

A thermodynamic criterion for the choice of flux and its validity in the NaBO₂-fluxed β -BaB₂O₄ crystal growth

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Growth of the β -BaB₂O₄ crystal

BaB₂O₄ polycrystalline raw material was synthesized through a solid-state reaction. Stoichiometric amounts of BaCO₃ (99.0%, Aladdin) and H₃BO₃ (99.5%, Sinopharm Chemical Reagent Co. Ltd.) were ground and mixed thoroughly in a mortar and loaded into a corundum crucible. Then, the mixture was heated at 850°C for 48 hours with an intermediate grind. NaBO₂ polycrystalline raw material was synthesized following a similar procedure, using Na₂CO₃ (99.8%, Sinopharm Chemical Reagent Co. Ltd.) and H₃BO₃ as the reactants and heating the mixture at 700°C.

BaB₂O₄ and NaBO₂ polycrystalline raw materials were mixed with a molar ratio of BaB₂O₄: NaBO₂ = 3:4, heated in a platinum crucible, and eventually melted into a uniform high temperature solution. The solution temperature was carefully adjusted until the saturation point (876 °C) was reached. After that, a BaB₂O₄ seed crystal was immersed in the saturated solution and started to grow. During the growth process, the cooling rate was kept at 1.0°C/day and the crystal rotation rate was kept at 5 rpm. Finally, transparent single crystals were obtained (Fig. S1a).

The phase of the obtained crystals was identified by the X-ray powder diffraction (XRPD) method. The XRPD patterns were collected on a computer-controlled TTR-III diffractometer equipped with graphite-monochromatized Cu K α radiation ($\lambda = 1.54056 \text{ \AA}$). A typical experimental XRPD pattern is shown in Fig. S1b, which is in good agreement with the standard pattern of β -BaB₂O₄ (JCPDS No. 24-86), indicating that the product is the β -BaB₂O₄ crystal.

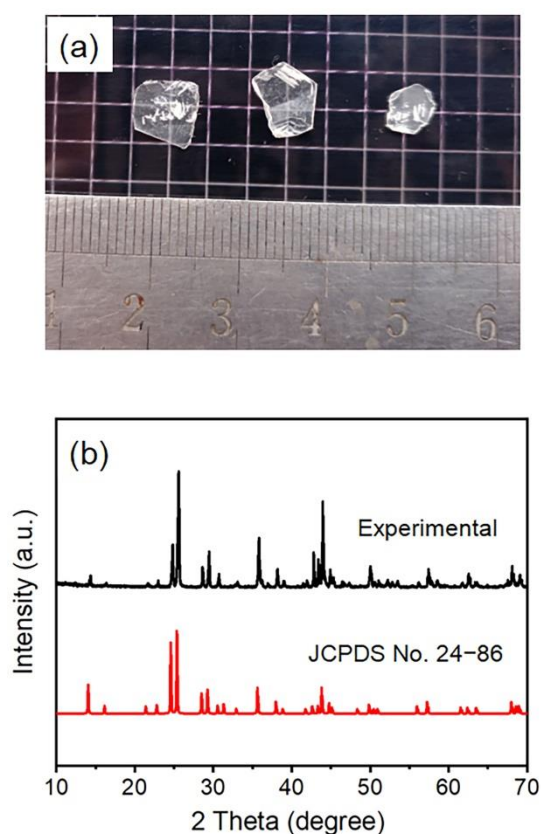


Fig. S1 (a) β -BaB₂O₄ crystals grown by the flux method. (b) A typical experimental XRPD pattern of the obtained crystals and the standard XRPD pattern of β -BaB₂O₄.

Computations for the band structures and thermodynamic properties of β -BaB₂O₄ and NaBO₂

The band structures and thermodynamic properties (*i.e.* the contribution of lattice vibration to the atomic chemical potential in this work) of β -BaB₂O₄ and NaBO₂ were performed using density functional theory (DFT) implemented in the Cambridge Sequential Total Energy Package (CASTEP).¹ The reported crystal structures of β -BaB₂O₄ and NaBO₂ were adopted as the initial structural models.^{2,3} Plane wave basis sets and norm-conserving pseudo-potentials were used in all computations. According to convergence tests, an energy cutoff of 750 eV was employed to expand the plane waves; a $2 \times 2 \times 2$ Monkhorst-Pack k -point grid was used to sum over the Brillouin zones. The valence electron configurations were $5s^25p^66s^2$ for barium, $2s^22p^63s^1$ for sodium, $2s^22p^1$ for boron, and $2s^22p^4$ for oxygen. The convergence thresholds between two neighboring optimization cycles for the total energy change, maximum displacement, maximum force, and maximum stress were set as 10^{-6} eV/atom, 0.001 Å, 0.03 eV/Å, and 0.05 GPa, respectively.

