Supplementary Information on "Long ranged order in a thermally driven system with temperature dependent interactions"

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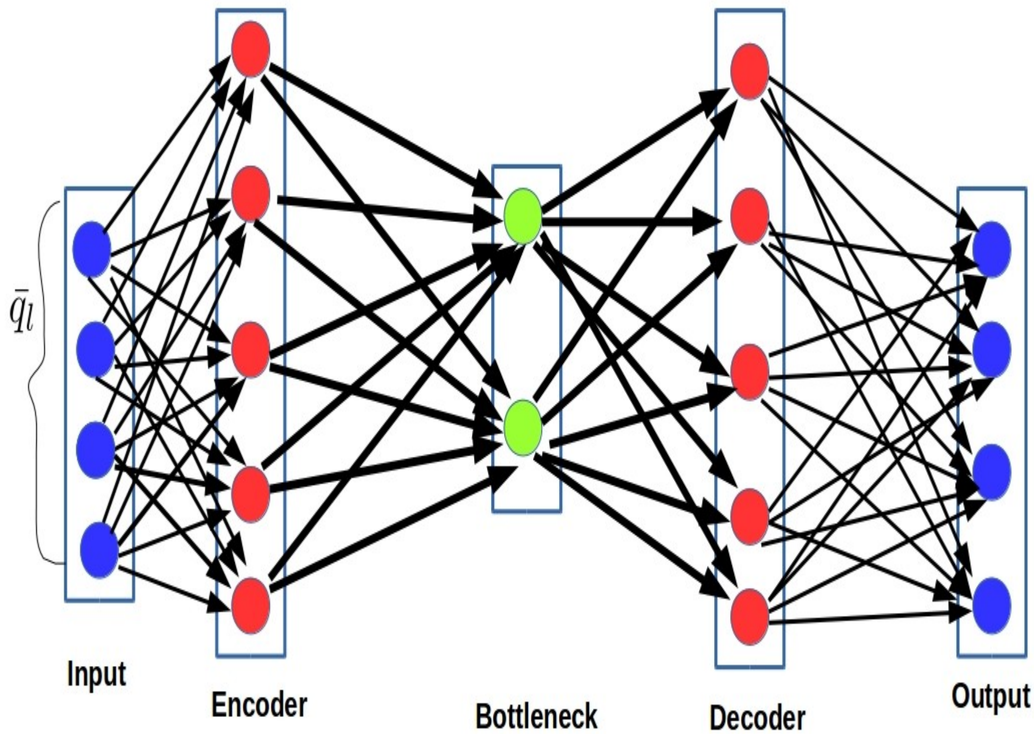
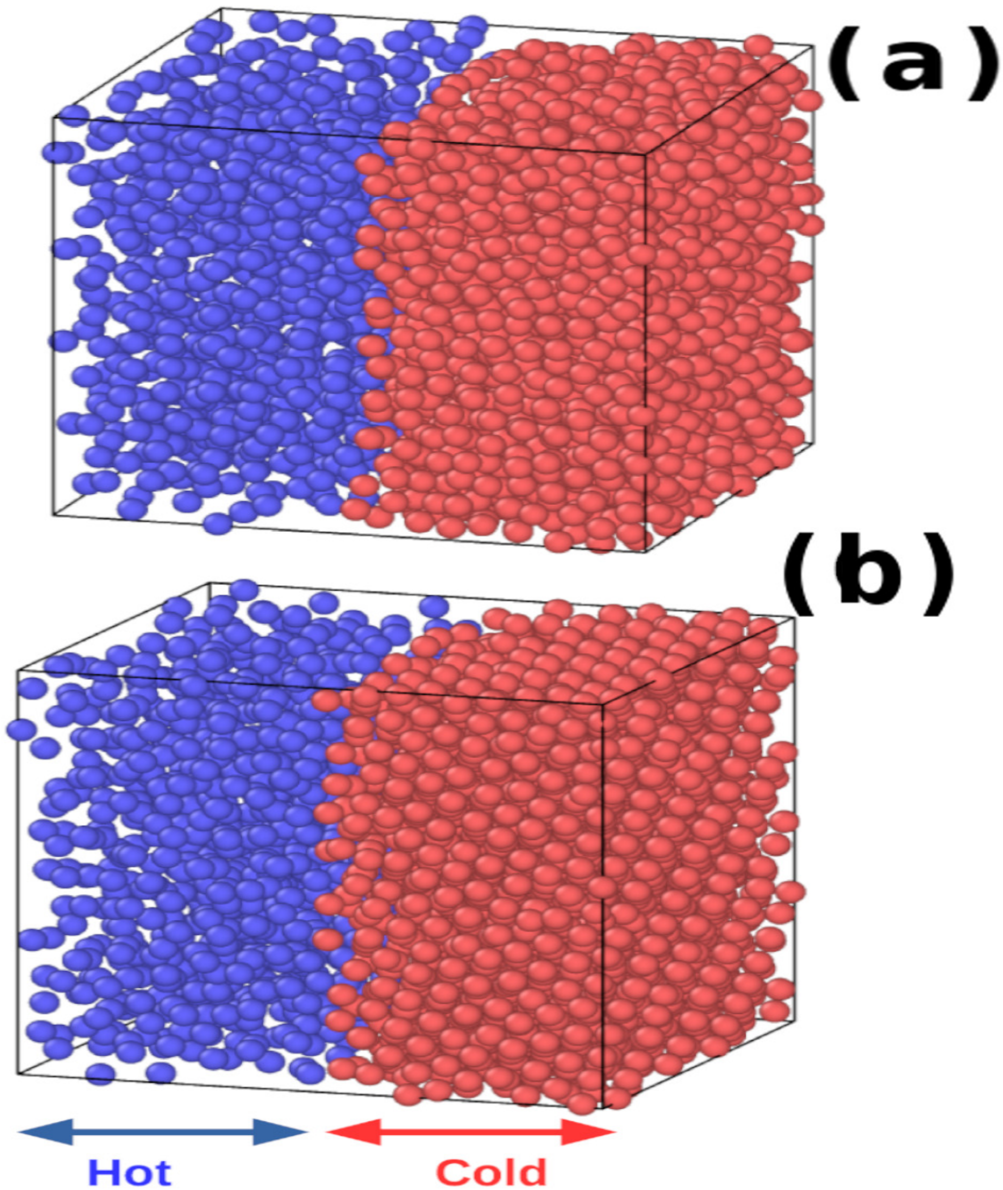
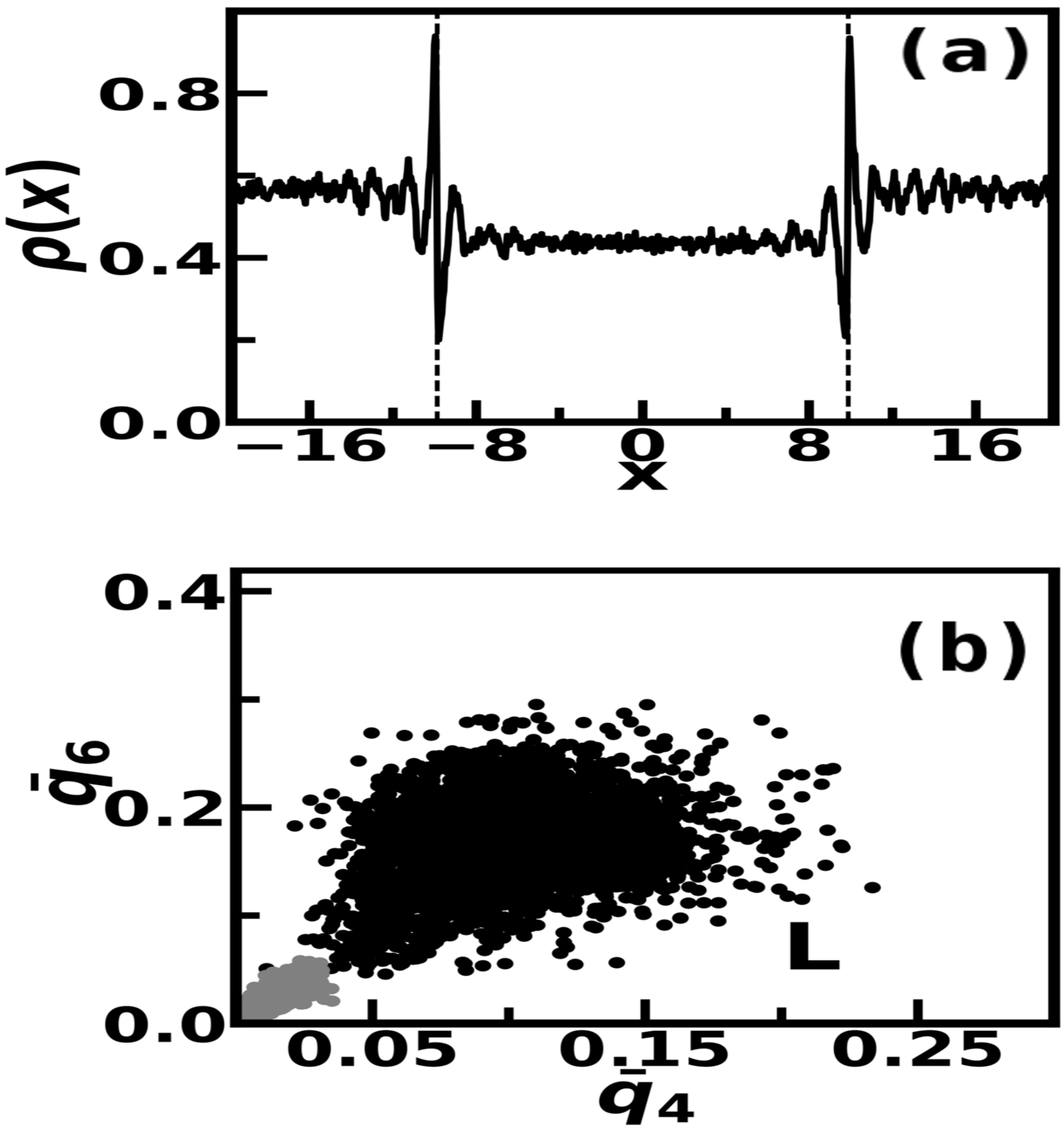


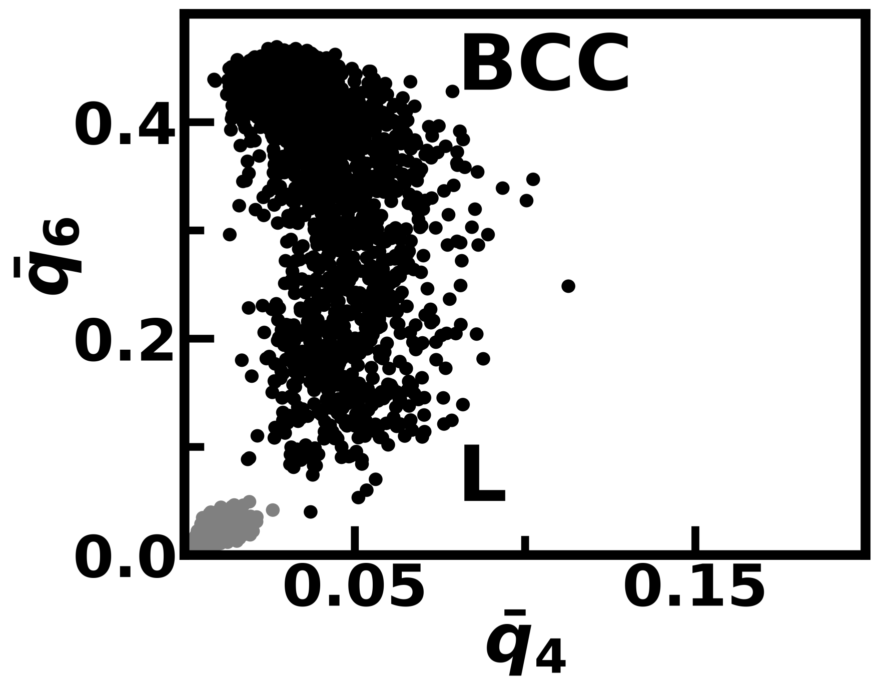
SI Fig S 1. A schematic diagram of neural network based auto encoder used in the unsupervised machine learning as in the Ref. [56]. An autoencoder is a neural network that is trained to perform the nonlinear identity mapping. Here the network inputs are reproduced at the output layer. Our network is consisting of two parts. 