

## Supplementary Information

### Tailoring the phase diagram of discotic mesogens

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## ISOTENSION-ISOTHERMAL ENSEMBLE

Monte Carlo simulations in the Isotension-Isothermal ensemble to a system of anisotropic particles requires some minor precautions. The formalism was developed by Parrinello and Rahman [1, 2] and subsequently incorporated to Monte Carlo frameworks by different authors, with the first publication usually attributed to Najafabadi [3]. The aim of this approach is to avoid any artificial stress on the anisotropic particles from the simulation box and the associated boundary conditions. To this end, the shape of the simulation box (side lengths and angles) are allowed to change by a means of a transformation of the space of coordinates and an appropriate acceptance rule for the volume moves [4]. The transformation of coordinates can be operated as follows:

$$\mathbf{r}_i = H\mathbf{s}_i \quad (1)$$

where  $H$  is a matrix whose columns are the vectors that define the edges of the simulation box,  $\mathbf{r}_i$  are the coordinates in the real space and  $\mathbf{s}_i$  are the coordinates in a cube of size 1. Conveniently, the volume of the system is then the determinant of this matrix ( $V = \det H$ ), therefore we assume it to be  $\det H > 0$ . During each try to change volume, a random element of the matrix  $H$  is changed to explore the phase space of the system. To compute the acceptance probability of each try we use the function  $W$  defined as follows [5]:

$$W = \Delta U + P\Delta V - N\kappa_B T \Delta(\log V) \quad (2)$$

where  $U$  is the total energy of the pairs interactions,  $V$  the volume,  $N$  the number of particles,  $\kappa_B$  the Boltzmann constant and  $T$  the temperature. Then, the acceptance probability  $acc$  can be expressed like:

$$\begin{aligned} acc &= 1 & \text{if } W \leq 0 \\ acc &= \exp(-W/\kappa_B T) & \text{if } W > 0 \end{aligned}$$

In order to correctly implement these transformations into a simulation code, we need to carefully address every calculation regarding distances, specially when working with anisotropic particles whose relative distance relies on their relative orientation. Meanwhile, the latter is naturally performed in the real space ( $r_i$ ). Any algorithm relying on the adjacency of particles, like the ones dealing with periodic boundary conditions or a cell lists, should use  $s_i$  particle coordinates. This transformation might change orientations of anisotropic particles in  $s_i$  coordinates, but will behave as usual in the space of  $r_i$  coordinates.

The Isobaric-Isothermal ensemble (NPT) implementation can be understood as a simplification of the isotension-isothermal ensemble, where only the diagonal elements of  $H$  matrix are varied starting from an orthogonal shape only varying the elements  $H_{11}$ ,  $H_{22}$  and  $H_{33}$ , which would represent the sides of the simulation box (for more details on this see reference [4]).

## CALCULATION OF NEMATIC AND CUBATIC ORDER PARAMETER AND CORRELATION FUNCTION

To classify the phases reported in the article, several correlation functions and order parameters have been calculated. The long-range orientation order was determined by the standard procedure of diagonalization of the following second rank symmetric tensor [6]:

$$Q_{\alpha\beta} = \frac{1}{2N} \left\langle \sum_1^N (3\hat{\mathbf{u}}_{i\alpha} \cdot \hat{\mathbf{u}}_{i\beta} - \delta_{\alpha\beta}) \right\rangle \quad (3)$$

where  $i$  indicates a generic particle,  $\hat{\mathbf{u}}_i$  denotes its unit orientation vector and  $\alpha$  and  $\beta$  indicate the components of the vector.  $\delta_{\alpha\beta}$  is the delta Kronecker. The nematic order parameter,  $S_2$ , is the largest eigenvalue resulting from the diagonalization of this tensor, while the nematic director vector that defines the preferred orientation of the particles is its associated eigenvector.

The cubic order parameter has been calculated following the procedure proposed by Duncan and coworkers [7]. The starting point of the method is the definition of a four-rank tensor

$$Q_{\alpha\beta\gamma\delta} = \frac{35}{8}u_\alpha u_\beta u_\gamma u_\delta - \frac{5}{8}(u_\alpha u_\beta \delta_{\gamma\delta} + u_\alpha u_\gamma \delta_{\beta\delta} + u_\alpha u_\delta \delta_{\beta\gamma} + u_\beta u_\gamma \delta_{\alpha\delta} + u_\beta u_\delta \delta_{\alpha\gamma} + u_\gamma u_\delta \delta_{\alpha\beta}) + \frac{1}{8}(\delta_{\alpha\beta} \delta_{\gamma\delta} + \delta_{\alpha\gamma} \delta_{\beta\delta} + \delta_{\alpha\delta} \delta_{\beta\gamma}) \quad (4)$$

The cubic order parameter,  $S_{cub}$  can be calculated solving the tensor equation

$$Q_{\alpha\beta\gamma\delta} Q_{\alpha\beta} = \frac{7S_4}{8} Q_{\gamma\delta} \quad (5)$$

where a second-rank tensor  $Q_{\alpha\beta}$  is defined. This tensor is symmetric and traceless. Due to these properties, and the symmetries on  $Q_{\alpha\beta\gamma\delta}$ , this tensor equation can be reduced to five coupled equations for  $(\alpha, \beta) = (x, x), (x, y), (x, z), (y, y)$  and  $(y, z)$ . From the diagonalization of the matrix associated to this system of equations  $S_{cub}$  can be calculated as  $S_4 = 8/7\Gamma$ , with  $\Gamma$  the largest eigenvalue resulting from the referred diagonalization.

In a perfect cubic phase (where the particles are oriented with the same probability in three mutually perpendicular directions) the cubic order parameter takes the value  $S_{cub} = 1$ , while it tends to  $S_{cub} = 0$  in an isotropic phase. In phases with high level of orientational alignment, as in nematic or columnar phases,  $S_{cub}$  becomes larger than 1.

To characterize phases and structures with some degree of positional order, a list of correlation functions have been calculated. For instance, the pair correlation function parallel to the nematic director, that informs about the possibility of arranging the particles in layers perpendicular to the nematic director, is defined as:

$$g_{\parallel}(r_{\parallel}) = \frac{1}{N_p \rho S_{\parallel}} \left\langle \sum_i \sum_{j \neq i} \delta(r_{\parallel} - r_{\parallel,ij}) \right\rangle \quad (6)$$

where  $\rho = N_p/V$ ,  $N_p$  is the number of particles,  $V$  the box volume,  $\delta$  is the Dirac delta.  $r_{\parallel,ij}$  is the projection on the nematic director of the distance between particles  $i$  and  $j$ .  $S_{\parallel}$  is the surface of a plane at a distance  $r_{\parallel}$  in the direction of the nematic director from the particle,  $i$  bounded by a sphere with radius half of the shorter side of the simulation box.  $g_{\parallel}^0(r_{\parallel})$  is similar to the previous one, but restricted to particles  $i$  and  $j$  with a perpendicular distance to the nematic director shorter than  $0.5/L + \sigma$ . This correlation function provides information about correlation to particles that belong to the same column.

