

Supplementary Material for

A Rayleigh-Bénard Convection Instability Analog in Vibrated Gas-Fluidized Granular Particles

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Table S1 lists the detailed parameters used in the base case simulation as shown in Fig. 4 (a). The Schaeffer model¹ that couples the frictional solids pressure formulation of Syamlal et al.¹ and the frictional solids viscosity formulation of Schaeffer² was used to account for the solids stress due to frictional contacts. The key input parameters of this model are the packing limit and the angle of internal friction. This model considers both particle-particle and particle-wall interactions. The Wen-Yu drag law³ was used to account for the momentum exchange between the gas phase and the solids phase.

Table S1. Detailed parameters used in the base case simulation as shown in Fig. 4 (a)

Quantity	Value
Bed width, mm	200
Bed depth, mm	10
Bed height, mm	500
CFD mesh size, mm	5
Gas viscosity, Pa·s	1.82×10^{-5}
Gas density, kg/m ³	1.2
Initial packing fraction	0.58
Initial packing height, mm	50
Packing limit	0.58
Particle size, mm	1.70
Particle density, kg/m ³	2500
U_{mf} , m/s	0.91
Angle of internal friction, °	30
Restitution coefficient	0.9
Vibration frequency, Hz	35
Vibration amplitude, mm	0.5
Drag law model	Wen-Yu ³
Viscous stress model	Lun et al. ⁴
Frictional stress model	Schaeffer model ¹
Inlet boundary condition	Fixed velocity, 0.82 m/s
Outlet boundary condition	Atmospheric pressure No slip for gas phase;
Wall boundary condition	Partial slip for solids phase: restitution coefficient of 0.8 and specularity coefficient of 0.4
Time step, s	1×10^{-4}

Movie S1 shows videos of the granular RBC analog reported here. Fig. S1 shows changes in the time-averaged particle velocity field when the normalized gas velocity is varied, but all other conditions are kept the same as in Fig. 1. Fig. S2 shows changes in the time-averaged particle velocity field when the vibration frequency is varied, but all other conditions are kept the same as in Fig. 1. Fig. S3 shows changes in the time-averaged particle velocity field when the vibration amplitude is varied, but all other conditions are kept the same as in Fig. 1. Fig. S4 shows that convection cells are formed when particle properties are varied. Fig. S5 shows the time-averaged vertical gas velocity in the simulation shown in Fig. 4 (a).

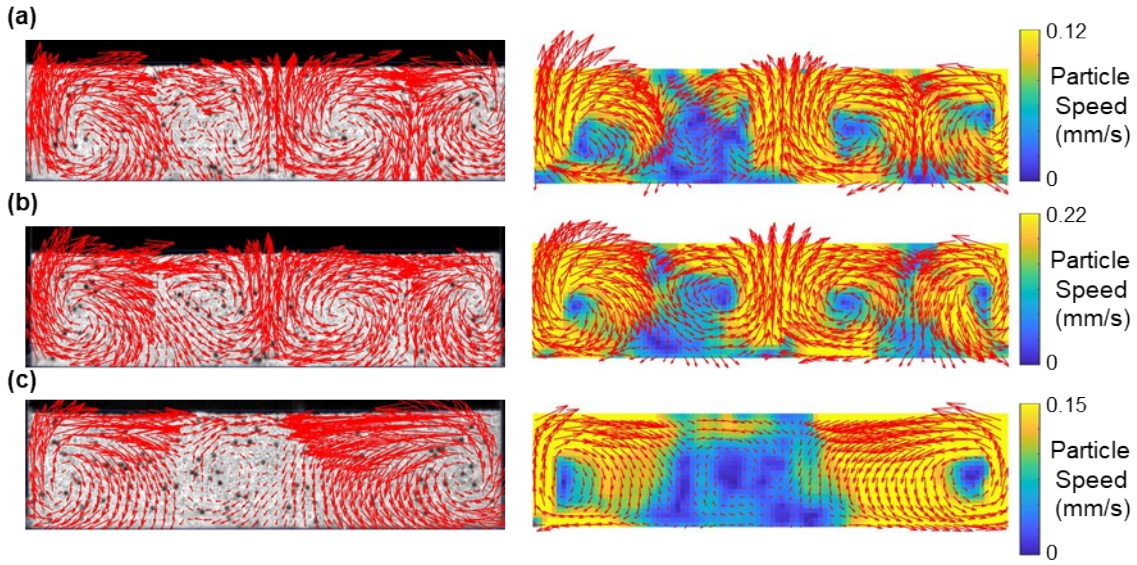


Fig. S1 Convection patterns of granular particles subjected to vertical vibration and gas flow in a pseudo-2D system (red arrow directions refer to velocity direction and lengths refer to speed). Vibration conditions: $f = 30$ Hz, $A = 0.25$ mm. Gas flow: $U/U_{mf} =$ (a) 0.78, (b) 0.81, and (c) 0.86. Particles: glass beads, $d_p = 1.7 \pm 0.15$ mm, $\rho_p = 2500$ kg/m³.

