Avalanche dynamics in sheared athermal particle packings occurs via localized bursts predicted by unstable linear response: Electronic Supplementary Information

1 D^2_{min} Computation

As described in the main text, the D^2_{min} is a measure of the local non-affine deformation between two configurations, $\vec{X}_1$ and $\vec{X}_2$:

$$D^2_{min,i}\left(\vec{X}_1,\vec{X}_2\right) = \sum_{j\in\partial i}\left(\vec{X}_{2i}-\vec{X}_{2j}-\mathbf{S}_i\left(\vec{X}_{2i}-\vec{X}_{2j}\right)\right)^2, \quad (1)$$

where ∂i is the set of particles that are defines as the neighborhood of particle i which we have determined in the main text via the distance from particle i in both configurations and S_i is the best-fit affine transformation around particle i such that any other transformation in place of S_i results in a larger value for the D^2 measure. This best-fit affine transformation can be found by calculating

$$X_{i,\alpha\beta} = \sum_{j\in\partial i}\left(\vec{X}_{2i}-\vec{X}_{2j}\right)_\alpha\left(\vec{X}_{1i}-\vec{X}_{1j}\right)_\beta \quad (2)$$

and

$$Y_{i,\alpha\beta} = \sum_{j\in\partial i}\left(\vec{X}_{1i}-\vec{X}_{1j}\right)_\alpha\left(\vec{X}_{1i}-\vec{X}_{1j}\right)_\beta. \quad (3)$$

The best-fit affine transformation is given by $S_i = X_iY_i^{-1}$ [1].

2 Cluster Mutual Information

In ESI Fig. 2, we show the relative mutual information between the clusters identified at different size thresholds where the x- and y- axes indicate the volume thresholds of the cluster sets to be compared, and the color indicates the relative mutual information. (1.m.8) We highlight using black lines the contours which indicate the size thresholds where this mutual information decreases below 95%. Note that the relative mutual information always takes a value between 0 and 1.

As discussed in the main text, we would like to identify regions where the confidence interval associated with the maximal mutual information changes rapidly and reaches a broad maximum. A challenge is that the information entropy, H, of the set of clusters identified at a particular threshold increases with increasing threshold as shown in ESI Fig 1 (B). This information entropy is found by computing the mutual information of a field with itself or computing the shannon entropy:

$$H(I) = -\sum_{x\in[I]} p_x \log_2(p_x). \quad (4)$$

To find an balance between a wide confidence interval for the relative mutual information while still maintaining low information entropy – with low information entropy at smaller threshold values – we study the ratio th_v^U/th_v^L between the value of the threshold associated with the upper curve th_v^U, highlighted in magenta in Fig S1(B), and the value of the threshold associated with the lower curve th_v^L, highlighted in green in Fig S1(B). This is equivalent to the width of the confidence interval for the relative mutual information on a log scale. This ratio is shown in the main text in Fig 2(C).

