebox (Vigor) or using Schlenk techniques. Tetrahydrofuran (THF), toluene and hexane were dried via solvent purification system (Braun). $\mathrm{Et}_{3} \mathrm{~N} \cdot \mathrm{HF}$ was obtained by mixing $\mathrm{Et}_{3} \mathrm{~N} \cdot 3 \mathrm{HF}$ with two equivalents of $\mathrm{Et}_{3} \mathrm{~N}$. [Cp, $\left.{ }_{3} \mathrm{Dy}\right]$ was prepared according to the literatures with some modification. All other reagents were commercially available and used as received. Elemental analysis was performed by Elementar Vario MICRO CUBE (Germany).

## X-ray crystallography

All crystals were manipulated under a nitrogen atmosphere and covered in grease. Data collections were performed at 180 K on an Agilent technologies Super Nova Atlas Dual System, with a (Mo K $\alpha=$ $0.71073 \AA$ ) microfocus source and focusing multilayer mirror optics. The structures were solved by direct methods and refined with the full-matrix least-squares technique based on $\mathrm{F}^{2}$ using the Olex2 program. ${ }^{[1]}$ All non-hydrogen atoms were refined anisotropically. All hydrogen atoms were placed at the calculation positions. The disordered solvent molecules were squeezed using the PLATON program. ${ }^{[2]}$

## Magnetic measurement

Samples were fixed by N-grease to avoid moving during measurement. Direct current susceptibility experiment was performed on Quantum Design MPMS XL-5 SQUID magnetometer on polycrystalline samples. Alternative current susceptibility measurements with frequencies ranging from 100 to 10000 Hz were performed on Quantum Design PPMS. All dc susceptibilities were corrected for diamagnetic
contribution from the sample holder, N -grease and diamagnetic contributions from the molecule using the pascal's constants.

1Dy: A THF solution $(10 \mathrm{~mL})$ of $\mathrm{Et}_{3} \mathrm{~N} \cdot \mathrm{HF}(42.2 \mathrm{mg}, 0.348 \mathrm{mmol})$ was added into the solution of [ $\mathrm{Cp}{ }^{\prime}{ }_{3} \mathrm{Dy}$ ] ( $200 \mathrm{mg}, 0.348 \mathrm{mmol}$ ) in THF under $-20^{\circ} \mathrm{C}$ and stirred for 24 h . The precipitates were filtered and collected. The raw product was dissolved in 2 mL toluene, stored under $-25^{\circ} \mathrm{C}$ for several days, yielding crystals. Yield: 54 mg . Anal. Calcd (\%) for $\mathrm{C}_{48} \mathrm{H}_{74} \mathrm{~F}_{3} \mathrm{Si}_{6} \mathrm{Dy}_{3}\left(\mathrm{C}_{7} \mathrm{H}_{8}\right)$ : C, 45.36; H, 5.67. Found: C, 45.82; H, 5.78.

2Dy: The preparing method was similar to 1Dy. The raw product was dissolved in hexane and recrystallized under $-25^{\circ} \mathrm{C}$. Anal. Calcd (\%) for $\mathrm{C}_{56} \mathrm{H}_{94} \mathrm{~F}_{6} \mathrm{Si}_{6} \mathrm{Dy}_{4} \mathrm{O}_{2}\left(\mathrm{C}_{6} \mathrm{H}_{14}\right)$ : C, 40.96; H, 5.99. Found: C, 40.61; H, 5.74.

3Dy: The preparing method was similar to 2Dy, where the molar ratio of [ $\mathrm{Cp},{ }_{3} \mathrm{Dy}$ ] and $\mathrm{Et}_{3} \mathrm{~N} \cdot \mathrm{HF}$ was changed to 1:1.5. Anal. Calcd (\%) for $\mathrm{C}_{64} \mathrm{H}_{110} \mathrm{~F}_{15} \mathrm{Si}_{6} \mathrm{Dy}_{7} \mathrm{O}_{4}\left(\mathrm{C}_{6} \mathrm{H}_{14}\right)$ : $\mathrm{C}, 32.08 ; \mathrm{H}, 4.77$. Found: C, 31.55; H, 4.51.