The pair correlation function perpendicular to the nematic director gives information about columnar packing. It is defined as:

$$g_{\perp}(r_{\perp}) = \frac{1}{N_p \rho S_{\perp}} \left\langle \sum_i \sum_{j \neq i} \delta(r_{\perp} - r_{\perp,ij}) \right\rangle \quad (7)$$

where now  $r_{\perp,ij}$  is the projection perpendicular to the nematic director of the distance between particles  $i$  and  $j$ .  $S_{\perp}$  is the surface of a plane at a distance  $r_{\perp}$  in the direction perpendicular to the nematic director from the particle  $i$  bounded by a sphere with radius half of the shorter side of the simulation box.

The orientational correlation function is defined as the average, for each intermolecular distance, of the second Legendre polynomial of the cosine of the angle between the orientation of the particles,  $g_2(r) = P_2(\hat{u}_i \cdot \hat{u}_j)$ . This function gives information about the averaged relative orientation of particles separated a distance  $r$ .  $g_2$  tends to one if the particles have similar orientation,  $-0.5$  if they are perpendicular and 0 if on average the particles are randomly oriented.

The angular distribution of the projection of the vector director of the particles in the plane perpendicular to the nematic director  $h(\varphi)$  is calculated as:

$$h(\varphi) = \frac{1}{N_p} \left\langle \sum_i \delta(\varphi - \varphi_i) \right\rangle \quad (8)$$

being  $\varphi_i$  is the azimuthal angle of the representation of the vector  $\hat{u}_i$  in spherical coordinates with respect to a

reference system with the z-axis in the direction of the nematic director vector.

### EVOLUTION OF THE CUBATIC ORDER PARAMETER IN ISOTHERMS WITH CUBATIC PHASE

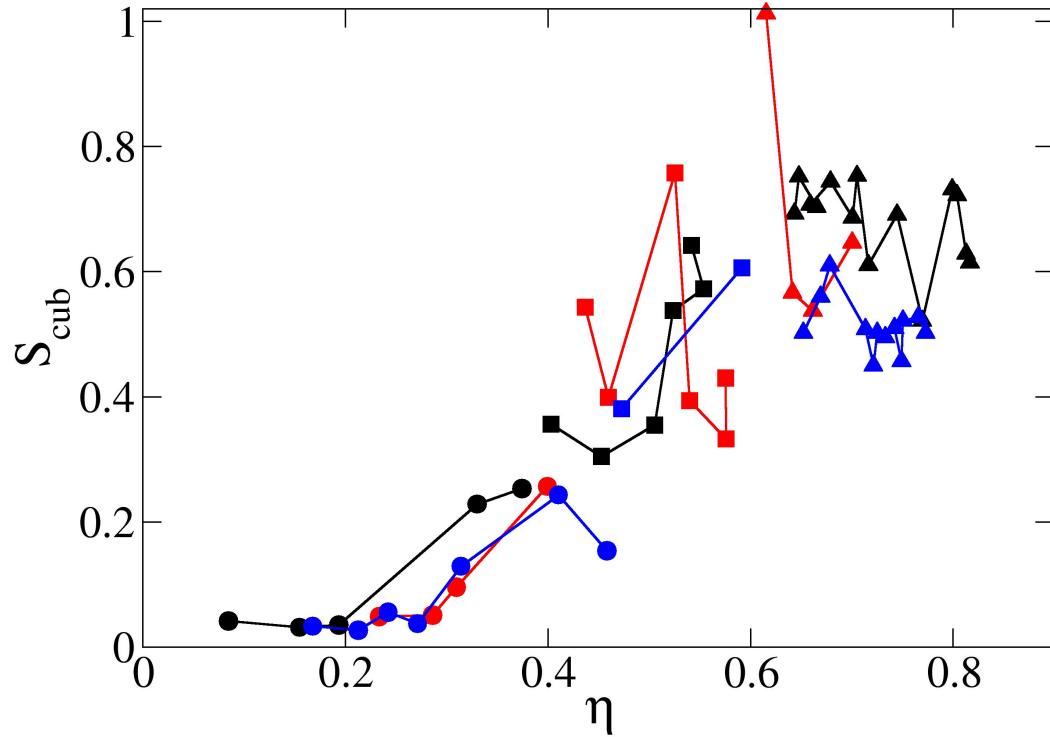


FIG. 1. Evolution of the cubatic order parameter for the  $U_{C_s}$  model at  $T^* = 0.5$  (black lines and symbols),  $T^* = 1$  (red lines and symbols) and the  $U_{C_w}$  model at  $T^* = 1$  (blue lines and symbols). Circles, squares and triangles correspond to states points in the I, Q and  $C_t$  phase respectively.

## TABLES

TABLE I. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Fs}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 5.85$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 11.7$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
75	0.199(2)	0.036(7)	I
100	0.230(2)	0.140(5)	I
140	0.281(2)	0.509(2)	N
180	0.324(3)	0.709(7)	N
200	0.339(2)	0.717(7)	N
225	0.359(3)	0.783(8)	N
250	0.375(3)	0.761(6)	N
275	0.392(3)	0.802(1)	N
300	0.409(3)	0.863(4)	N
325	0.420(3)	0.715(7)	N
350	0.434(3)	0.755(9)	N
375	0.448(3)	0.824(5)	N
400	0.463(3)	0.761(5)	N
450	0.541(4)	0.960(2)	$C_d$
500	0.568(4)	0.956(3)	$C_d$
600	0.609(4)	0.976(4)	$C_o$
700	0.644(3)	0.989(1)	$C_o$
800	0.667(3)	0.984(2)	$C_o$
900	0.693(4)	0.996(3)	$C_o$
1000	0.711(4)	0.997(3)	$C_o$
1250	0.748(3)	0.998(9)	$C_o$
1500	0.770(3)	0.999(5)	$C_o$
1750	0.785(3)	0.999(3)	$C_o$
2000	0.792(4)	0.994(5)	$C_o$
2250	0.800(2)	0.973(4)	$C_t$
2500	0.808(2)	0.948(2)	$C_t$
2750	0.817(2)	0.961(2)	$C_t$
3000	0.823(2)	0.930(3)	$C_t$
3250	0.830(2)	0.924(1)	$C_t$
3500	0.837(2)	0.942(2)	$C_t$
4000	0.847(2)	0.912(3)	$C_t$
5000	0.865(1)	0.903(2)	$C_t$
6000	0.880(2)	0.912(1)	$C_t$

