

## Supporting Information

### Ultra-low thermal conductivity in B<sub>2</sub>O<sub>3</sub> composited SiGe bulk with enhanced thermoelectric performance at medium temperature region

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#### 1. The Lorentz calculation

Because the charge carriers transport both heat and charge,  $\kappa_e$  is commonly estimated using the measured  $\sigma$  by the Wiedemann-Franz law:  $\kappa_e = L\sigma T$ , where  $L$  is the Lorentz number. Once  $\kappa_e$  is known,  $\kappa_l$  is computed by subtracting the  $\kappa_e$  from the total thermal conductivity,  $\kappa = \kappa_e + \kappa_l$ .

For a single parabolic band,  $L$  and  $S$  are both functions of reduced chemical potential ( $\eta$ ) and carrier scattering factor ( $\lambda$ ) only, the value of  $L$  is calculated according to the following formula:

$$L = \left(\frac{k_B}{e}\right)^2 \frac{(1 + \lambda)(3 + \lambda)F_{\lambda}(\eta)F_{\lambda+2}(\eta) - (2 + \lambda)^2 F_{\lambda+1}(\eta)^2}{(1 + \lambda)^2 F_{\lambda}(\eta)^2}$$

$$S = \frac{k_B}{e} \left( \frac{(2 + \lambda)F_{\lambda+1}(\eta)}{(1 + \lambda)F_{\lambda}(\eta)} - \eta \right)$$

Where  $F_j(\eta)$  represents the Fermi integral,

$$F_j(\eta) = \int_0^{\infty} \frac{\epsilon^j d\epsilon}{1 + \text{Exp}[\epsilon - \eta]}$$

The Lorentz calculation values of different B<sub>2</sub>O<sub>3</sub> adding amounts are shown in Fig. S1.

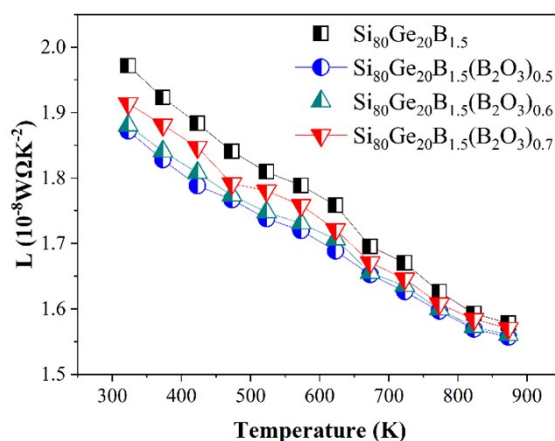


Fig. S1 Lorentz calculated value.

## 2. Effect of B<sub>2</sub>O<sub>3</sub> on crystallization

In the main text, it can be seen from XRD pattern, the stronger peaks intensities with the increase of the B<sub>2</sub>O<sub>3</sub> content indicate that adding of B<sub>2</sub>O<sub>3</sub> promotes the crystallization of SiGe. In addition to XRD characterization, we prepared samples without and added with B<sub>2</sub>O<sub>3</sub> with the same sintering method and sintering time, and then characterized by SEM. As shown in Fig. S1(a), many holes and poor compactness in the sample without B<sub>2</sub>O<sub>3</sub> are disclosed under the premise of the same preparation process. Figure S1(b) shows the sample added with B<sub>2</sub>O<sub>3</sub>, which has good porosity and compactness, further illustrating that B<sub>2</sub>O<sub>3</sub> can promote crystallization.