4Dy: A toluene solution ( 10 mL ) containing 0.610 mmol of $\mathrm{H}_{2} \mathrm{O}$ was slowly added to the solution of [Cp' ${ }_{3} \mathrm{Dy}$ ] ( $350 \mathrm{mg}, 0.610 \mathrm{mmol}$ ) in toluene under $-20^{\circ} \mathrm{C}$ and stirred overnight. The precipitates were collected by filtration and dried under vacuum. Then the solid was dissolved in 2 mL toluene and stored under $-25^{\circ} \mathrm{C}$ for several days to yield colorless single crystals. Yield: 83 mg . Anal. Calcd (\%) for $\mathrm{C}_{32} \mathrm{H}_{54} \mathrm{Si}_{4} \mathrm{Dy}_{2} \mathrm{O}_{2}:$ C, $42.75 ; \mathrm{H}, 6.12$. Found: C, 42.32; H, 5.99.

## Ab initio calculations

The two, one and four types of Dy ${ }^{\text {III }}$ fragments of complexes 1Dy,2Dy and 4Dy were calculated, respectively. Complete-active-space self-consistent field (CASSCF) calculations on individual Dy ${ }^{\text {III }}$
fragments of the model structures extracted from the compound on the basis of single-crystal X-ray determined geometry have been carried out using MOLCAS 8.2 program package. ${ }^{[3]}$ Each Dy ${ }^{\text {III }}$ fragment was calculated with experimentally determined structure of the corresponding compound while replacing the other $\mathrm{Dy}^{\text {III }}$ ions by diamagnetic $\mathrm{Lu}^{\text {III }}$ ions. The basis sets for all atoms are atomic natural orbitals from the MOLCAS ANO-RCC library: ANO-RCC-VTZP for Dy ${ }^{\text {III }}$ ion; VTZ for close C and O in 4Dy, C and F in 1Dy and 2Dy; VDZ for distant atoms. The calculations employed the second order Douglas-Kroll-Hess Hamiltonian, where scalar relativistic contractions were taken into account in the basis set and the spin-orbit couplings were handled separately in the restricted active space state interaction (RASSI-SO) procedure. For individual Dy ${ }^{\text {III }}$ fragment, active electrons in 7 active spaces include all $f$ electrons (CAS (9 in 7)) in the CASSCF calculation. To exclude all the doubts, we calculated all the roots in the active space. We have mixed the maximum number of spinfree state which was possible with our hardware (all from 21 sextets, 128 from 224 quadruplets, 130 from 490 doublets). Single-Aniso program was used to obtain energy levels, $\boldsymbol{g}$ tensors, $m_{J}$ values, magnetic axes, etc. based on the above CASSCF/RASSI calculations. ${ }^{[4,5]}$

Table S1: Crystallographic data and refinement for complexes 1-4.

|  | 1Dy | 2Dy | 3Dy | 4Dy |
| :---: | :---: | :---: | :---: | :---: |
| Formula | $\mathrm{C}_{48} \mathrm{H}_{74} \mathrm{Dy}_{3} \mathrm{~F}_{3}$ | $\mathrm{C}_{56} \mathrm{H}_{94} \mathrm{Dy}_{4} \mathrm{~F}_{6} \mathrm{O}_{2} \mathrm{Si}_{6} \cdot\left(\mathrm{C}_{6}\right.$ | $\mathrm{C}_{64} \mathrm{H}_{110} \mathrm{Dy}_{7} \mathrm{~F}_{15} \mathrm{O}_{4} \mathrm{~S}_{\mathrm{i} 6} \cdot\left(\mathrm{C}_{6}\right.$ | $\mathrm{C}_{32} \mathrm{H}_{54} \mathrm{Dy}_{2} \mathrm{O}_{2}$ |
|  | $\mathrm{Si}_{6} \cdot\left(\mathrm{C}_{7} \mathrm{H}_{8}\right)$ | $\left.\mathrm{H}_{14}\right)$ | $\left.\mathrm{H}_{14}\right)$ | $\mathrm{Si}_{4}$ |
| Mr | 1456.25 | 1818.02 | 2620.72 | 908.11 |
| cryst syst | triclinic | monoclinic | Triclinic | monoclinic |
| space group | $P \overline{1}$ | $P 2_{1} / c$ | $P \overline{1}$ | $P 2_{1} / c$ |
| $a, \AA$ | $12.1349(4)$ | $13.8238(3)$ | $14.4849(3)$ | $13.1068(2)$ |
| $b, \AA$ | $12.1825(3)$ | $25.4525(6)$ | $16.1855(4)$ | $23.7324(3)$ |
| $c, \AA$ | $22.6447(7)$ | $22.0905(5)$ | $20.3478(4)$ | $25.4803(5)$ |
| $\alpha$, deg | $81.832(2)$ | 90 | $76.1439(19)$ | 90 |
| $\beta, \operatorname{deg}$ | $74.493(3)$ | $90.064(2)$ | $75.9937(18)$ | $102.1981(17)$ |
| $\gamma, \operatorname{deg}$ | $69.021(3)$ | 90 | $78.922(2)$ | 90 |
| $V, \AA^{3}$ | $3007.96(17)$ | $7772.5(3)$ | $4448.10(18)$ | $7746.9(2)$ |
| Z | 2 | 4 | 2 | 8 |
| $T, \mathrm{~K}$ | 180 | 180 | 180 | 180 |
| $\mu$, mm | 3.844 | 3.939 | 5.952 | 3.976 |
| $\lambda, \AA$ | 0.71073 | 0.71073 | 0.71073 | 0.71073 |
| GOF | 1.055 | 1.056 | 1.070 | 1.128 |
| $R_{\text {int }}$ | 0.0574 | 0.0729 | 0.0453 | 0.0576 |
| $R_{1}, w R_{2}[I>$ | 0.0346, | $0.0545,0.1333$ | $0.0327,0.0578$ | 0.0340, |
| $2 \sigma(I)]$ | 0.0678 |  |  | 0.0667 |
| $R_{1}, w R_{2}[$ all | 0.0546, | $0.0781,0.1535$ | $0.0598,0.0683$ | 0.0556, |
| data] | 0.0781 |  |  | 0.0770 |

