Electronic supplementary information for: "On the cross-streamline lift of microswimmers in viscoelastic flows"

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I. PROBLEM FORMULATION

The schematic in Fig. 1 (a) shows a neutrally buoyant spherical microswimmer suspended in pressure-driven flow of a polymeric fluid between two walls. In order to derive the expressions for the lift velocities, we work in a reference frame that translates with the swimmer $(\tilde{x}, \tilde{y}, \tilde{z})$. For simplicity, we temporarily drop the tilde ˜ notation. Fig. 1 (b) shows the non-dimensional description, where $s = d/2w$ and $s/\kappa = d/a$ is the distance from the bottom wall normalized by the particle radius a.

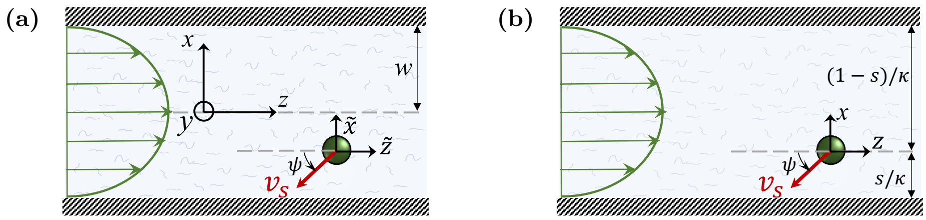

FIG. 1: (a) A spherical microswimmer self-propels with velocity $\boldsymbol{v}_s = v_s \boldsymbol{p}$ in a planar Poiseuille flow inside a channel with half width w. The coordinate frame $\{\tilde{x}, \tilde{y}, \tilde{z}\}$ co-moves with the swimmer. (b) Schematic with all lengths normalized by particle radius a. The tilde notation of the coordinates is shown to be dropped for brevity.

The inertia-less hydrodynamics of the disturbance field is governed by the continuity ($\nabla \cdot \boldsymbol{V} = 0$) and momentum equation in the co-moving swimmer frame $\{\tilde{x}, \tilde{y}, \tilde{z}\}$ as[1]:

$$\nabla \cdot (-P\mathbf{I} + 2\mathbf{E} + \mathrm{Wi}\mathbf{S}) = \mathbf{0}, \tag{1}$$

where $\mathbf{S} = 4\mathbf{E} \cdot \mathbf{E} + 2\delta \overset{\triangle}{\mathbf{E}}$. Here, $\mathbf{E}$ is the rate of strain tensor and $\triangle$ denotes the lower-convected time derivative:

$$\overset{\triangle}{\mathbf{E}} = \frac{\partial \mathbf{E}}{\partial t} + \boldsymbol{V} \cdot \nabla \mathbf{E} + \mathbf{E} \cdot \nabla \boldsymbol{V}^{\dagger} + \nabla \boldsymbol{V} \cdot \mathbf{E}, \tag{2}$$

where the time derivative can be neglected for the current problem of steady pressure-driven flow [1]. We split the actual velocity field into the background flow field $\boldsymbol{v}^{\infty}$ and the disturbance field $\boldsymbol{v}$ (*i.e.* $\boldsymbol{V} = \boldsymbol{v} + \boldsymbol{v}^{\infty}$). Similarly, pressure and rate of strain tensor is split: $P = p + p^{\infty}$ and $\mathbf{E} = \mathbf{e} + \mathbf{e}^{\infty}$, respectively. Substituting this in (1), we obtain the following:

$$\nabla \cdot [-(p + p^{\infty})\mathbf{I} + 2(\mathbf{e} + \mathbf{e}^{\infty}) + \mathrm{Wi}(\mathbf{s} + \mathbf{s}^{\infty})] = \mathbf{0}. \tag{3}$$