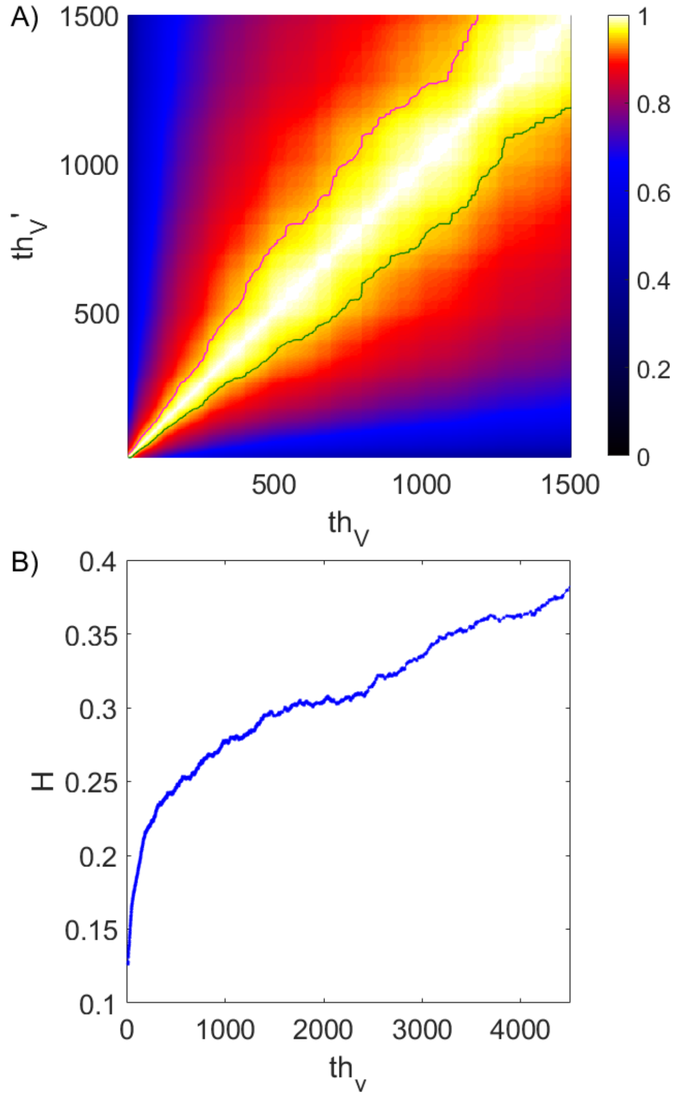

Fig. 1 A) The colorscale represents the relative mutual information between the clusters identified at one volume threshold th_v, x-axis, compared to those at a different volume threshold th'_v, y-axis. For each volume threshold th_v, we identify two other volume thresholds, th_v^U and th_v^L, at which the relative mutual information of the clusters at th_v compared to the clusters th_v^U and th_v^L decreases to 95%. We order these thresholds such that $th_v^L < th_v < th_v^U$. The contour generated by this upper threshold, th_v^U, as a function of volume is plotted with the pink curve, while the contour generated by the lower threshold, th_v^L, is plotted with the green curve. B) The information entropy of the sets of clusters identified at each volume threshold th_v.

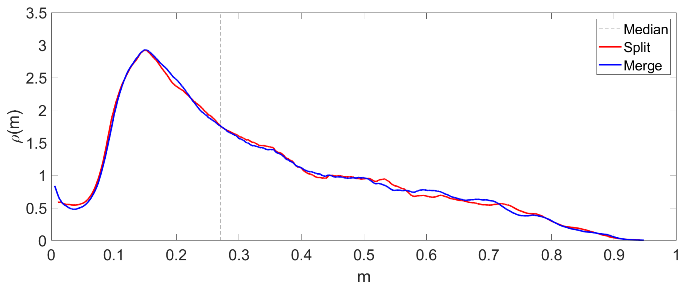

Fig. 2 The probability distribution ρ of the relative mutual information between soft spots identified in each time frame separated by the time window of 0.1 natural time units or less averaged over 180 avalanches. These overlaps are separated into cases where the soft spot splits into two soft spots (Split) and where two soft spots merge into one (Merge) which have nearly identical distributions.

3 Soft Spot Temporal Overlap

By computing the smooth derivative of the cumulative distribution of the relative mutual information between soft spots, m, shown in Fig. 8, we show the probability distribution of m in ESI Fig. 2. This smooth derivative has been calculated at each point in the cumulative distribution by selecting all points with relative mutual information within 0.03 of the point in question and computing the best fit slope.

One possible interpretation of these data is that there is a peak of low-information m-values centered around $m \approx 0.15$ which corresponds to spots that don't overlap much and should not be merged together while there is a broader, nearly flat distribution of higher m-values that correspond to soft spots that should be identified and merged together. The "cusp" between these two distributions could be estimated as the black dashed line, which happens to be the median and is the value we chose.

Notes and references

1 M. Falk and J. Langer, *Physical Review E*, 1998, **57**, 7192.