Table S2: Selected Distances $(\AA)$ and Angles $\left({ }^{\circ}\right)$ for complex 1Dy.

|  | Dy1 | Dy2 | Dy3 |
| :--- | :---: | :---: | :---: |
| Dy-F | (F1)2.204(2) | (F1)2.195(2) | (F2)2.202(2) |
| Dy-F | (F3)2.197(2) | (F2)2.206(2) | (F3)2.201(2) |
| Dy-C(Cp'1) | $2.667(5)$ | $2.631(5)$ | $2.651(5)$ |
|  | $2.680(5)$ | $2.630(5)$ | $2.670(5)$ |
|  | $2.642(5)$ | $2.651(5)$ | $2.625(5)$ |
|  | $2.645(5)$ | $2.659(5)$ | $2.641(5)$ |
|  | $2.681(5)$ | $2.654(5)$ | $2.681(5)$ |
| Average | $2.663(5)$ | $2.645(5)$ | $2.653(5)$ |
| Dy-C(Cp'2) | $2.667(5)$ | $2.651(5)$ | $2.653(5)$ |
|  | $2.636(5)$ | $2.657(5)$ | $2.658(5)$ |
|  | $2.634(5)$ | $2.649(5)$ | $2.669(5)$ |
|  | $2.647(5)$ | $2.658(5)$ | $2.647(5)$ |
| Average | $2.661(5)$ | $2.654(5)$ | $2.652(5)$ |
| Angle of F-Dy-F | $2.649(5)$ | $2.654(5)$ | $2.656(5)$ |
| Angle of Centroid(Cp'1)-Dy-Centroid(Cp') | $86.2(1)$ | $87.5(1)$ | $85.9(1)$ |
| Dihedral angle of the two Cp' | $130.8(1)$ | $130.5(1)$ | $129.6(1)$ |
|  | $50.8(1)$ | $50.1(1)$ | $50.6(1)$ |
| Angle of Dy-F-Dy |  |  |  |

Table S3: Selected Distances $(\AA)$ and Angles $\left({ }^{\circ}\right)$ for complex 2Dy.

|  | Dy3 | Dy4 |
| :--- | :---: | :---: |
| Dy-F | (F3)2.191(6) | (F6)2.199(6) |
| Dy-F | (F4)2.172(5) | (F5)2.195(5) |
| Dy-C(Cp'1) | $2.678(1)$ | $2.645(1)$ |
|  | $2.644(1)$ | $2.657(1)$ |
|  | $2.662(1)$ | $2.672(1)$ |
|  | $2.670(1)$ | $2.697(1)$ |
|  | $2.663(1)$ | $2.650(1)$ |
| Average | $2.663(1)$ | $2.664(1)$ |
| Dy-C(Cp'2) | $2.677(1)$ | $2.682(1)$ |
|  | $2.678(1)$ | $2.669(1)$ |
|  | $2.658(1)$ | $2.646(1)$ |
|  | $2.658(1)$ | $2.680(1)$ |
| Average | $2.645(1)$ | $2.681(1)$ |
| Angle of F-Dy-F | $2.663(1)$ | $2.671(1)$ |
| Angle of Centroid(Cp')-Dy-Centroid(Cp') | $90.6(1)$ | $88.4(1)$ |
| Dihedral angle of the two Cp' | $131.6(1)$ | $128.3(1)$ |
|  | $48.7(1)$ | $50.5(1)$ |