Here,

$$\mathbf{s} = 4(\mathbf{e} \cdot \mathbf{e} + \mathbf{e}^{\infty} \cdot \mathbf{e} + \mathbf{e} \cdot \mathbf{e}^{\infty}) + 2\delta \Big(\boldsymbol{v} \cdot \nabla \mathbf{e} + \mathbf{e} \cdot \nabla \boldsymbol{v}^{\dagger} + \nabla \boldsymbol{v} \cdot \mathbf{e} + \boldsymbol{v}^{\infty} \cdot \nabla \mathbf{e} + \mathbf{e} \cdot \nabla \boldsymbol{v}^{\infty \dagger} + \nabla \boldsymbol{v}^{\infty} \cdot \mathbf{e} + \boldsymbol{v} \cdot \nabla \mathbf{e}^{\infty} + \mathbf{e}^{\infty} \cdot \nabla \boldsymbol{v}^{\dagger} + \nabla \boldsymbol{v} \cdot \mathbf{e}^{\infty} \Big), \tag{4a}$$

$$\mathbf{s}^{\infty} = 4(\mathbf{e}^{\infty} \cdot \mathbf{e}^{\infty}) + 2\delta \left(\boldsymbol{v}^{\infty} \cdot \nabla \mathbf{e}^{\infty} + \mathbf{e}^{\infty} \cdot \nabla \boldsymbol{v}^{\infty \dagger} + \nabla \boldsymbol{v}^{\infty} \cdot \mathbf{e}^{\infty} \right). \tag{4b}$$

[1] We follow a quasi-steady description because the time scales associated with cross-stream motions both due to swimming ($a/v_s \sim 1$s) and viscoelastic lift are much larger than the characteristic vorticity diffusion time ($a^2/\nu \sim 10^{-4}$s).

In (3), the combination of terms $-\nabla p^{\infty} + 2\nabla \cdot \mathbf{e}^{\infty} + \mathrm{Wi}\nabla \cdot \mathbf{s}^{\infty}$ is zero, as it forms the momentum equation of pressure driven flow of a second-order fluid. Its solution is the undisturbed Poiseuille flow velocity. In the frame of reference translating with the particle, it is obtained as:

$$\boldsymbol{v}^{\infty} = \left(\alpha + \beta x + \gamma x^2\right) \boldsymbol{e}_z - \boldsymbol{U}_p, \tag{5}$$

where $\boldsymbol{U}_p$ is the total velocity of the swimmer, *i.e.*, swimming velocity $\boldsymbol{v}_s$ plus advection due to the Poiseuille flow and the lift velocities. The constants α, β and γ are:

$$\alpha = 4s\left(1-s\right)/\kappa,\ \beta = 4\left(1-2s\right),\ \gamma = -4\kappa, \tag{6}$$

where β and γ represent the shear and curvature of the background flow, respectively.

Compactly, one can describe the final governing equations as:

$$\begin{aligned} \nabla \cdot \boldsymbol{v} &= 0, \quad -\nabla p + \nabla^2 \boldsymbol{v} = -\mathrm{Wi}\left(\nabla \cdot \mathbf{s}\right), \text{ where} \\ \mathbf{s} &= 4(\mathbf{e}\cdot\mathbf{e} + \mathbf{w}) + 2\,\delta(\overset{\triangle}{\mathbf{e}} + \overset{\triangle}{\mathbf{w}}). \end{aligned} \tag{7}$$

Here, $\mathbf{e}$ is the rate of strain tensor for the disturbance flow $(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^{\dagger})/2$, whereas $\mathbf{w}$, $\overset{\triangle}{\mathbf{e}}$ and $\overset{\triangle}{\mathbf{w}}$ are the different parts of the perturbation $\mathbf{s}$ of the polymeric tensor due to the disturbance flow field $\boldsymbol{v}$:

$$\begin{aligned} \mathbf{w} &= \mathbf{e}^{\infty}\cdot\mathbf{e} + \mathbf{e}\cdot\mathbf{e}^{\infty}, \\ \overset{\triangle}{\mathbf{e}} &= \boldsymbol{v}\cdot\nabla\mathbf{e} + \mathbf{e}\cdot\nabla\boldsymbol{v}^{\dagger} + \nabla\boldsymbol{v}\cdot\mathbf{e}, \\ \overset{\triangle}{\mathbf{w}} &= \boldsymbol{v}^{\infty}\cdot\nabla\mathbf{e} + \mathbf{e}\cdot\nabla\boldsymbol{v}^{\infty\dagger} + \nabla\boldsymbol{v}^{\infty}\cdot\mathbf{e} + \boldsymbol{v}\cdot\nabla\mathbf{e}^{\infty} + \mathbf{e}^{\infty}\cdot\nabla\boldsymbol{v}^{\dagger} + \nabla\boldsymbol{v}\cdot\mathbf{e}^{\infty}, \end{aligned} \tag{8}$$