1) An encoder network, which performs a nonlinear projection of the input data onto a low-dimensional subspace; 2) a decoder network that attempts to reconstruct the input data from the low-dimensional projection. We use $Q_l = (\bar{q}_l)$ $(l = 1, .., 8)$ as our input set. The number of input and output nodes are same here, d=8. Output is specified by the dimensionality of the input vectors. Here, we set the number of nodes in the encoder and decoder hidden layers to 10d and use a hyperbolic tangent as the activation function, whereas a linear activation function is used for bottle neck and output layers as mentioned in the reference. The bottleneck layer contains the low-dimensional projection to be learned by the encoder $Y(i)$. We vary number of nodes in the bottleneck, train the network, calculate Mean Square Error(MSE) and plot as a function of node number. We obtain optimum node number by detecting elbow of the curve called L-method proposed by Salvador and Chan. The dimension for the lower dimension subspace comes out c=2 for our case. We train the system and project the input in lower dimension in the bottleneck in two dimension. Then we use Gaussian Mixture Model(GMM) with Bayesian Information Criteria (BIC) followed by entropy based clustering method to cluster the data. It is a probabilistic method that assumes that the observed data are generated from a mixture of a finite number of Gaussian distributions. Applying GMM in our data set we find that each data point(each particle) is a member of every cluster with some membership percentage, and we assign that point to a particular cluster with maximum percentage.