|  | Dy2 |  | Dy1 |
| :--- | :---: | :---: | :---: |
| Dy-F4 | $2.232(6)$ | Dy-F3 | $2.219(6)$ |
| Dy-F6 | $2.251(6)$ | Dy-F5 | $2.228(6)$ |
| Dy-F1 | $2.242(5)$ | Dy-F1 | $2.268(5)$ |
| Dy-F2 | $2.223(5)$ | Dy-F2 | $2.204(5)$ |
| Dy-O1 | $2.396(7)$ | Dy-O10 | $2.409(6)$ |
| Dy-Cp, | $2.706(1)$ | Dy-Cp, | $2.728(1)$ |
|  | $2.707(1)$ |  | $2.709(1)$ |
|  | $2.698(9)$ |  | $2.704(1)$ |
|  | $2.683(1)$ |  | $2.687(1)$ |
|  | $2.707(1)$ |  | $2.710(1)$ |
| Average | $2.700(1)$ | average | $2.707(1)$ |
| Angle of F1-Dy2-Cp, | $170.9(1)$ | Angle of F1-Dy1-Cp, | $173.6(1)$ |


| Angle of Dy-F-Dy | Dy3-F4-Dy2 | Dy2-F6-Dy4 | Dy4-F5-Dy1 | Dy1-F3-Dy3 |
| :---: | :---: | :---: | :---: | :---: |
|  | $142.9(3)$ | $143.6(3)$ | $143.1(3)$ | $141.0(3)$ |

Table S4: Selected Distances $(\AA)$ and Angles $\left({ }^{\circ}\right)$ for complex 3Dy.

|  | Dy1 |
| :--- | :---: |
| Dy-F1 | $2.274(3)$ |
| Dy-F5 | $2.268(3)$ |
| Dy-F6 | $2.340(2)$ |
| Dy-C(Cp'1) | $2.668(5)$ |
|  | $2.665(5)$ |
|  | $2.675(5)$ |
|  | $2.685(5)$ |
|  | $2.671(5)$ |
| Average | $2.672(5)$ |
| Dy-C(Cp'2) | $2.670(5)$ |
|  | $2.667(5)$ |
|  | $2.687(5)$ |
|  | $2.684(5)$ |
| Average | $2.684(5)$ |
| Angle of F1-Dy-F5 | $2.678(5)$ |
| Angle of Centroid(Cp')-Dy-Centroid(Cp') | $133.9(1)$ |
| Dihedral angle of the two Cp' | $135.5(1)$ |


|  | Dy2 |  | Dy5 |
| :--- | :---: | :---: | :---: |
| Dy-F1 | $2.186(2)$ | Dy-F4 | $2.194(2)$ |
| Dy-F2 | $2.190(2)$ | Dy-F5 | $2.187(2)$ |
| Dy-F6 | $2.308(3)$ | Dy-F6 | $2.299(3)$ |
| Dy-F7 | $2.183(2)$ | Dy-F10 | $2.189(2)$ |
| Dy-F11 | $2.195(2)$ | Dy-F14 | $2.181(2)$ |
| Dy-O81 | $2.403(4)$ | Dy-O90 | $2.400(4)$ |
| Dy-O85 | $2.370(5)$ | Dy-O75 | $2.375(3)$ |
|  |  |  |  |
|  | Dy3 |  | Dy4 |
| Dy-F2 | $2.208(2)$ | Dy-F3 | $2.194(3)$ |
| Dy-F3 | $2.196(3)$ | Dy-F4 | $2.213(2)$ |
| Dy-F8 | $2.197(2)$ | Dy-F9 | $2.206(2)$ |
| Dy-F12 | $2.207(2)$ | Dy-F13 | $2.191(2)$ |
| Dy-F15 | $2.550(3)$ | Dy-F15 | $2.515(3)$ |
| Dy-Cp' | $2.654(7)$ | Dy-Cp' | $2.678(7)$ |
|  | $2.672(7)$ |  | $2.663(7)$ |
|  | $2.645(7)$ |  | $2.662(7)$ |
|  | $2.663(7)$ |  | $2.659(7)$ |
|  | $2.666(7)$ |  | $2.664(7)$ |