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TABLE II. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Fs}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 11.7$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 11.7$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
100	0.162(5)	0.031(8)	I
200	0.228(1)	0.057(1)	I
250	0.257(2)	0.232(2)	I
300	0.284(2)	0.333(5)	I
350	0.314(2)	0.602(8)	N
400	0.339(2)	0.716(3)	N
450	0.359(2)	0.783(6)	N
500	0.380(4)	0.826(4)	N
550	0.392(4)	0.842(6)	N
600	0.408(2)	0.873(3)	N
650	0.421(2)	0.843(1)	N
700	0.435(3)	0.878(6)	N
750	0.446(3)	0.874(1)	N
800	0.458(3)	0.821(6)	N
850	0.471(3)	0.923(3)	N
900	0.512(4)	0.922(3)	$C_d$
1000	0.556(4)	0.956(2)	$C_d$
1200	0.603(4)	0.965(2)	$C_d$
1500	0.649(4)	0.978(2)	$C_o$
2000	0.702(3)	0.966(7)	$C_o$
2500	0.738(2)	0.980(3)	$C_o$
3000	0.766(3)	0.989(3)	$C_o$
3500	0.787(3)	0.991(2)	$C_o$
4000	0.805(3)	0.991(3)	$C_o$
4500	0.819(3)	0.992(1)	$C_o$
5500	0.842(2)	0.987(2)	$C_o$
6000	0.851(2)	0.980(2)	$C_t$
7000	0.866(2)	0.954(2)	$C_t$

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TABLE III. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Fs}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 23.4$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 11.7$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
50	0.080(4)	0.032(1)	I
200	0.164(3)	0.043(1)	I
300	0.197(4)	0.041(8)	I
550	0.275(7)	0.292(2)	I
620	0.295(5)	0.541(1)	N
670	0.307(3)	0.531(2)	N
800	0.344(3)	0.721(8)	N
850	0.356(3)	0.749(1)	N
900	0.361(4)	0.764(9)	N
1200	0.415(3)	0.869(4)	N
1300	0.432(2)	0.868(1)	N
1400	0.444(3)	0.904(5)	N
1500	0.454(3)	0.890(5)	N
1700	0.479(3)	0.905(9)	N
1900	0.531(4)	0.933(4)	N
2200	0.583(4)	0.960(3)	$C_d$
2500	0.617(4)	0.966(3)	$C_d$
3000	0.658(4)	0.971(2)	$C_o$
3500	0.690(4)	0.974(2)	$C_o$
4000	0.716(4)	0.970(3)	$C_o$
4500	0.737(3)	0.980(1)	$C_o$
5000	0.755(3)	0.967(7)	$C_o$
5500	0.771(3)	0.972(2)	$C_o$
6000	0.786(3)	0.978(3)	$C_o$
6500	0.798(3)	0.982(2)	$C_o$
7000	0.810(3)	0.986(3)	$C_o$
8000	0.830(3)	0.985(1)	$C_o$
9000	0.846(3)	0.981(3)	$C_o$
10000	0.861(2)	0.988(1)	$C_o$
11000	0.874(2)	0.989(8)	$C_o$
12500	0.889(2)	0.977(2)	$C_o$

TABLE IV. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Fw}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 1.375$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 2.75$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
15	0.203(4)	0.062(9)	I
20	0.234(2)	0.102(9)	I
30	0.293(2)	0.474(8)	N
34	0.315(3)	0.640(2)	N
41	0.345(3)	0.750(7)	N
45	0.351(2)	0.638(4)	N
50	0.374(2)	0.788(3)	N
55	0.387(2)	0.789(7)	N
60	0.402(3)	0.784(3)	N
63	0.407(2)	0.803(3)	N
67	0.419(3)	0.775(3)	N
74	0.433(2)	0.727(5)	N
80	0.452(3)	0.717(5)	N
81	0.467(2)	0.807(4)	N
82	0.512(4)	0.874(6)	N
83	0.540(3)	0.930(10)	N
85	0.580(4)	0.971(2)	$C_d$
87	0.587(4)	0.967(7)	$C_d$
92	0.603(4)	0.979(1)	$C_d$
95	0.608(4)	0.980(2)	$C_o$
99	0.617(4)	0.981(2)	$C_o$
107	0.633(3)	0.983(1)	$C_o$
115	0.645(3)	0.985(1)	$C_o$
125	0.658(3)	0.987(7)	$C_o$
136	0.672(3)	0.986(5)	$C_o$
143	0.681(3)	0.986(4)	$C_o$
150	0.691(3)	0.987(4)	$C_o$
155	0.688(3)	0.988(7)	$C_o$
160	0.691(3)	0.989(8)	$C_o$
165	0.695(3)	0.986(5)	$C_o$
175	0.706(3)	0.989(6)	$C_o$
200	0.717(2)	0.974(10)	$C_o$
300	0.752(2)	0.878(1)	$C_t$
350	0.765(2)	0.895(3)	$C_t$
400	0.776(2)	0.886(1)	$C_t$
450	0.785(2)	0.853(1)	$C_t$
500	0.793(2)	0.844(7)	$C_t$
550	0.802(2)	0.889(2)	$C_t$
600	0.807(2)	0.855(7)	$C_t$
800	0.829(1)	0.801(2)	$C_t$
1000	0.847(1)	0.832(5)	$C_t$
1300	0.866(1)	0.799(2)	$C_t$
1500	0.878(1)	0.834(2)	$C_t$
2000	0.899(1)	0.816(1)	$C_t$
2500	0.916(1)	0.805(7)	$C_t$



TABLE V. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Fw}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 2.75$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 2.75$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
33	0.196(7)	0.036(7)	I
37	0.209(1)	0.023(9)	I
45	0.229(2)	0.047(7)	I
53	0.248(1)	0.047(1)	I
72	0.305(2)	0.682(3)	N
86	0.330(2)	0.712(7)	N
128	0.394(3)	0.847(7)	N
140	0.408(2)	0.841(2)	N
161	0.432(3)	0.859(8)	N
175	0.446(2)	0.803(1)	N
180	0.455(2)	0.715(9)	N
190	0.480(3)	0.871(9)	N
200	0.528(3)	0.921(4)	$C_d$
210	0.547(4)	0.958(3)	$C_d$
220	0.562(4)	0.963(2)	$C_d$
230	0.572(4)	0.962(2)	$C_d$
250	0.594(3)	0.965(3)	$C_d$
275	0.615(3)	0.971(1)	$C_d$
300	0.634(3)	0.974(2)	$C_o$
336	0.657(3)	0.981(7)	$C_o$
350	0.665(3)	0.983(9)	$C_o$
375	0.677(3)	0.984(8)	$C_o$
400	0.689(3)	0.986(2)	$C_o$
450	0.708(3)	0.988(7)	$C_o$
500	0.725(3)	0.988(1)	$C_o$
550	0.741(3)	0.987(9)	$C_o$
600	0.753(3)	0.988(7)	$C_o$
700	0.770(3)	0.942(8)	$C_t$
800	0.786(2)	0.936(5)	$C_t$
900	0.800(2)	0.914(8)	$C_t$
1000	0.811(2)	0.936(1)	$C_t$
1200	0.831(2)	0.955(2)	$C_t$
1500	0.851(2)	0.911(3)	$C_t$
2000	0.878(2)	0.867(2)	$C_t$
2500	0.898(1)	0.880(2)	$C_t$