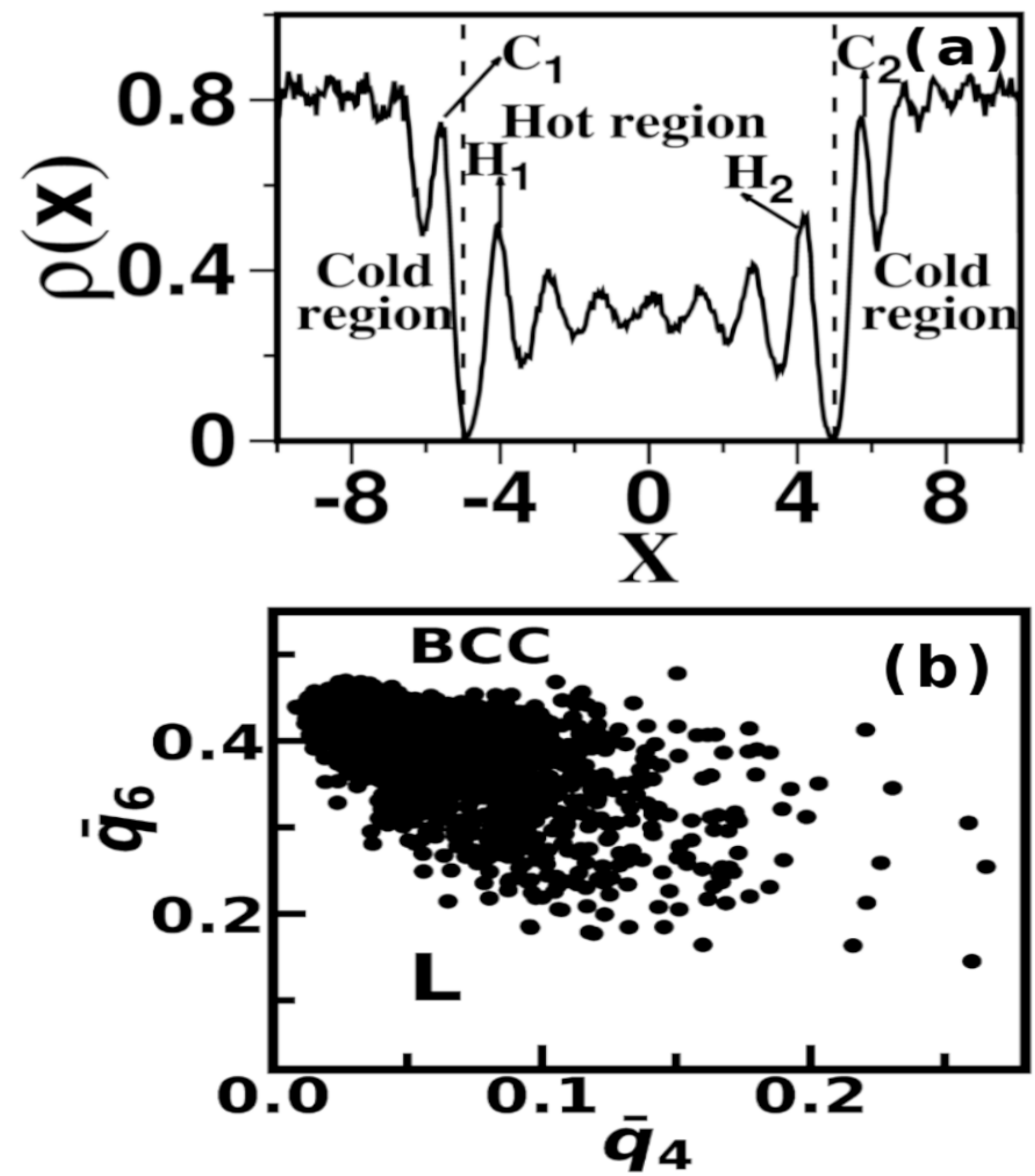
SI Fig S 2. Snapshots of steady state configurations of half of the box ($0 < x < L_x/2,$) for two different times. Blue particles have interaction parameters corresponding to hot temperature T_H and red particles have parameters corresponding to cold temperature T_c. Panel (a) is shown for lower time ($6\tau_H$) configuration (b) higher time ($540\tau_H$) configuration . We observe ordering of red particles in the cold region at large time.