| Average | $2.660(7)$ |
| :--- | :--- | :--- | :--- |
| Angle of F15-Dy-Cp, | $178.1(1)$ |$\quad$ Angle of F15-Dy-Cp, $\quad 177.7(1)$


|  | Dy6 |  | Dy7 |
| :--- | :---: | :--- | :---: |
| F7 | $2.190(3)$ | F11 | $2.191(2)$ |
| F8 | $2.192(3)$ | F12 | $2.190(3)$ |
| F9 | $2.197(2)$ | F13 | $2.193(3)$ |
| F10 | $2.184(2)$ | F14 | $2.190(3)$ |
| F15 | $2.815(2)$ | F15 | $2.695(2)$ |
| Cp' | $2.649(6)$ | Cp' | $2.656(6)$ |
|  | $2.629(6)$ |  | $2.679(6)$ |
|  | $2.664(6)$ |  | $2.698(6)$ |
|  | $2.679(6)$ |  | $2.686(6)$ |
|  | $2.663(6)$ |  | $2.633(6)$ |
| Average | $2.656(6)$ |  | $2.670(6)$ |
| Angle of F15-Dy-Cp' | $177.3(1)$ | Angle of F15-Dy-Cp' | $175.1(1)$ |

Table S5: Selected Distances $(\AA)$ and Angles $\left({ }^{\circ}\right)$ for one of the molecules in the asymmetric unit of complex 4Dy.

|  | Dy1 | Dy2 |
| :--- | :---: | :---: |
| Dy-O1 | $2.260(2)$ | $2.237(2)$ |
| Dy-O2 | $2.261(2)$ | $2.232(2)$ |
| Dy-C(Cp'1) | $2.669(5)$ | $2.684(4)$ |
|  | $2.650(5)$ | $2.669(4)$ |
|  | $2.639(4)$ | $2.650(4)$ |
|  | $2.656(4)$ | $2.671(4)$ |
|  | $2.681(5)$ | $2.674(4)$ |
| Average | $2.659(4)$ | $2.669(4)$ |
| Dy-C(Cp'2) | $2.646(4)$ | $2.671(4)$ |
|  | $2.652(4)$ | $2.677(4)$ |
|  | $2.653(4)$ | $2.666(4)$ |
|  | $2.675(4)$ | $2.668(4)$ |
| Average | $2.664(4)$ | $2.668(4)$ |
| Angle of O1-Dy-O2 | $2.658(4)$ | $2.670(4)$ |
| Angle of Centroid (Cp' 1)- | $74.36(1)$ | $75.37(1)$ |
| Dy-Centroid (Cp') | $127.4(1)$ | $129.9(1)$ |
| Dihedral angle of the two | $57.8(1)$ |  |
| Cp' |  | $49.6(1)$ |

Table S6: Calculated energy levels $\left(\mathrm{cm}^{-1}\right), \boldsymbol{g}\left(g_{\mathrm{x}}, g_{\mathrm{y}}, g_{\mathrm{z}}\right)$ tensors and $m_{J}$ values of the lowest Kramers doublets (KDs) of individual Dy ${ }^{\text {III }}$ fragments of complexes 1Dy,2Dy and 4Dy.