TABLE VI. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Fw}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 5.5$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 2.75$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
55	0.176(2)	0.036(1)	I
65	0.192(2)	0.038(8)	I
70	0.200(2)	0.033(9)	I
75	0.205(2)	0.038(1)	I
100	0.236(1)	0.047(2)	I
125	0.266(2)	0.257(9)	I
150	0.302(3)	0.524(2)	N
175	0.326(2)	0.703(5)	N
250	0.388(3)	0.845(8)	N
280	0.406(3)	0.859(6)	N
320	0.429(3)	0.887(4)	N
371	0.456(3)	0.859(6)	N
399	0.469(3)	0.864(8)	N
415	0.526(4)	0.938(3)	$C_d$
430	0.536(3)	0.955(3)	$C_d$
450	0.551(4)	0.957(3)	$C_d$
470	0.563(4)	0.956(2)	$C_d$
500	0.580(4)	0.958(2)	$C_d$
600	0.624(4)	0.973(2)	$C_d$
700	0.657(4)	0.977(1)	$C_o$
864	0.697(4)	0.979(2)	$C_o$
892	0.704(3)	0.980(1)	$C_o$
971	0.720(3)	0.979(3)	$C_o$
999	0.725(3)	0.978(1)	$C_o$
1200	0.756(3)	0.989(9)	$C_o$
1300	0.769(3)	0.987(2)	$C_o$
1500	0.792(3)	0.988(2)	$C_o$
2000	0.834(3)	0.987(7)	$C_o$

TABLE VII. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Es}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 5.65$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 11.3$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
51	0.172(2)	0.036(1)	I
54	0.177(2)	0.058(2)	I
70	0.207(3)	0.287(4)	I
90	0.299(6)	0.900(8)	N
110	0.367(3)	0.964(1)	N
130	0.404(4)	0.977(1)	N
140	0.421(4)	0.983(6)	N
150	0.435(3)	0.984(1)	N
160	0.445(6)	0.984(1)	N
180	0.489(4)	0.984(5)	N
200	0.530(4)	0.981(1)	N
225	0.580(5)	0.997(1)	S
250	0.630(2)	0.997(7)	S
275	0.639(3)	0.996(1)	S
375	0.649(3)	0.997(10)	S
400	0.771(3)	0.998(10)	$C_o$
420	0.778(3)	0.998(7)	$C_o$
450	0.785(3)	0.999(8)	$C_o$
500	0.794(3)	0.999(7)	$C_o$
550	0.804(3)	0.999(7)	$C_o$
650	0.819(3)	0.999(7)	$C_o$
700	0.825(3)	0.999(5)	$C_o$
750	0.831(3)	0.999(5)	$C_o$
800	0.837(2)	0.999(5)	$C_o$
900	0.847(2)	0.999(4)	$C_o$

TABLE VIII. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Es}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 11.3$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 11.3$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
90	0.161(1)	0.037(1)	I
110	0.175(2)	0.036(8)	I
250	0.265(1)	0.195(6)	I
390	0.365(3)	0.895(3)	N
550	0.427(7)	0.962(2)	N
600	0.446(3)	0.966(1)	N
650	0.463(3)	0.970(2)	N
700	0.477(3)	0.976(2)	N
725	0.635(6)	0.992(4)	$C_d$
750	0.655(6)	0.993(5)	$C_d$
900	0.712(5)	0.995(2)	$C_o$
1000	0.734(4)	0.996(2)	$C_o$
1200	0.768(4)	0.997(2)	$C_o$
1500	0.805(4)	0.998(2)	$C_o$
2500	0.878(2)	0.999(6)	$C_o$
2800	0.892(2)	0.999(6)	$C_o$
3300	0.913(2)	0.999(5)	$C_o$

TABLE IX. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Es}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 22.6$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 11.3$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
200	0.167(2)	0.023(7)	I
340	0.216(2)	0.039(10)	I
400	0.232(1)	0.044(2)	I
430	0.243(10)	0.105(4)	I
550	0.276(3)	0.235(4)	I
610	0.296(2)	0.471(3)	N
850	0.359(6)	0.757(1)	N
950	0.376(3)	0.838(4)	N
1000	0.385(3)	0.844(1)	N
1100	0.403(3)	0.861(6)	N
1200	0.418(3)	0.875(7)	N
1300	0.432(3)	0.901(4)	N
1400	0.446(3)	0.811(10)	N
1500	0.464(3)	0.939(3)	N
1600	0.554(6)	0.967(2)	$C_d$
1700	0.580(4)	0.971(1)	$C_d$
1900	0.629(5)	0.985(9)	$C_d$
2200	0.682(5)	0.989(9)	$C_d$
2500	0.724(5)	0.992(5)	$C_o$
3000	0.774(4)	0.995(4)	$C_o$
3500	0.810(4)	0.996(3)	$C_o$
4000	0.839(4)	0.997(2)	$C_o$