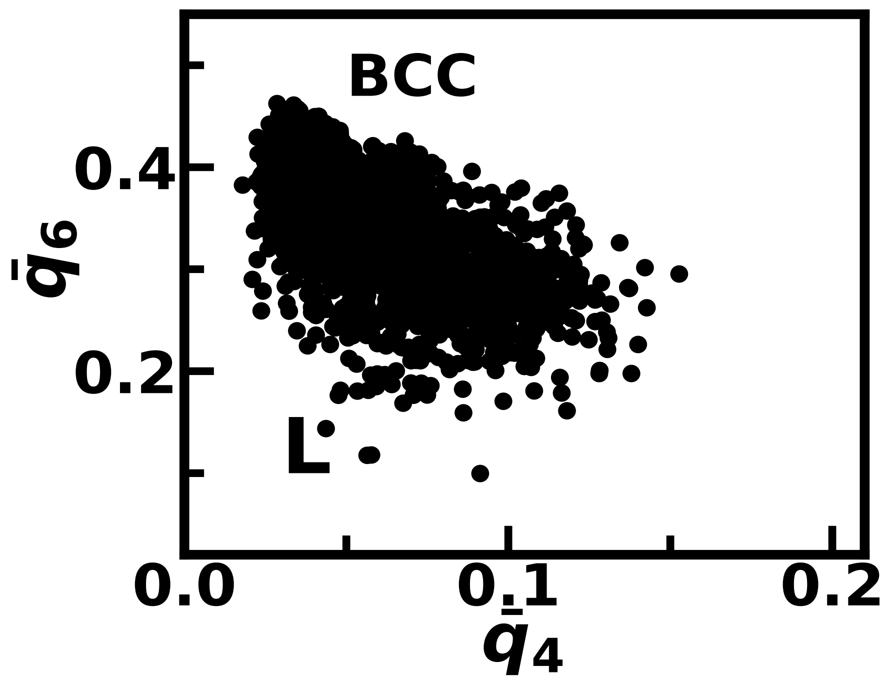
SI Fig S 3. Data for the system where we keep interaction parameters same in both region as for hot temperature but maintain different temperature in different region(case 2). We show (a) Density profile $\rho(x)$. (b) Scatter plot in $\bar{q}_4 - \bar{q}_6$ plane at large time in cold region(black dot) and hot region(grey dot). Ordering is liquid like in both regions.