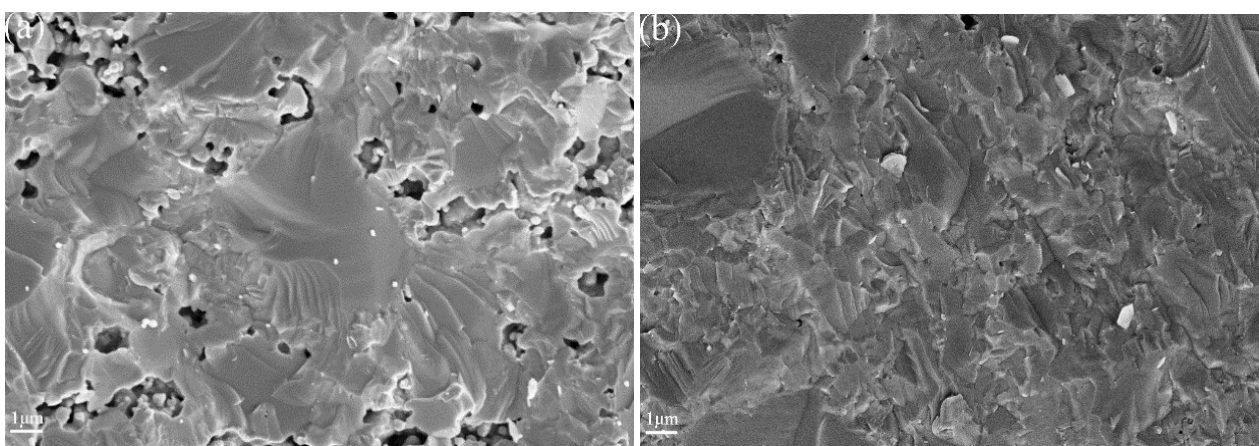


Fig. S2 (a) The SEM images of the sample without B<sub>2</sub>O<sub>3</sub>, (b) the SEM images of the sample added with B<sub>2</sub>O<sub>3</sub>

## 3. The EPMA measurement

EPMA works by bombarding micro-volume samples with focused electron beams and collecting X-rays emitted by various elements. Since the wavelengths of X-rays are characteristic of the emitting material, the sample components can be easily identified by recording wavelength dispersion spectra.

Figure. S3 is the EPMA test of Si<sub>80</sub>Ge<sub>20</sub>B<sub>1.5</sub>(B<sub>2</sub>O<sub>3</sub>)<sub>0.6</sub> sample. It can be seen from the figure that the composition of point 1 and point 2 is different, and it can be judged that there are two phases. In addition, the sample was tested by surface scanning, and it was found that B element was distributed sporadically. It was inferred that it was probably a nano-second-phase containing B<sub>2</sub>O<sub>3</sub>, which needed to be further confirmed by TEM analysis.

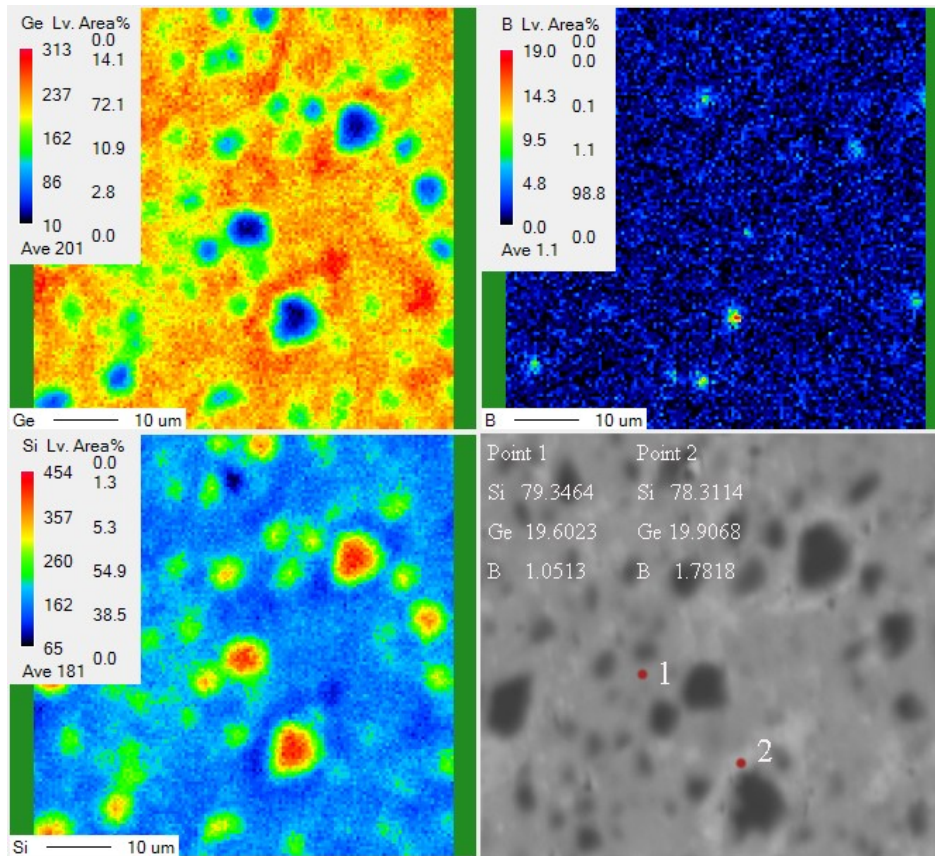


Fig. S3 The EPMA measurement of  $\text{Si}_{80}\text{Ge}_{20}\text{B}_{1.5}(\text{B}_2\text{O}_3)_{0.6}$  Sample

#### 4. The TEM observation

We have carried out TEM analysis in other different areas, and we can also find micropores, nano-second-phase, it is proved from the side that the micropores and the nano-second-phase are uniformly distributed.