| KDs | 4Dy |  |  |  |  |  | $\begin{aligned} & \hline \text { 1Dy } \\ & \hline \text { Dy1 } \end{aligned}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Dy1 |  |  | Dy2 |  |  |  |  |  |
|  | $E / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ | $E / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ | $E / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ |
| 1 | 0.0 | $\begin{gathered} 0.091 \\ 3.69 \\ 15.532 \\ \hline \end{gathered}$ | $\pm 15 / 2$ | 0.0 | $\begin{aligned} & \hline 0.2378 \\ & 0.8919 \\ & 17.299 \end{aligned}$ | $\pm 15 / 2$ | 0.0 | $\begin{gathered} 0.112 \\ 0.139 \\ 18.472 \\ \hline \end{gathered}$ | $\pm 15 / 2$ |
| 2 | 30.0 | $\begin{gathered} 0.062 \\ 3.644 \\ 14.552 \end{gathered}$ | $\pm 1 / 2$ | 2.96 | $\begin{gathered} \hline 0.2176 \\ 0.804 \\ 17.778 \end{gathered}$ | $\pm 7 / 2$ | 9.04 | $\begin{gathered} 1.406 \\ 1.680 \\ 16.158 \end{gathered}$ | $\pm 9 / 2$ |
| 3 | 79.3 | $\begin{gathered} 0.361 \\ 0.456 \\ 16.174 \end{gathered}$ | $\pm 7 / 2$ | 58.2 | $\begin{gathered} 0.309 \\ 0.680 \\ 15.544 \end{gathered}$ | $\pm 5 / 2$ | 55.8 | $\begin{gathered} 1.283 \\ 2.029 \\ 13.392 \\ \hline \end{gathered}$ | $\pm 13 / 2$ |
| 4 | 114.4 | $\begin{gathered} 0.404 \\ 0.870 \\ 15.866 \\ \hline \end{gathered}$ | $\pm 13 / 2$ | 124.5 | $\begin{gathered} 0.431 \\ 0.457 \\ 16.004 \\ \hline \end{gathered}$ | $\pm 13 / 2$ | 115.5 | $\begin{gathered} 0.070 \\ 1.219 \\ 15.204 \\ \hline \end{gathered}$ | $\pm 3 / 2$ |
| 5 | 169.4 | $\begin{gathered} 4.160 \\ 5.363 \\ 10.411 \end{gathered}$ | $\pm 11 / 2$ | 185.3 | $\begin{gathered} 3.793 \\ 5.158 \\ 11.289 \end{gathered}$ | $\pm 11 / 2$ | 161.9 | $\begin{gathered} 3.502 \\ 5.736 \\ 11.753 \end{gathered}$ | $\pm 11 / 2$ |
| 6 | 223.9 | $\begin{gathered} \hline 2.505 \\ 4.039 \\ 11.134 \end{gathered}$ | $\pm 9 / 2$ | 241.1 | $\begin{gathered} 2.507 \\ 4.081 \\ 11.265 \end{gathered}$ | $\pm 9 / 2$ | 209.7 | 2.000 <br> 4.168 <br> 10.573 | $\pm 7 / 2$ |
| 7 | 291.6 | $\begin{gathered} 0.576 \\ 1.164 \\ 15.767 \end{gathered}$ | $\pm 3 / 2$ | 317.3 | $\begin{gathered} \hline 0.461 \\ 0.912 \\ 15.780 \end{gathered}$ | $\pm 3 / 2$ | 288.8 | $\begin{gathered} 0.345 \\ 0.465 \\ 15.496 \end{gathered}$ | $\pm 5 / 2$ |
| 8 | 473.7 | $\begin{gathered} 0.046 \\ 0.076 \\ 19.623 \\ \hline \end{gathered}$ | $\pm 5 / 2$ | 540.3 | $\begin{aligned} & \hline 0.0445 \\ & 0.0814 \\ & 19.571 \end{aligned}$ | $\pm 1 / 2$ | 499.9 | $\begin{gathered} 0.052 \\ 0.086 \\ 19.635 \\ \hline \end{gathered}$ | $\pm 1 / 2$ |
|  |  |  |  |  |  |  |  |  |  |
| KDs | 2Dy |  |  |  |  |  |  |  |  |
|  | Dy1 |  |  | Dy3 |  |  | Dy2 |  |  |
|  | $E / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ | $E / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ | $E / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ |
| 1 | 0.0 | $\begin{gathered} 1.082 \\ 3.705 \\ 15.419 \\ \hline \end{gathered}$ | $\pm 15 / 2$ | 0.0 | $\begin{gathered} 0.074 \\ 0.261 \\ 19.390 \end{gathered}$ | $\pm 15 / 2$ | 0.0 | $\begin{gathered} 0.533 \\ 0.741 \\ 17.744 \end{gathered}$ | $\pm 15 / 2$ |
| 2 | 18.2 | $\begin{gathered} 1.742 \\ 2.366 \\ 12.839 \\ \hline \end{gathered}$ | $\pm 5 / 2$ | 112.9 | $\begin{gathered} 0.887 \\ 1.454 \\ 18.023 \\ \hline \end{gathered}$ | $\pm 1 / 2$ | 33.6 | $\begin{gathered} 1.084 \\ 2.908 \\ 14.459 \\ \hline \end{gathered}$ | $\pm 7 / 2$ |
| 3 | 68.5 | $\begin{gathered} 0.123 \\ 2.123 \\ 12.862 \\ \hline \end{gathered}$ | $\pm 13 / 2$ | 168.9 | $\begin{gathered} 0.166 \\ 0.727 \\ 16.106 \\ \hline \end{gathered}$ | $\pm 13 / 2$ | 75.3 | $\begin{aligned} & \hline 2.053 \\ & 5.314 \\ & 9.468 \\ & \hline \end{aligned}$ | $\pm 13 / 2$ |
| 4 | 103.6 | 0.262 | $\pm 3 / 2$ | 242.3 | 8.212 | $\pm 9 / 2$ | 113.5 | 1.801 | $\pm 9 / 2$ |