TABLE X. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Ew}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 1.115$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 2.23$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
7	0.157(2)	0.032(9)	I
11	0.212(4)	0.439(4)	N
13	0.264(4)	0.744(1)	N
17	0.321(3)	0.874(5)	N
19	0.340(3)	0.895(3)	N
21	0.361(3)	0.911(2)	N
23	0.378(4)	0.935(5)	N
25	0.386(3)	0.940(2)	N
27	0.399(3)	0.945(2)	N
29	0.408(4)	0.948(2)	N
31	0.418(3)	0.950(1)	N
33	0.427(3)	0.960(2)	N
35	0.438(3)	0.964(1)	N
37	0.439(3)	0.941(5)	N
39	0.454(3)	0.968(2)	N
41	0.461(2)	0.967(1)	N
43	0.467(3)	0.971(1)	N
45	0.475(3)	0.968(10)	N
48	0.485(3)	0.979(1)	N
51	0.500(3)	0.982(6)	N
54	0.506(4)	0.982(1)	N
57	0.523(5)	0.987(1)	N
60	0.536(4)	0.989(8)	N
63	0.693(3)	0.992(3)	$C_o$
66	0.701(3)	0.992(2)	$C_o$
69	0.706(3)	0.992(3)	$C_o$
75	0.709(3)	0.992(3)	$C_o$
80	0.719(3)	0.991(3)	$C_o$
85	0.720(3)	0.992(2)	$C_o$
90	0.726(2)	0.991(2)	$C_o$
95	0.731(3)	0.991(2)	$C_o$
100	0.735(2)	0.992(2)	$C_o$
110	0.739(2)	0.993(2)	$C_o$
120	0.747(2)	0.993(2)	$C_o$
130	0.755(2)	0.992(4)	$C_o$
140	0.759(2)	0.993(2)	$C_o$
150	0.764(2)	0.992(2)	$C_o$
160	0.769(2)	0.993(2)	$C_o$
180	0.777(2)	0.993(2)	$C_o$
200	0.785(2)	0.993(2)	$C_o$
225	0.793(2)	0.994(2)	$C_o$
250	0.801(2)	0.994(2)	$C_o$
275	0.806(2)	0.973(3)	$C_o$
300	0.814(2)	0.994(2)	$C_o$
325	0.820(2)	0.990(2)	$C_o$
350	0.826(2)	0.992(2)	$C_o$
375	0.831(2)	0.992(2)	$C_o$
400	0.835(2)	0.981(3)	$C_o$
500	0.851(2)	0.989(4)	$C_o$
600	0.861(1)	0.986(5)	$C_o$
700	0.869(3)	0.896(4)	$C_t$

TABLE XI. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Ew}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 2.23$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 2.23$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
33	0.224(3)	0.251(6)	I
39	0.251(3)	0.459(3)	N
50	0.298(2)	0.720(5)	N
66	0.347(3)	0.864(5)	N
81	0.380(3)	0.894(3)	N
95	0.404(3)	0.924(3)	N
100	0.414(2)	0.936(2)	N
110	0.425(2)	0.926(3)	N
120	0.440(3)	0.926(5)	N
140	0.469(3)	0.963(2)	N
160	0.563(4)	0.983(7)	N
180	0.643(4)	0.989(7)	$C_d$
200	0.669(4)	0.991(5)	$C_o$
220	0.687(4)	0.993(6)	$C_o$
240	0.702(3)	0.993(5)	$C_o$
260	0.715(3)	0.994(4)	$C_o$
280	0.727(3)	0.993(5)	$C_o$
300	0.738(3)	0.994(4)	$C_o$
330	0.752(3)	0.994(6)	$C_o$
360	0.765(3)	0.994(3)	$C_o$
390	0.774(3)	0.993(5)	$C_o$
420	0.784(3)	0.993(3)	$C_o$
450	0.791(2)	0.994(4)	$C_o$
500	0.803(3)	0.995(2)	$C_o$
550	0.813(2)	0.994(3)	$C_o$
600	0.823(2)	0.994(2)	$C_o$
650	0.831(2)	0.994(2)	$C_o$
700	0.838(2)	0.994(4)	$C_o$
750	0.845(2)	0.994(3)	$C_o$
800	0.852(2)	0.993(4)	$C_o$
1200	0.884(1)	0.919(5)	$C_t$

TABLE XII. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Ew}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 4.46$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 2.23$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
60	0.162(8)	0.285(1)	I
71	0.174(9)	0.303(5)	I
80	0.181(1)	0.364(5)	I
140	0.336(3)	0.796(6)	N
150	0.343(3)	0.802(6)	N
160	0.353(2)	0.813(8)	N
170	0.366(3)	0.862(7)	N
180	0.374(3)	0.855(5)	N
200	0.390(3)	0.869(8)	N
220	0.407(3)	0.903(4)	N
240	0.420(3)	0.901(5)	N
260	0.435(3)	0.910(4)	N
280	0.448(3)	0.921(4)	N
300	0.461(3)	0.939(2)	N
320	0.524(4)	0.956(4)	$C_d$
340	0.550(4)	0.970(2)	$C_d$
360	0.568(4)	0.972(2)	$C_d$
380	0.587(4)	0.974(1)	$C_d$
400	0.603(4)	0.980(3)	$C_d$
430	0.623(4)	0.980(1)	$C_d$
460	0.642(4)	0.983(1)	$C_o$
490	0.658(4)	0.985(8)	$C_o$
520	0.674(4)	0.988(1)	$C_o$
550	0.688(4)	0.988(8)	$C_o$
580	0.700(4)	0.990(8)	$C_o$
610	0.711(4)	0.990(6)	$C_o$
640	0.722(4)	0.992(7)	$C_o$
670	0.731(3)	0.991(6)	$C_o$
700	0.740(4)	0.992(8)	$C_o$
730	0.748(3)	0.993(5)	$C_o$
750	0.754(4)	0.993(7)	$C_o$
800	0.765(4)	0.993(6)	$C_o$
850	0.776(4)	0.994(6)	$C_o$
900	0.786(4)	0.995(4)	$C_o$
950	0.795(4)	0.994(5)	$C_o$
1000	0.803(3)	0.994(5)	$C_o$
1100	0.819(3)	0.995(4)	$C_o$
1200	0.832(3)	0.995(3)	$C_o$
1300	0.844(3)	0.995(5)	$C_o$
1500	0.861(4)	0.984(2)	$C_o$

TABLE XIII. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{C_s}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 0.5$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
1	0.085(1)	0.021(6)	I
3	0.153(2)	0.021(7)	I
5	0.193(2)	0.021(8)	I
6	0.327(1)	0.075(5)	I
7	0.376(2)	0.089(1)	I
9	0.406(3)	0.138(5)	Q
11	0.456(3)	0.131(3)	Q
12	0.471(2)	0.107(4)	Q
14	0.504(3)	0.037(6)	Q
17	0.527(2)	0.055(5)	Q
20	0.557(2)	0.052(3)	Q
24	0.639(3)	0.206(4)	$C_t$
26	0.655(4)	0.210(7)	$C_t$
28	0.666(4)	0.207(6)	$C_t$
30	0.661(3)	0.234(5)	$C_t$
32	0.679(2)	0.254(4)	$C_t$
36	0.708(2)	0.237(4)	$C_t$
43	0.723(3)	0.246(5)	$C_t$
50	0.743(3)	0.254(7)	$C_t$
80	0.798(2)	0.285(5)	$C_t$
86	0.805(2)	0.280(5)	$C_t$
93	0.812(2)	0.286(3)	$C_t$
100	0.820(2)	0.297(5)	$C_t$
150	0.862(2)	0.339(6)	$C_t$
200	0.896(2)	0.371(8)	$C_t$
250	0.919(2)	0.406(7)	$C_t$
300	0.939(2)	0.438(2)	$C_t$
400	0.972(2)	0.486(1)	$C_t$