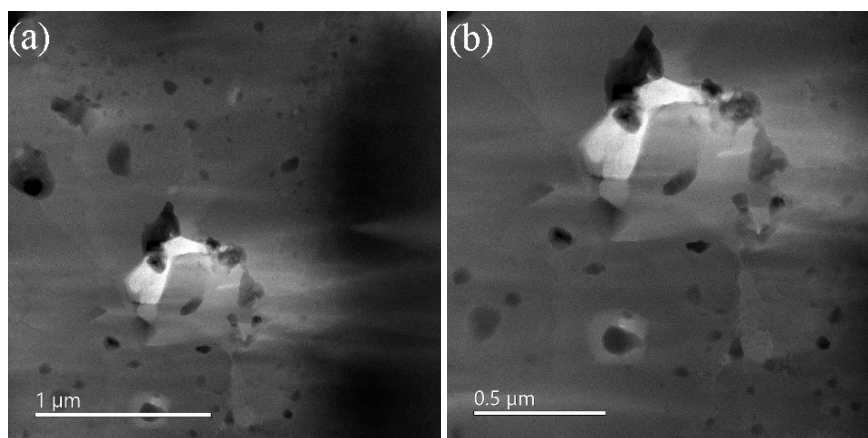


Fig. S4 TEM image of the micropores and nano-second-phase.

## 5. Fitting calculation of lattice thermal conductivity

We use the Callaway model to calculate the lattice thermal conductivity,<sup>1-3</sup> and the details are as follows.

$$k_l = \frac{k_B}{2\pi^2\nu} \left( \frac{k_B T}{\hbar} \right)^3 \int_0^{\theta_D/T} \frac{x^4 e^x}{\tau_c^{-1} (e^x - 1)^2} dx$$

where  $x = \hbar\omega/(k_B T)$ ,  $k_B$ ,  $\omega$ ,  $\hbar$ ,  $\theta_D$ ,  $\nu$  and  $\tau_c^{-1}$  are the reduced phonon frequency, Boltzmann constant, phonon frequency, reduced Planck constant, Debye temperature, sound velocity and phonon-scattering relaxation time, respectively.

The relaxation time is affected by point defect scattering, phonon-phonon scattering, stacking fault and electron-phonon scattering, second phase, micropore and grain boundary scattering, as shown in the following formula.

$$\tau_c^{-1} = A\omega^4 + B\omega^2 T \exp(-\theta_D/3T) + C\omega^2 + \frac{\nu}{d}$$

Among them,  $A\omega^4$  corresponds to the relaxation time of the scattering of point defects, relaxation time of phonon-phonon scattering corresponding to  $B\omega^2 T \exp(-\theta_D/3T)$ ,  $C\omega^2$  corresponds to the relaxation time of the stacking fault and electron-phonon scattering,  $\frac{\nu}{d}$  corresponds to the relaxation time of the second phase, micropore and grain boundary scattering.

Combined with the actual test results and literature values,<sup>4-8</sup> we get the parameters of the following table. The  $d$  is obtained from the grain size distribution and micropore size distribution of TEM images, and the  $B$  is obtained by fitting with  $k_l$  known in previous studies. The point defect scattering is not considered, but only phonon-phonon scattering, electron-phonon scattering and boundary scattering are considered, by fitting the experimental  $k_l$ ,  $C$  can be obtained when  $B$  is determined. On the premise that both  $B$ ,  $C$  and  $d$  are determined, combined with the actual test results,  $A$  is calculated.

Table S1

Composition	A(s <sup>3</sup> )	B(s/K)	C(s)	$\frac{\nu}{d}$ (s <sup>-1</sup> )
Si <sub>80</sub> Ge <sub>20</sub> B <sub>1.5</sub> (B <sub>2</sub> O <sub>3</sub> ) <sub>0.6</sub>	$3500 \times 10^{-45}$	$37 \times 10^{-20}$	$0.4 \times 10^{-17}$	$8 \times 10^9$

## References

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