|  |  | $\begin{gathered} \hline 2.462 \\ 14.200 \end{gathered}$ |  |  | $\begin{aligned} & 7.751 \\ & 4.147 \end{aligned}$ |  |  | $\begin{array}{r} 5.166 \\ 11.134 \end{array}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 174.2 | $\begin{aligned} & \hline 3.117 \\ & 4.252 \\ & 12.911 \end{aligned}$ | $\pm 11 / 2$ | 305.6 | $\begin{gathered} 10.571 \\ 5.646 \\ 0.048 \end{gathered}$ | $\pm 5 / 2$ | 182.4 | $\begin{gathered} 1.081 \\ 2.859 \\ 14.162 \end{gathered}$ | $\pm 11 / 2$ |
| 6 | 208.1 | $\begin{gathered} \hline 3.071 \\ 5.296 \\ 10.194 \end{gathered}$ | $\pm 9 / 2$ | 352.0 | $\begin{gathered} 1.842 \\ 3.778 \\ 11.414 \end{gathered}$ | $\pm 7 / 2$ | 207.3 | $\begin{aligned} & \hline 8.218 \\ & 7.813 \\ & 3.220 \end{aligned}$ | $\pm 5 / 2$ |
| 7 | 255.6 | $\begin{gathered} \hline 0.707 \\ 0.985 \\ 15.158 \end{gathered}$ | $\pm 7 / 2$ | 415.2 | $\begin{gathered} 1.466 \\ 2.102 \\ 14.024 \end{gathered}$ | $\pm 3 / 2$ | 242.9 | $\begin{gathered} 1.132 \\ 1.673 \\ 14.309 \end{gathered}$ | $\pm 3 / 2$ |
| 8 | 384.5 | $\begin{gathered} 0.085 \\ 0.164 \\ 19.517 \\ \hline \end{gathered}$ | $\pm 1 / 2$ | 457.4 | $\begin{gathered} 0.187 \\ 0.935 \\ 16.792 \end{gathered}$ | $\pm 11 / 2$ | 354.8 | $\begin{gathered} 0.102 \\ 0.218 \\ 19.403 \end{gathered}$ | $\pm 1 / 2$ |
|  |  | 2Dy |  |  |  |  |  |  |  |
| KDs |  | Dy4 |  |  |  |  |  |  |  |
|  | $E / \mathrm{cm}^{-1}$ | $g$ | $m_{J}$ |  |  |  |  |  |  |
| 1 | 0.0 | $\begin{gathered} 0.078 \\ 0.231 \\ 19.541 \\ \hline \end{gathered}$ | $\pm 15 / 2$ |  |  |  |  |  |  |
| 2 | 115.7 | $\begin{gathered} 0.850 \\ 1.066 \\ 18.472 \end{gathered}$ | $\pm 1 / 2$ |  |  |  |  |  |  |
| 3 | 203.8 | $\begin{gathered} \hline 1.378 \\ 1.780 \\ 14.294 \\ \hline \end{gathered}$ | $\pm 13 / 2$ |  |  |  |  |  |  |
| 4 | 246.9 | 10.163 <br> 7.362 <br> 2.952 | $\pm 7 / 2$ |  |  |  |  |  |  |
| 5 | 314.3 | $\begin{gathered} 1.209 \\ 3.535 \\ 11.427 \end{gathered}$ | $\pm 3 / 2$ |  |  |  |  |  |  |
| 6 | 363.6 | $\begin{gathered} 1.748 \\ 2.679 \\ 13.285 \\ \hline \end{gathered}$ | $\pm 9 / 2$ |  |  |  |  |  |  |
| 7 | 400.6 | $\begin{gathered} 1.439 \\ 1.870 \\ 13.998 \\ \hline \end{gathered}$ | $\pm 5 / 2$ |  |  |  |  |  |  |
| 8 | 508.4 | $\begin{gathered} \hline 0.095 \\ 0.210 \\ 18.870 \\ \hline \end{gathered}$ | $\pm 11 / 2$ |  |  |  |  |  |  |