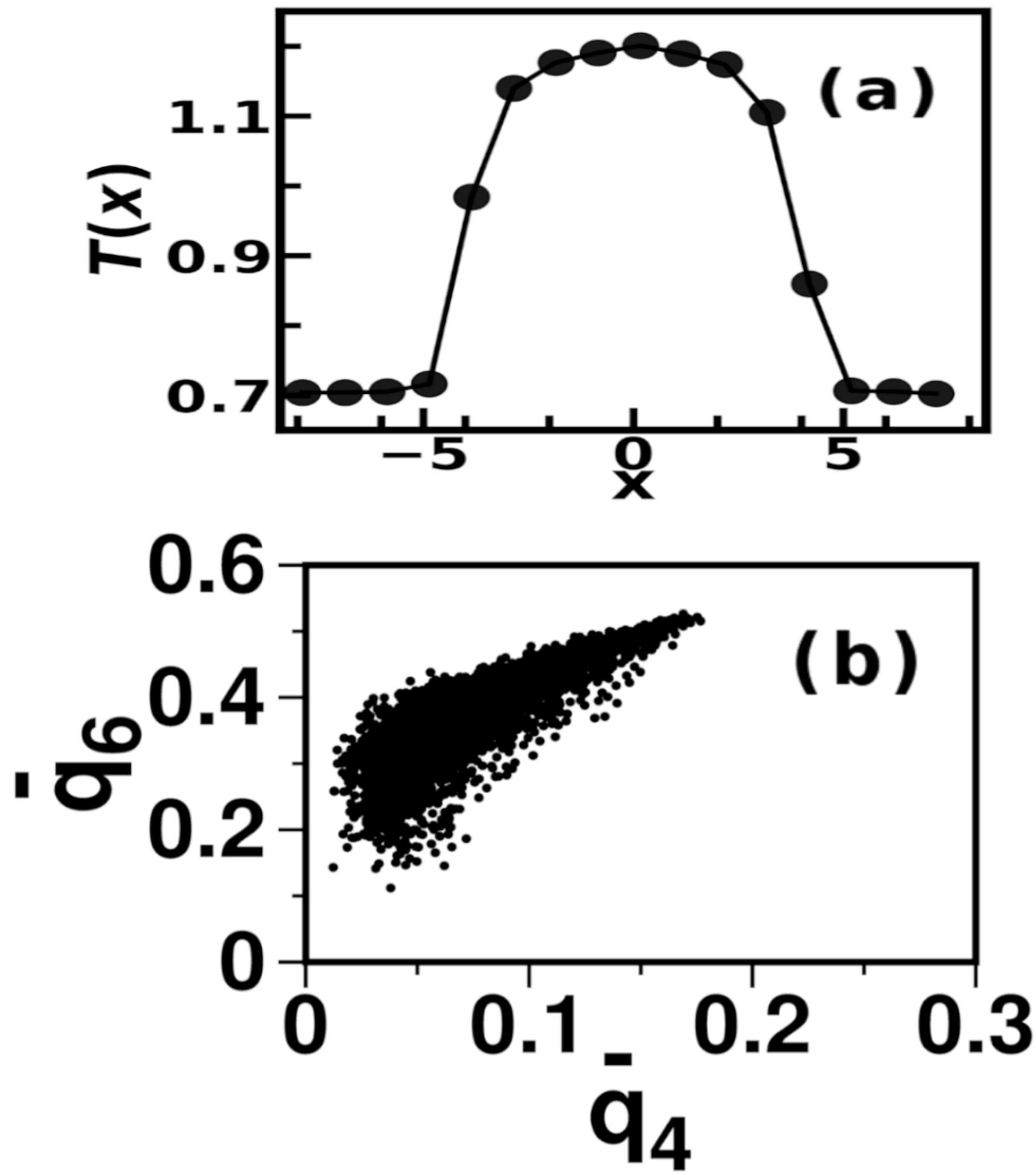
SI Fig S 4. Scatter plot in $\bar{q}_4 - \bar{q}_6$ plane at large time in cold region(black) and hot region(grey dot) for the system where we take diffusivity as per the local temperature without considering the temperature dependent viscosity (case 3). The interaction parameters are temperature dependent.



SI Fig S 5. Data for the system with lower number(4000) of particles. (a) Density profile $\rho(x)$. We observe that the density profile shows larger fluctuations compare to the large system case(main text). (b) Scatter plot in $\bar{q}_4 - \bar{q}_6$ plane at large time in cold region(black dot). We observe crystal order at large time as in larger system reported in the text.



SI Fig S 6. Results from MD simulations: Scatter plot in $\bar{q}_4 - \bar{q}_6$ plane at large time in cold region. Ordering is same as the BD calculations in the main text.



SI Fig S 7. Results from MD simulation of a Lennard Jones(LJ) system. Panel (a) shows temperature profile along x direction. (b) Scatter plot in $\bar{q}_4 - \bar{q}_6$ plane at large time in cold region. Here we observe ordering (coexistence of HCP and FCC phase with liquid).