where $\mathbf{e}^{\infty}$ is the rate of strain tensor for the undisturbed flow, $\overset{\triangle}{\mathbf{e}}$ is the lower convected derivative of $\mathbf{e}$ (also known as the Rivlin-Eriksen tensor), $\mathbf{w}$ is the 'interaction tensor' (arising from the interaction between background flow and disturbance field), and $\overset{\triangle}{\mathbf{w}}$ is its lower convected derivative.

The above equations are non-dimensionalized using a, κv_m, $\mu\kappa v_m/a$ as the characteristic scales for length, velocity, and pressure, respectively. The definitions of these dimensional parameters a (particle size), $\kappa = a/2w$, and v_m (maximum flow velocity) are consistent with the communication article.

The boundary conditions of the disturbance flow field are

$$\boldsymbol{v} = \boldsymbol{v}_{\theta} + \boldsymbol{\Omega}_s \times \boldsymbol{r} - \boldsymbol{v}^{\infty} \quad \text{at } r = 1, \tag{9a}$$

$$\boldsymbol{v} = 0 \quad \text{at walls}, \tag{9b}$$

$$\boldsymbol{v} \to \boldsymbol{0} \quad \text{as } \{y, z\} \to \infty. \tag{9c}$$

Here, the walls are located at $x = -s/\kappa$ and $x = (1-s)/\kappa$, and $\boldsymbol{v}_{\theta}$ represents a prescribed tangential surface velocity of the spherical microswimmer, which determines its swimming motion. In Sect. III we will introduce we will introduce for $\boldsymbol{v}_{\theta}$ the surface flow field of a neutral squirmer, which serves as a model microswimmer [2, 3].

II. PERTURBATION EXPANSION

We find the viscoelastic lift or migration velocities at $O(\mathrm{Wi})$ using a regular perturbation expansion. For small values of Wi, the disturbance field variables are expanded as:

$$\xi = \xi_0 + \mathrm{Wi}\,\xi_1 + \cdots\,. \tag{10}$$

Here, ξ is a generic field variable which represents velocity $(\boldsymbol{v})$, pressure (p), translational $(\boldsymbol{U}_p)$ and angular particle velocity $(\boldsymbol{\Omega}_p)$. We substitute (10) in the equations governing the disturbance field

(7) and obtain the problem at $O(1)$ (*i.e.* Stokes problem) as

$$\left.\begin{aligned} \nabla\cdot\boldsymbol{v}_0 &= 0,\\ \nabla^2\boldsymbol{v}_0 - \nabla p_0 &= \mathbf{0},\\ \boldsymbol{v}_0 &= \boldsymbol{v}_\theta + \boldsymbol{\Omega}_{p0}\times\boldsymbol{r} - \boldsymbol{v}_0^\infty \quad \text{at } r=1,\\ \boldsymbol{v}_0 &= 0 \quad \text{at walls},\\ \boldsymbol{v}_0 &\to \mathbf{0} \quad \text{as } \{y,z\}\to\infty. \end{aligned}\right\} \tag{11}$$

and at $O(\text{Wi})$ as:

$$\left.\begin{aligned} \nabla\cdot\boldsymbol{v}_1 &= 0,\\ \nabla^2\boldsymbol{v}_1 - \nabla p_1 &= -\nabla\cdot\mathbf{s}_0,\\ \boldsymbol{v}_1 &= \boldsymbol{U}_{p1} + \boldsymbol{\Omega}_{p1}\times\boldsymbol{r} \quad \text{at } r=1,\\ \boldsymbol{v}_1 &= \mathbf{0} \quad \text{at walls},\\ \boldsymbol{v}_1 &\to \mathbf{0} \quad \text{as } \{y,z\}\to\infty. \end{aligned}\right\} \tag{12}$$

In (11), $\boldsymbol{v}_0^\infty = \left(\alpha+\beta x+\gamma x^2\right)\boldsymbol{e}_z - \boldsymbol{U}_{p0}$, where $\boldsymbol{U}_{p0}$ is the particle velocity in leading order in the Stokes regime.