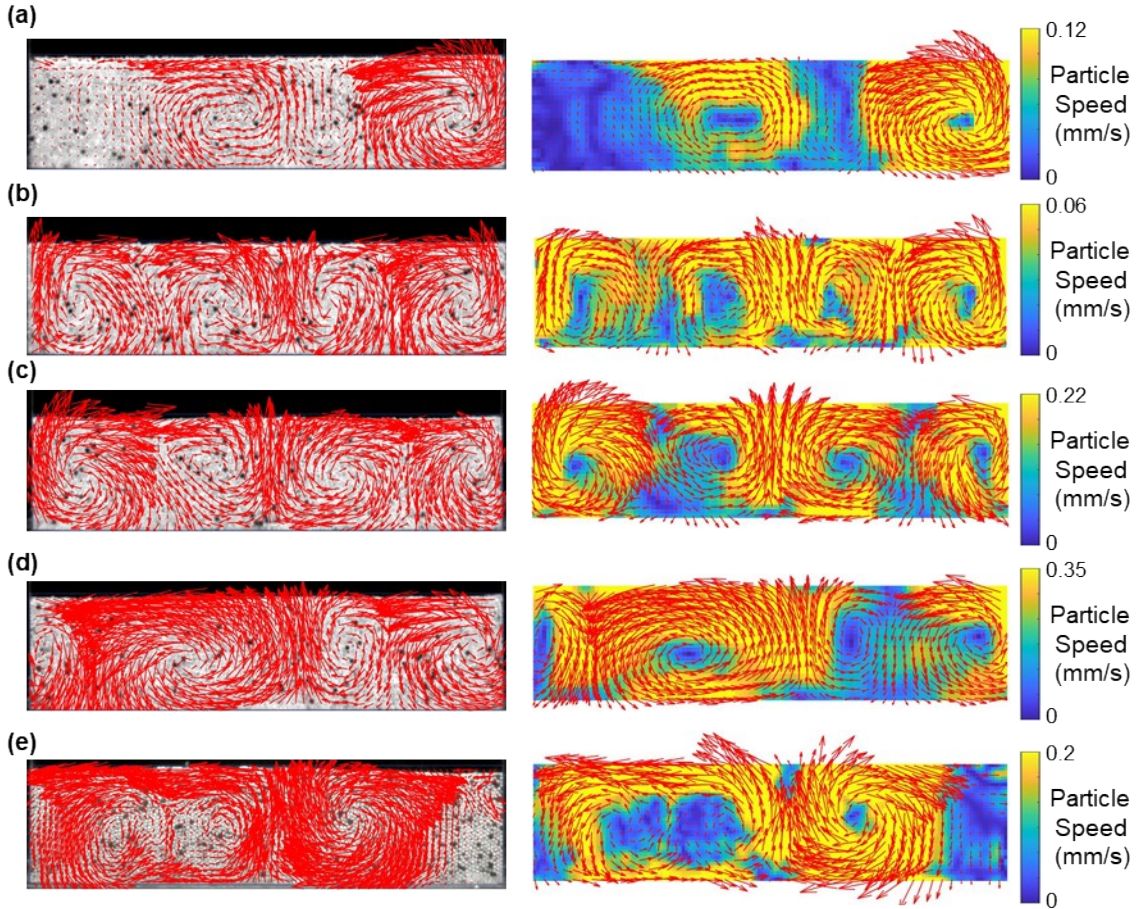


Fig. S2 Convection patterns of granular particles subjected to vertical vibration and gas flow in a pseudo-2D system (red arrow directions refer to velocity direction and lengths refer to speed). Vibration conditions: $f =$ (a) 20 Hz, (b) 25 Hz, (c) 30 Hz, (d) 35 Hz, and (e) 40 Hz, $A = 0.25$ mm. Gas flow: $U/U_{mf} = 0.81$. Particles: glass beads, $d_p = 1.7 \pm 0.15$ mm, $\rho_p = 2500$ kg/m³.