TABLE XIV. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{C_s}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 1.0$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
17	0.231(3)	0.053(1)	I
25	0.282(2)	0.062(3)	I
30	0.310(3)	0.224(3)	I
43	0.403(4)	0.326(3)	I
45	0.444(3)	0.348(1)	Q
47	0.464(4)	0.250(1)	Q
50	0.529(3)	0.227(4)	Q
55	0.541(3)	0.093(4)	Q
60	0.574(4)	0.100(7)	Q
65	0.585(4)	0.148(10)	Q
80	0.641(3)	0.386(3)	$C_t$
90	0.670(3)	0.388(2)	$C_t$
100	0.699(3)	0.367(4)	$C_t$
120	0.737(3)	0.367(5)	$C_t$
136	0.762(3)	0.381(1)	$C_t$
150	0.779(3)	0.379(7)	$C_t$
170	0.801(3)	0.429(2)	$C_t$
190	0.820(3)	0.440(4)	$C_t$
210	0.837(3)	0.444(2)	$C_t$
220	0.844(3)	0.443(3)	$C_t$
236	0.855(3)	0.432(4)	$C_t$
250	0.864(3)	0.444(6)	$C_t$
300	0.893(2)	0.465(9)	$C_t$
400	0.935(2)	0.529(9)	$C_t$
900	1.034(2)	0.540(1)	$C_t$

TABLE XV. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{C_s}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 2.0$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
25	0.197(2)	0.033(9)	I
30	0.216(2)	0.059(9)	I
40	0.251(2)	0.175(3)	I
67	0.339(3)	0.635(1)	N
80	0.373(3)	0.743(2)	N
99	0.417(3)	0.816(7)	N
127	0.473(3)	0.785(3)	N
130	0.482(3)	0.660(8)	N
146	0.544(5)	0.742(8)	N
150	0.559(5)	0.905(4)	$C_d$
155	0.569(4)	0.894(2)	$C_d$
160	0.600(4)	0.702(2)	$C_d$
170	0.615(5)	0.883(5)	$C_d$
174	0.614(4)	0.943(6)	$C_d$
180	0.635(4)	0.822(7)	$C_d$
200	0.663(4)	0.897(6)	$C_d$
220	0.704(4)	0.623(8)	$C_d$
248	0.735(4)	0.635(2)	$C_t$
262	0.751(4)	0.657(1)	$C_t$
276	0.764(4)	0.691(2)	$C_t$
280	0.767(4)	0.673(4)	$C_t$
290	0.773(3)	0.645(6)	$C_t$
310	0.793(3)	0.609(4)	$C_t$
350	0.823(4)	0.605(1)	$C_t$

TABLE XVI. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Cw}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 0.5$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
3	0.165(2)	0.027(1)	I
5	0.209(2)	0.047(2)	I
7	0.242(3)	0.155(3)	I
9	0.271(2)	0.193(3)	I
11	0.321(4)	0.268(2)	I
13	0.416(3)	0.152(1)	I
15	0.462(2)	0.104(7)	I
19	0.477(2)	0.132(5)	Q
21	0.591(3)	0.294(3)	Q
23	0.626(3)	0.297(7)	$C_t$
25	0.638(3)	0.299(4)	$C_t$
27	0.649(3)	0.309(5)	$C_t$
29	0.660(3)	0.319(5)	$C_t$
31	0.664(3)	0.312(5)	$C_t$
33	0.671(3)	0.313(3)	$C_t$
35	0.679(2)	0.322(7)	$C_t$
37	0.681(2)	0.332(3)	$C_t$
39	0.691(2)	0.320(5)	$C_t$
41	0.692(2)	0.336(10)	$C_t$
43	0.700(2)	0.326(4)	$C_t$
45	0.698(2)	0.330(3)	$C_t$
48	0.708(2)	0.329(3)	$C_t$
51	0.715(2)	0.346(5)	$C_t$
54	0.718(2)	0.337(6)	$C_t$
57	0.721(2)	0.344(3)	$C_t$
60	0.725(2)	0.346(4)	$C_t$
63	0.733(2)	0.344(8)	$C_t$
66	0.732(2)	0.379(5)	$C_t$
69	0.739(2)	0.353(5)	$C_t$
75	0.746(2)	0.372(4)	$C_t$
80	0.751(2)	0.362(4)	$C_t$
85	0.758(2)	0.385(3)	$C_t$
90	0.765(2)	0.383(3)	$C_t$
95	0.767(2)	0.407(4)	$C_t$
100	0.772(2)	0.407(6)	$C_t$
110	0.780(2)	0.420(4)	$C_t$
120	0.788(2)	0.425(4)	$C_t$
130	0.796(2)	0.437(4)	$C_t$
140	0.801(2)	0.440(6)	$C_t$
150	0.809(2)	0.463(6)	$C_t$
160	0.814(2)	0.459(5)	$C_t$
180	0.824(2)	0.484(3)	$C_t$
200	0.834(2)	0.493(4)	$C_t$
250	0.855(1)	0.536(3)	$C_t$
275	0.865(1)	0.570(4)	$C_t$
300	0.873(2)	0.584(5)	$C_t$
325	0.881(1)	0.598(2)	$C_t$
350	0.888(1)	0.632(5)	$C_t$
375	0.895(1)	0.639(6)	$C_t$
400	0.901(1)	0.641(3)	$C_t$
500	0.922(1)	0.680(3)	$C_t$