Ho and Leal [1], in their seminal work, used the reciprocal theorem to derive a volume integral expression for the lift velocity associated with the $O(\text{Wi})$ equations (12):

$$\mathcal{F} \equiv \text{Wi}\,\boldsymbol{U}_{p1}\cdot\boldsymbol{e}_x = -\frac{1}{6\pi}\text{Wi}\int_{V_f}\mathbf{s}_0 : \tilde{\nabla}\boldsymbol{v}^t\,\text{dV}. \tag{13}$$

The auxiliary or test field ($\boldsymbol{v}^t$, p^t) is associated with a sphere moving in the positive x-direction (towards the upper wall) with unit velocity in a quiescent fluid:

$$\boldsymbol{v}^t(\boldsymbol{r}) = \frac{3}{4}\left(\boldsymbol{e}_x + \frac{x\boldsymbol{r}}{r^2}\right)\frac{1}{r} + \frac{1}{4}\left(\boldsymbol{e}_x - \frac{3x\boldsymbol{r}}{r^2}\right)\frac{1}{r^3}. \tag{14}$$

The reciprocal theorem makes it relatively easy to find lift velocities at $O(\text{Wi})$, as we can solve the creeping flow problem (11) using well-established methods [4, 5] and directly substitute its solution in (13). In other words, we do not need to solve the $O(\text{Wi})$ problem (12) to obtain the $O(\text{Wi})$ lift.

III. VISCOELASTIC LIFT VELOCITY: SOURCE-DIPOLE SWIMMER

We now use expression (13) for evaluating the swimming lift of a source-dipole swimmer. We use the method of reflections to solve for (11). Assuming that the small particle is not too close to the walls (i.e. $s \gg \kappa$), the disturbance field ($\boldsymbol{v}_0$, p_0) is sought as successive reflections: $\xi = \xi^{(1)} + \xi^{(2)} + \cdots$. Here, $\xi^{(i)}$ represents the i^{th} reflection, where the odd reflections satisfy boundary conditions at the particle surface and even reflections satisfy the wall boundary conditions. Accounting for each successive pair of reflections increases the accuracy by $O(\kappa)$ [6]. Furthermore, it is shown by Choudhary et al. [7][p. 18] and Ho and Leal [1][p. 792] that wall effects do not add to the leading order (in κ) of the volume integral. Thus, it suffices to include the first reflection and neglect wall effects.

We explicitly choose the axisymmetric neutral squirmer, which has the surface velocity field $\boldsymbol{v}_\theta = B_1\sin\theta\boldsymbol{e}_\theta$, where θ is the polar angle and $\boldsymbol{e}_\theta$ the corresponding base vector. The swimming velocity is directly related to this squirmer coefficient: $v_s = 2B_1/3$ [8, 9]. The solution to the $O(1)$ Stokes problem (11) can be divided in swimming and passive disturbances. From the squirmer model [2], we obtain:

$$\begin{aligned} \boldsymbol{v}_0^{\text{swim}} &= \frac{\tilde{v}_s\boldsymbol{p}}{2r^3}\cdot\left[\frac{3\boldsymbol{r}\boldsymbol{r}}{r^2} - \mathbf{I}\right]\\ &= \frac{\tilde{v}_s}{2r^3}\left[\cos\psi\left(\frac{3z\boldsymbol{r}}{r^2} - \boldsymbol{e}_z\right) + \sin\psi\left(\frac{3x\boldsymbol{r}}{r^2} - \boldsymbol{e}_x\right)\right], \end{aligned} \tag{15}$$

where $\tilde{v}_s$ is non-dimensionalized *i.e.* $\tilde{v}_s = v_s/(v_m\kappa)$. Using Lamb's general solution [4], we obtain the passive disturbance