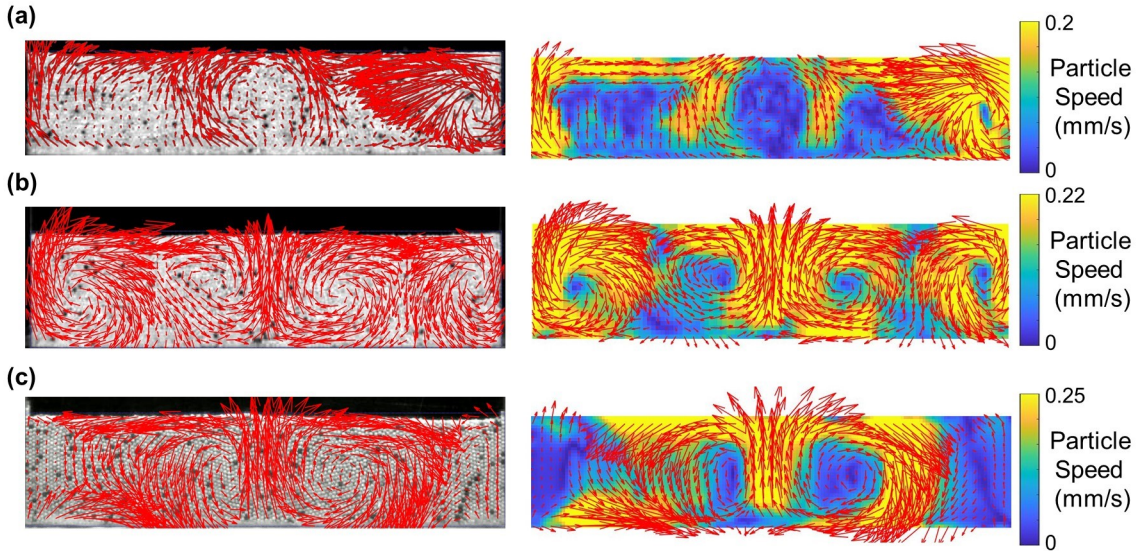


Fig. S3 Convection patterns of granular particles subjected to vertical vibration and gas flow in a pseudo-2D system (red arrow directions refer to velocity direction and lengths refer to speed). Vibration conditions: $f = 30$ Hz, $A =$ (a) 0.15 mm, (b) 0.25 mm, and (c) 0.5 mm. Gas flow: $U/U_{mf} = 0.81$. Particles: glass beads, $d_p = 1.7 \pm 0.15$ mm, $\rho_p = 2500$ kg/m³.

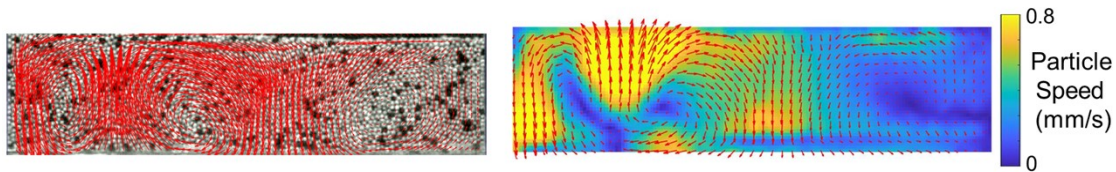


Fig. S4 Convection patterns of granular particles subjected to vertical vibration and gas flow in a pseudo-2D system (red arrow directions refer to velocity direction and lengths refer to speed). Vibration conditions: $f = 5$ Hz, $A = 12$ mm. Gas flow: $U/U_{mf} = 0.81$. Particles: mustard seeds, $d_p = 2.2 \pm 0.35$ mm, $\rho_p = 1090$ kg/m³.

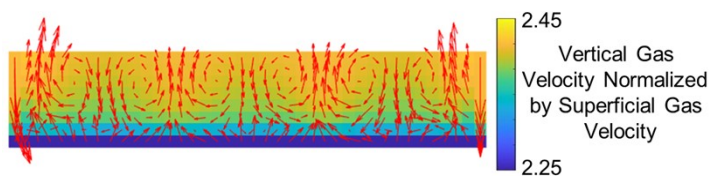


Fig. S5 TFM simulations of convection (red arrows) of particles subject to vertical vibration and gas flow in a pseudo-2D system. Colors: time-averaged vertical gas velocity normalized by superficial gas velocity. Same conditions as Fig. 4 (a).

References:

- 1 M. Syamlal, W. Rogers and T. J. O'Brien, *MFLX documentation theory guide*, USDOE Morgantown Energy Technology Center, WV (United States), 1993.
- 2 D. G. Schaeffer, *Journal of Differential Equations*, 1987, **66**, 19.
- 3 C. Y. Wen and Y. H. Yu, *AIChE Journal*, 1966, **12**, 610–612.
- 4 C. K. K. Lun, S. B. Savage, D. J. Jeffrey and N. Chepurniy, *Journal of Fluid Mechanics*, 1984, **140**, 223–256.