TABLE XVII. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Cw}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 1.0$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
10	0.189(4)	0.036(9)	I
15	0.223(2)	0.053(1)	I
18	0.243(2)	0.061(3)	I
21	0.263(2)	0.195(4)	I
24	0.283(2)	0.367(2)	I
27	0.299(2)	0.404(4)	N
30	0.317(2)	0.503(9)	N
33	0.333(2)	0.537(2)	N
36	0.345(3)	0.565(2)	N
39	0.359(2)	0.597(1)	N
42	0.372(3)	0.564(9)	N
45	0.384(2)	0.532(7)	N
48	0.402(3)	0.533(6)	N
57	0.482(3)	0.279(2)	$C_d$
60	0.494(6)	0.290(8)	$C_d$
63	0.511(3)	0.257(2)	$C_d$
66	0.555(4)	0.464(4)	$C_t$
69	0.566(4)	0.452(1)	$C_t$
72	0.598(4)	0.459(10)	$C_t$
75	0.616(3)	0.427(2)	$C_t$
78	0.623(3)	0.420(9)	$C_t$
81	0.630(3)	0.431(8)	$C_t$
85	0.641(3)	0.446(10)	$C_t$
90	0.651(3)	0.431(6)	$C_t$
95	0.663(3)	0.443(8)	$C_t$
100	0.672(3)	0.443(6)	$C_t$
110	0.688(3)	0.440(9)	$C_t$
120	0.702(3)	0.457(8)	$C_t$
140	0.727(3)	0.463(5)	$C_t$
160	0.748(3)	0.476(7)	$C_t$
180	0.765(3)	0.509(7)	$C_t$
200	0.780(2)	0.543(6)	$C_t$
220	0.793(2)	0.564(8)	$C_t$
240	0.804(2)	0.573(5)	$C_t$
260	0.814(2)	0.601(7)	$C_t$
280	0.823(2)	0.598(6)	$C_t$
300	0.832(2)	0.619(10)	$C_t$
330	0.845(2)	0.662(9)	$C_t$
360	0.856(2)	0.691(5)	$C_t$
390	0.867(2)	0.712(3)	$C_t$
420	0.875(2)	0.715(8)	$C_t$
500	0.898(2)	0.769(2)	$C_t$
1000	0.958(2)	0.629(8)	$C_t$

TABLE XVIII. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{C_w}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 2.0$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
35	0.232(2)	0.056(3)	I
45	0.265(2)	0.249(2)	I
55	0.300(2)	0.536(3)	N
65	0.325(2)	0.590(2)	N
85	0.374(3)	0.779(2)	N
100	0.402(3)	0.816(7)	N
110	0.420(3)	0.765(8)	N
120	0.438(3)	0.736(1)	N
130	0.457(3)	0.725(6)	N
140	0.477(3)	0.725(4)	N
150	0.538(4)	0.920(7)	$C_d$
160	0.563(4)	0.918(3)	$C_d$
170	0.583(4)	0.912(5)	$C_d$
180	0.598(4)	0.923(3)	$C_d$
200	0.632(4)	0.887(3)	$C_d$
220	0.659(4)	0.851(6)	$C_t$
240	0.684(4)	0.829(1)	$C_t$
260	0.701(3)	0.824(7)	$C_t$
280	0.720(3)	0.790(7)	$C_t$
300	0.735(3)	0.817(2)	$C_t$
320	0.748(3)	0.790(3)	$C_t$
340	0.760(3)	0.799(1)	$C_t$
360	0.773(3)	0.812(3)	$C_t$
380	0.784(3)	0.812(6)	$C_t$
400	0.793(3)	0.795(5)	$C_t$
430	0.806(3)	0.824(9)	$C_t$
460	0.820(3)	0.847(8)	$C_t$
490	0.830(3)	0.856(4)	$C_t$
520	0.841(3)	0.841(4)	$C_t$
550	0.853(3)	0.892(5)	$C_t$
570	0.856(3)	0.854(8)	$C_t$
580	0.860(3)	0.881(10)	$C_t$
610	0.871(2)	0.915(5)	$C_t$
800	0.905(4)	0.840(2)	$C_t$

TABLE XIX. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{T_s}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 4.35$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 8.66$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
15	0.104(10)	0.020(7)	I
25	0.136(1)	0.027(6)	I
35	0.162(1)	0.019(4)	I
45	0.184(1)	0.028(1)	I
55	0.206(1)	0.033(6)	I
66	0.229(2)	0.049(3)	I
75	0.245(2)	0.068(10)	I
85	0.267(2)	0.240(9)	I
95	0.306(3)	0.646(2)	N
100	0.334(5)	0.801(2)	N
150	0.426(5)	0.917(3)	N
190	0.498(4)	0.953(2)	N
225	0.539(4)	0.916(2)	N
230	0.550(3)	0.962(2)	N
275	0.599(3)	0.947(1)	N
315	0.647(1)	0.979(1)	N
345	0.673(4)	0.980(1)	N
360	0.687(4)	0.983(10)	N
375	0.760(6)	0.686(2)	$C_b$
385	0.791(6)	0.389(8)	$C_b$
400	0.827(6)	0.325(7)	$C_b$
415	0.841(6)	0.328(8)	$C_b$
450	0.878(6)	0.300(6)	$C_b$
500	0.903(6)	0.323(1)	$C_t$
550	0.943(6)	0.337(2)	$C_t$
600	0.979(6)	0.391(9)	$C_t$

TABLE XX. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Ts}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 8.7$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 8.66$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
95	0.191(1)	0.036(1)	I
110	0.207(2)	0.039(8)	I
120	0.218(2)	0.044(6)	I
125	0.222(1)	0.036(8)	I
145	0.244(2)	0.115(2)	I
160	0.267(2)	0.394(7)	I
170	0.285(2)	0.506(1)	I
180	0.312(2)	0.770(3)	N
280	0.429(3)	0.904(7)	N
330	0.474(3)	0.924(3)	N
390	0.520(4)	0.951(2)	N
500	0.593(4)	0.961(2)	N
600	0.650(4)	0.977(9)	N
700	0.700(5)	0.981(1)	N
800	0.747(4)	0.985(5)	N
900	0.846(5)	0.988(5)	$C_o$
1000	0.905(4)	0.989(6)	$C_o$
1100	0.950(2)	0.990(5)	$C_o$

TABLE XXI. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Ts}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 17.4$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 8.66$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
100	0.137(1)	0.025(7)	I
250	0.226(2)	0.071(2)	I
300	0.260(2)	0.343(1)	I
400	0.350(3)	0.809(9)	N
450	0.380(3)	0.833(6)	N
550	0.436(3)	0.897(5)	N
580	0.450(3)	0.919(3)	N
610	0.464(3)	0.921(3)	N
670	0.491(3)	0.936(3)	N
700	0.503(3)	0.938(2)	N
730	0.516(3)	0.943(3)	N
750	0.524(3)	0.948(2)	N
800	0.543(3)	0.951(3)	N
950	0.595(3)	0.964(1)	N
1000	0.611(3)	0.969(10)	N
1200	0.671(5)	0.973(2)	N
1300	0.698(4)	0.979(9)	N
1400	0.723(5)	0.981(7)	N
1750	0.826(6)	0.987(9)	$C_o$
1900	0.897(5)	0.988(5)	$C_o$
2000	0.928(6)	0.989(5)	$C_o$