$$\begin{aligned}
\boldsymbol{v}_0^{\text{passive}} &= B\left(-\boldsymbol{e}_z + \frac{3z\boldsymbol{r}}{r^2}\right)\frac{1}{r^3} + D\frac{zx\boldsymbol{r}}{r^5} + E\left(x\boldsymbol{e}_z + z\boldsymbol{e}_x - \frac{5xz\boldsymbol{r}}{r^2}\right)\frac{1}{r^5} \\
&+ F\left(\boldsymbol{e}_z - \frac{2x^2\boldsymbol{e}_z + z\boldsymbol{r}}{r^2} + \frac{2xz\boldsymbol{e}_x}{r^2}\right)\frac{1}{r^3} + G\left(\boldsymbol{e}_z - \frac{5x^2\boldsymbol{e}_z + 10xz\boldsymbol{e}_x + 13z\boldsymbol{r}}{r^2} + \frac{75zx^2\boldsymbol{r}}{r^4}\right)\frac{1}{r^3} \\
&+ H\left(\boldsymbol{e}_z - \frac{5x^2\boldsymbol{e}_z + 10xz\boldsymbol{e}_x + 5z\boldsymbol{r}}{r^2} + \frac{35zx^2\boldsymbol{r}}{r^4}\right)\frac{1}{r^5}\,, \qquad (16)
\end{aligned}$$

where the coefficients are defined as:

$$B = \frac{\gamma}{15},\ D = -\frac{5\beta}{2},\ E = -\frac{\beta}{2},\ F = \frac{\gamma}{3},\ G = -\frac{7\gamma}{120},\ H = \frac{\gamma}{8}. \qquad (17)$$

The terms multiplying the coefficients B, D, E represent source-dipole, stresslet, and octupole singularities, respectively [5, 10]. The other disturbances (terms multiplying F, G, H) are further singularities in the multipole expansion, which arise due to the curvature γ in the background flow field together with the source dipole.

The tensor $\mathbf{e}^\infty$ is yet unknown for the Poiseuille flow of Eq. (5) in zeroth order of Wi. To calculate it, we note that the total velocity of the force-free swimmer in the Stokes regime is $\boldsymbol{U}_{p\,0} = \tilde{\boldsymbol{v}}_s + (\alpha + \gamma/3)\boldsymbol{e}_z$ (the second part is obtained by using Faxen's laws [5]). To complete the expression of $\boldsymbol{v}_0^\infty$, we substitute $\boldsymbol{U}_{p\,0}$ in (5), and obtain:

$$\boldsymbol{v}_0^\infty = (\beta x + \gamma x^2 - \gamma/3)\boldsymbol{e}_z - \tilde{\boldsymbol{v}}_s \qquad (18)$$

which gives $[\mathbf{e}^\infty]_{xz} = [\mathbf{e}^\infty]_{zx} = (\beta + 2x\gamma)/2$.

Now, we evaluate the volume integral in Eq. (13). Since the source-dipole field of the neutral swimmer decays quickly away from the swimmer ($\sim 1/r^3$), we can neglect the wall corrections in the volume integral of Eq. (13). In the context of an electrophoretic source-dipole disturbance, Choudhary et al. [11] showed that the error generated from this neglection is dispensable. Integrating over the infinite space, we obtain the swimming lift velocity in units of v_s as

$$\mathcal{F} = \text{Wi}\,[\,(5/9)\beta\gamma(1+3\delta)\bar{v}_m\kappa\ +\ (1/4)\beta(1+\delta)\cos\psi\,]\,, \qquad (19)$$

expressed in the co-moving frame of the swimmer. The first component is the passive lift velocity (identical to that obtained by Ho and Leal [1]), and the second component is the swimming-lift velocity that arises due to the source-dipole disturbance created by the neutral swimmer.