Table S7: Exchange energies $\left(\mathrm{cm}^{-1}\right)$ and main values of the $g_{z}$ for the lowest two exchange doublets of complex 4Dy, the lowest four exchange doublets of complex 1Dy and the lowest eight exchange doublets of complex 2Dy

|  | 4Dy |  | 1Dy |  | 2Dy |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $E / \mathrm{cm}^{-1}$ | $g_{z}$ | $E / \mathrm{cm}^{-1}$ | $g_{z}$ | E/cm ${ }^{-1}$ | $g_{z}$ |
| 1 | 0.0 | 38.045 | 0.0 | 18.476 | 0.0 | 29.751 |
| 2 | 3.0 | 2.337 | 1.7 | 15.304 | 0.1 | 37.800 |
| 3 |  |  | 1.7 | 52.240 | 0.1 | 39.282 |
| 4 |  |  | 1.7 | 18.468 | 0.2 | 34.499 |
| 5 |  |  |  |  | 0.6 | 7.830 |
| 6 |  |  |  |  | 2.3 | 24.042 |
| 7 |  |  |  |  | 2.9 | 27.782 |
| 8 |  |  |  |  | 4.7 | 53.707 |

Figure S1: Variable-field-variable-temperature magnetization measurement for 1Dy.


Figure S2: Variable-field-variable-temperature magnetization measurement for 2Dy.


Figure S3: Variable-field-variable-temperature magnetization measurement for 3Dy.


Figure S4: Variable-field-variable-temperature magnetization measurement for 4Dy.


Figure S5 Temperature dependence of ac susceptibility in the absence of de field for 1Dy.


Figure S6: Temperature dependence of ac susceptibility under 1.5 kOe dc field for 1Dy.


Figure $\mathbf{S 7}$ Temperature dependence of ac susceptibility in the temperature range of 2 K to 10 K in the absence of dc field for 2Dy.


Figure S8: Cole-Cole plots fitting for the determination of the temperature dependence of $\tau$ for 1Dy in the absence of dc field (a) and (b) under 1.5 kOe dc field.


Figure S9: Cole-Cole plots fitting for the determination of the temperature dependence of $\tau$ for 2Dy in the absence of dc field (a) and (b) under 2.0 kOe dc field.


Figure S10: Frequency dependence of ac susceptibility for 3Dy in the absence of de field.


Figure S11. Temperature and frequency dependence of ac susceptibility for 4Dy in the absence of dc field (a) and under 2.5 kOe dc field (b).


Figure S12: Magnetization blocking barrier for individual Dy ${ }^{\text {III }}$ fragments in 1Dy.

The thick black lines represent the Kramers doublets as a function of their magnetic moment along the magnetic axis. The green lines correspond to diagonal quantum tunneling of magnetization (QTM); the blue line represent off-diagonal relaxation process. The numbers at each arrow stand for the mean absolute value of the corresponding matrix element of transition magnetic moment.


Figure S13: Magnetization blocking barriers for individual Dy ${ }^{\text {III }}$ fragments in 2Dy.

The thick black lines represent the Kramers doublets as a function of their magnetic moment along the magnetic axis. The green lines correspond to diagonal quantum tunneling of magnetization (QTM); the blue line represent off-diagonal relaxation process. The numbers at each arrow stand for the mean absolute value of the corresponding matrix element of transition magnetic moment. (a and $b$ represent Dy1 and Dy2, c and d represent Dy3 and Dy4, respectively)


Figure S14: Magnetization blocking barriers for individual Dy ${ }^{\text {III }}$ fragments (Dy1 and Dy2) in 4Dy.
The thick black lines represent the Kramers doublets as a function of their magnetic moment along the magnetic axis. The green lines correspond to diagonal quantum tunneling of magnetization (QTM); the blue line represent off-diagonal relaxation process. The numbers at each arrow stand for the mean absolute value of the corresponding matrix element of transition magnetic moment.



Figure S15: Ab initio calculated easy axis for 1Dy.


Figure S16: Ab initio calculated easy axis for 2Dy.


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