TABLE XXII. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Tw}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 0.875$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1.76$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
7	0.184(2)	0.040(1)	I
10	0.215(2)	0.025(8)	I
15	0.257(2)	0.063(8)	I
21	0.298(2)	0.217(4)	I
28	0.341(2)	0.254(2)	I
29	0.346(2)	0.210(2)	I
30	0.353(2)	0.258(2)	I
31	0.364(2)	0.162(4)	I
32	0.367(3)	0.202(4)	I
35	0.391(2)	0.088(3)	I
37	0.408(1)	0.083(5)	I
40	0.439(2)	0.143(6)	U
42	0.451(3)	0.185(6)	U
44	0.462(3)	0.127(6)	U
49	0.486(2)	0.198(4)	U
51	0.508(3)	0.099(3)	U
54	0.519(3)	0.133(4)	U
56	0.588(4)	0.394(5)	$C_t$
60	0.635(5)	0.381(5)	$C_t$
62	0.622(4)	0.412(6)	$C_t$
63	0.641(3)	0.402(6)	$C_t$
65	0.647(3)	0.418(9)	$C_t$
68	0.651(3)	0.395(8)	$C_t$
70	0.665(3)	0.387(5)	$C_t$
83	0.687(3)	0.384(9)	$C_t$
86	0.693(3)	0.418(4)	$C_t$
93	0.701(2)	0.412(5)	$C_t$
100	0.709(2)	0.382(4)	$C_t$
125	0.733(2)	0.438(3)	$C_t$
150	0.754(2)	0.435(4)	$C_t$
200	0.781(2)	0.459(9)	$C_t$
225	0.793(2)	0.462(4)	$C_t$
250	0.802(2)	0.489(4)	$C_t$
300	0.819(2)	0.522(6)	$C_t$
350	0.833(2)	0.558(4)	$C_t$
450	0.856(2)	0.580(4)	$C_t$
600	0.882(1)	0.575(5)	$C_t$
700	0.896(1)	0.591(6)	$C_t$
900	0.918(1)	0.633(3)	$C_t$
1175	0.940(1)	0.589(2)	$C_t$



TABLE XXIII. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{T_w}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 1.75$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1.76$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
11	0.147(2)	0.032(1)	I
14	0.165(2)	0.021(9)	I
20	0.195(2)	0.031(8)	I
35	0.254(2)	0.073(3)	I
42	0.279(3)	0.270(3)	I
45	0.288(2)	0.272(3)	I
50	0.302(3)	0.280(4)	I
55	0.321(2)	0.465(3)	N
70	0.362(2)	0.680(2)	N
78	0.380(2)	0.666(1)	N
90	0.407(2)	0.583(8)	N
95	0.416(3)	0.752(10)	N
97	0.425(3)	0.538(7)	N
100	0.427(3)	0.629(2)	N
104	0.435(3)	0.669(1)	N
107	0.447(2)	0.402(1)	N
110	0.448(3)	0.627(9)	N
115	0.472(3)	0.466(1)	N
117	0.471(3)	0.442(9)	N
120	0.482(3)	0.394(1)	N
125	0.494(3)	0.381(2)	N
129	0.506(4)	0.335(2)	N
135	0.551(3)	0.622(7)	$C_t$
140	0.566(4)	0.608(8)	$C_t$
145	0.588(3)	0.662(1)	$C_t$
150	0.596(4)	0.661(8)	$C_t$
160	0.611(3)	0.675(6)	$C_t$
180	0.643(3)	0.610(7)	$C_t$
190	0.655(3)	0.601(1)	$C_t$
200	0.664(3)	0.640(5)	$C_t$
240	0.706(3)	0.568(1)	$C_t$
260	0.718(3)	0.604(4)	$C_t$
286	0.738(3)	0.579(6)	$C_t$
300	0.744(3)	0.582(5)	$C_t$
400	0.790(2)	0.587(6)	$C_t$
500	0.820(2)	0.642(7)	$C_t$
700	0.860(2)	0.660(6)	$C_t$
900	0.888(2)	0.670(4)	$C_t$
1000	0.899(2)	0.660(5)	$C_t$
1200	0.919(2)	0.665(3)	$C_t$
1500	0.941(2)	0.659(5)	$C_t$

TABLE XXIV. Monte Carlo simulation results for the equation of state and phase behaviour of the  $U_{Tw}$  model described in the main text at  $T^* = k_B T / \epsilon_0 = 3.5$ . The first column correspond to the pressure in reduced units ( $P^* = P(L + \sigma)^3 / \epsilon_0$ ).  $\eta = \rho \nu_0$  is the packing fraction, with  $\rho$  as the particle density and  $\nu_0$  as the volume of the particles.  $S_2$  is the nematic order parameter. The values in brackets denote the statistical uncertainty (one standard deviation) in the last digit. The last column indicates the phase corresponding to each state. In this case  $P^{**} = P^* / 1.76$ .

$P^*$	$\eta$	$S_2$	Phase/Structure
10	0.093(1)	0.017(8)	I
30	0.165(1)	0.021(6)	I
40	0.195(2)	0.388(4)	I
50	0.210(1)	0.040(8)	I
60	0.229(2)	0.053(13)	I
70	0.248(1)	0.065(7)	I
90	0.283(2)	0.200(13)	I
100	0.306(2)	0.564(2)	N
140	0.363(3)	0.732(6)	N
150	0.377(3)	0.774(7)	N
160	0.388(3)	0.804(6)	N
186	0.417(2)	0.841(7)	N
235	0.463(3)	0.803(1)	N
263	0.508(3)	0.811(5)	N
270	0.537(3)	0.873(3)	$C_d$
280	0.551(4)	0.851(3)	$C_d$
285	0.560(4)	0.920(3)	$C_d$
291	0.566(4)	0.932(4)	$C_d$
310	0.590(3)	0.825(1)	$C_d$
330	0.604(3)	0.936(2)	$C_d$
350	0.622(4)	0.879(3)	$C_d$
380	0.641(4)	0.888(5)	$C_d$
410	0.660(3)	0.845(5)	$C_t$
430	0.671(4)	0.888(5)	$C_t$
450	0.681(3)	0.858(3)	$C_t$
460	0.686(4)	0.891(5)	$C_t$
472	0.692(3)	0.871(4)	$C_t$
500	0.705(4)	0.886(4)	$C_t$
600	0.742(3)	0.878(2)	$C_t$
600	0.742(3)	0.882(9)	$C_t$
700	0.774(3)	0.800(6)	$C_t$
800	0.799(3)	0.800(4)	$C_t$
900	0.819(3)	0.782(8)	$C_t$
1000	0.836(3)	0.836(5)	$C_t$
1100	0.853(3)	0.776(7)	$C_t$
1200	0.866(3)	0.826(9)	$C_t$
1500	0.899(2)	0.753(4)	$C_t$