IV. VISCOELASTIC LIFT VELOCITY: FORCE-DIPOLE SWIMMER

As before, $\boldsymbol{v}_0$ is the combination of flow fields due to swimming and the passive disturbance. The latter is identical to (16); for the swimmer, we take the force-dipole field from the studies on flagellated microswimmers [12, 13], where $\mathcal{P}$ is the dipole strength normalized with $8\pi\mu a^2 v_s$:

$$\begin{aligned}
\boldsymbol{v}_0^{(1)swim} &= \mathcal{P}\tilde{v}_s\boldsymbol{r}\left[\frac{-1}{r^3} + 3\frac{(\boldsymbol{r}\cdot\boldsymbol{p})^2}{r^5}\right] \\
&= \mathcal{P}\tilde{v}_s\cos^2\psi\left(\frac{-\boldsymbol{r}}{r^3} + \frac{3z^2\boldsymbol{r}}{r^5}\right) + \mathcal{P}\tilde{v}_s\sin^2\psi\left(\frac{-\boldsymbol{r}}{r^3} + \frac{3x^2\boldsymbol{r}}{r^5}\right) + \mathcal{P}\tilde{v}_s\sin 2\psi\left(\frac{3xz\,\boldsymbol{r}}{r^5}\right). \qquad (20)
\end{aligned}$$

Substituting the above equation together with Eq. (16) into Eq. (13) and integrating over the infinite domain, we obtain (in the units of v_s):

$$\mathcal{F} = \text{Wi}\,[\,(5/9)\beta\gamma(1+3\delta)\bar{v}_m\kappa\ -\ (2/3)\mathcal{P}\gamma(1+3\delta)\sin 2\psi\,]\,. \qquad (21)$$

The second component is the additional swimming-lift velocity that will be experienced by the force-dipole swimmer. Note that it depends on the curvature γ of the Poiseuille flow.

V. INERTIAL LIFT VELOCITIES IN THE CHANNEL FRAME

Here we provide the final expressions of the swimming and passive lift velocities in the channel frame of reference, which is used in the communication article. The conversion requires a transformation of particle-wall distance s/κ to the channel x coordinate (see Fig. 1); for x in units of w we then have $s = (1+x)/2$.

1. Swimming lift: Neutral microswimmer
Using the definition of β (6) and $s = (1+x)/2$ in (19), yields:

$$\mathcal{F}_{\text{swim}} = -\text{Wi}(1+\delta)\, x \cos\psi. \tag{22}$$

2. Swimming lift: Pusher/puller microswimmer
Using the definition of γ (6) in (21), yields:

$$\mathcal{F}_{\text{swim}} = \text{Wi}(8/3)(1+3\delta)\, \mathcal{P}\kappa \sin 2\psi \tag{23}$$

3. Passive lift:
Using the definition of β, γ (6) and $s = (1+x)/2$ in the passive lift component, yields:

$$\mathcal{F}_{\text{passive}} = \text{Wi}(80/9)(1+3\delta)x\, \bar{v}_m \kappa^2. \tag{24}$$

VI. PARTICLE DRIFT AND ROTATION MODIFICATION

To calculate the viscoelastic modification to drift and rotational velocity of the swimmer, we use the following two test fields (respectively):

$$\boldsymbol{v}^t(\boldsymbol{r}) = \frac{3}{4}\left(\boldsymbol{e}_z + \frac{z\boldsymbol{r}}{r^2}\right)\frac{1}{r} + \frac{1}{4}\left(\boldsymbol{e}_z - \frac{3z\boldsymbol{r}}{r^2}\right)\frac{1}{r^3}. \tag{25}$$

$$\boldsymbol{v}^t(\boldsymbol{r}) = \frac{x\boldsymbol{e}_z - z\boldsymbol{e}_x}{r^3}. \tag{26}$$

A. Source-dipole swimmer

Substituting (25) in the volume integral (13), we obtain the drift modification as:

$$U_{drift} = \frac{1}{4}\text{Wi}\,\beta\, \tilde{v}_s(1+\delta)\sin\psi. \tag{27}$$

Substituting (26) in the volume integral for $O(\text{Wi})$ rotational correction: $-\frac{\text{Wi}}{8\pi}\int_{V_f} \mathbf{s}_0 : \tilde{\nabla}\boldsymbol{v}^t\, \text{dV}$, we obtain the rotation modification as:

$$\boldsymbol{\Omega}_1 = \frac{\text{Wi}}{2}\,\gamma\, \tilde{v}_s \sin\psi\, \boldsymbol{e}_y \tag{28}$$

B. Force-dipole swimmer

Similarly, for force-dipole swimmer, we obtain

$$U_{drift} = \frac{\text{Wi}}{12}\,\gamma\, \tilde{v}_s(1+\delta)(-3+5\cos\psi), \qquad \boldsymbol{\Omega}_1 = \frac{\text{Wi}}{2}\,\beta\, \tilde{v}_s(1+3\delta)\cos 2\psi. \tag{29}$$

Since these effects are an order of magnitude (in Wi) smaller than the flow speed and flow vorticity (respectively), they do not alter the swimmer dynamics.

VII. FURTHER RESULTS ON FOCUSING TIME

Fig. 2 shows the variation of focusing time with swimmer size and flow rate strength. For both source-dipole and force-dipole swimmers, the focusing time decreases with increase in κ and $\bar{v}_m$. Interestingly for a source-dipole swimmer, Fig. 2 (a) shows that for a relatively small size ($\kappa < 0.1$) the focusing time increases with flow rate in an intermediate range (approximately $5 < \bar{v}_m < 10$). This occurs because in this range of flow rate the swimmer has to escape the tumbling state to attain centerline focusing, as can be observed in Fig. 2 (c). As the flow rate further increases, the migration velocity also increases which results in reduction of focusing time.

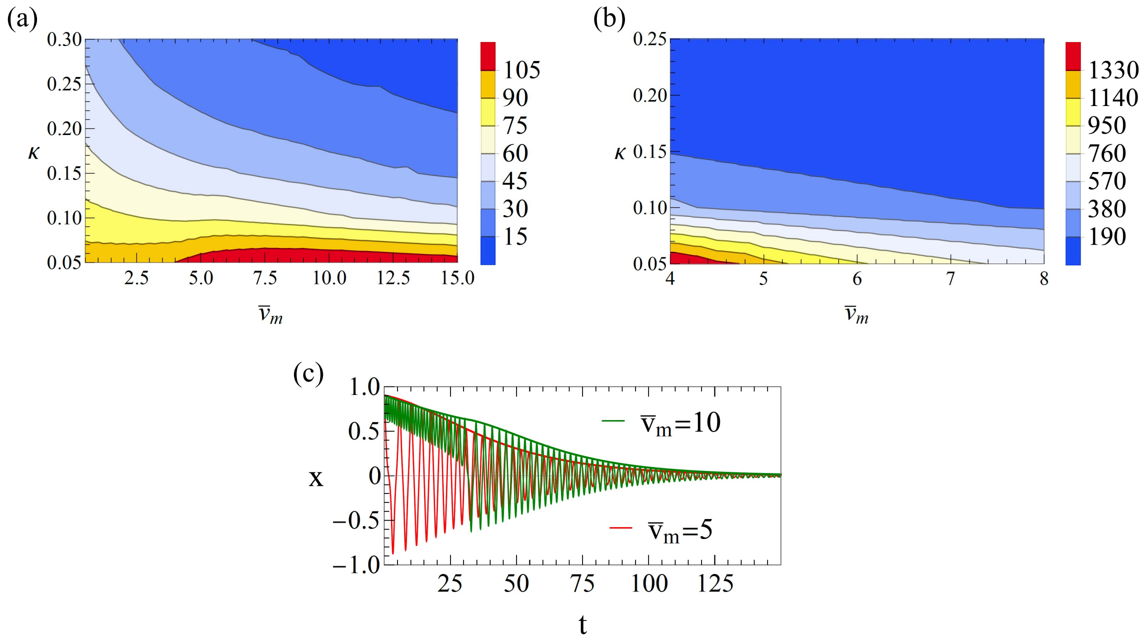

FIG. 2: (a) Contour plot for focusing time of a source-dipole swimmer that varies with κ and $\bar{v}_m$. (b) Contour plot for force-dipole swimmer for $\bar{v}_m > 4$. (c) Temporal trajectories of a source-dipole swimmer for $\kappa = 0.05$. Other parameters: $x_0 = 0.9$, $\psi_0 = 0$, Wi $= 0.1$.

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