The Perdew–Burke–Ernzerhof (PBE) functional of the generalized gradient approximation (GGA) was chosen to describe the exchange–correlation energy in the β -BaB₂O₄/NaBO₂ structural optimization and in the thermodynamic property study.⁴ On the basis of the GGA-PBE optimized structures (with the benchmark volume at 0 K and different expanded volumes at different temperatures), the Heyd–Scuseria–Ernzerhof 06 (HSE06) screened hybrid functional were employed to get accurate band structures.⁵

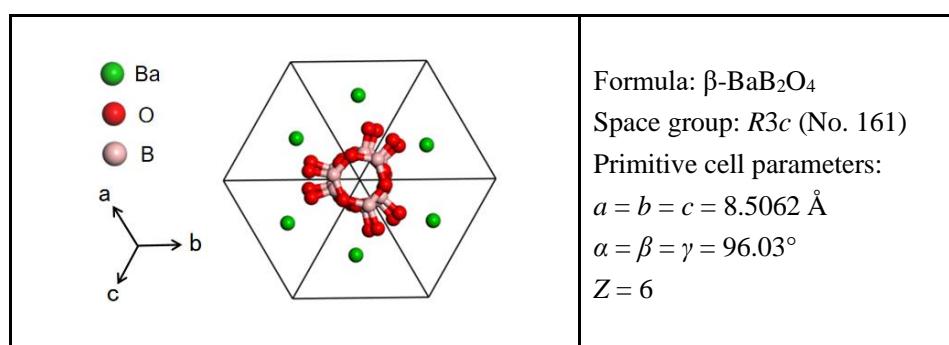


Fig. S2 Optimized structural model and primitive cell parameters of β -BaB₂O₄.

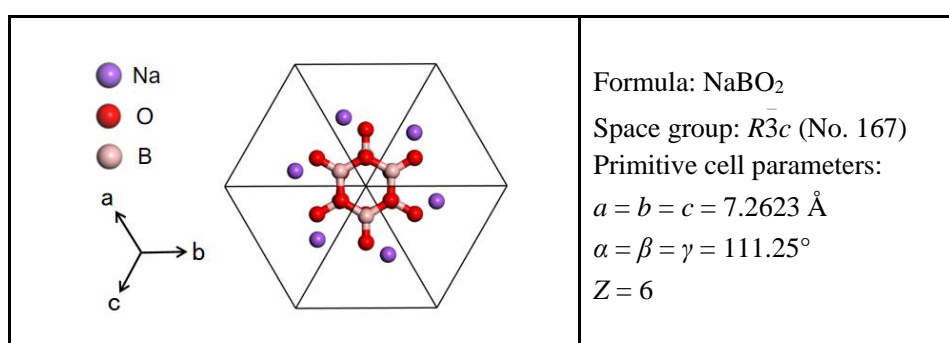


Fig. S3 Optimized structural model and primitive cell parameters of NaBO₂.

Estimation of the atomic chemical potentials of β -BaB₂O₄ and NaBO₂

In density functional theory, the atomic chemical potential (ACP) of a solid (with the chemical formula of $M_1^{n_1} \dots M_i^{n_i} \dots M_n^{n_n}$) is defined as the negative of its electronegativity, and can be estimated through the following formula⁶

$$\mu = - \left[\prod_{i=1}^n (\chi_{M_i}^{n_i}) \right]^{1/n} \quad (1)$$

where, μ is the atomic chemical potential of the solid, χ_{M_i} is the electronegativity of the i th constituent atom, n is the total number of atoms in the chemical formula ($n = n_1 + n_2 + \dots + n_n$). As shown in Table 1, the calculated ACPs of β -BaB₂O₄ and NaBO₂ are -5.45 eV and -5.14 eV, respectively, and the calculated ACP difference is 0.315 eV.

Table. S1 Atomic chemical potentials (ACPs) of β -BaB₂O₄ and NaBO₂

Crystal	Atom	χ_M^a	ACP (eV)	ACP difference (eV)
β -BaB ₂ O ₄	Ba	2.40	-5.45	0.31 eV
	B	4.29		
	O	7.54		
NaBO ₂	Na	2.85	-5.14	
	B	4.29		
	O	7.54		

^a χ_M is the electronegativity of the constituent atom on Mulliken scale.⁷

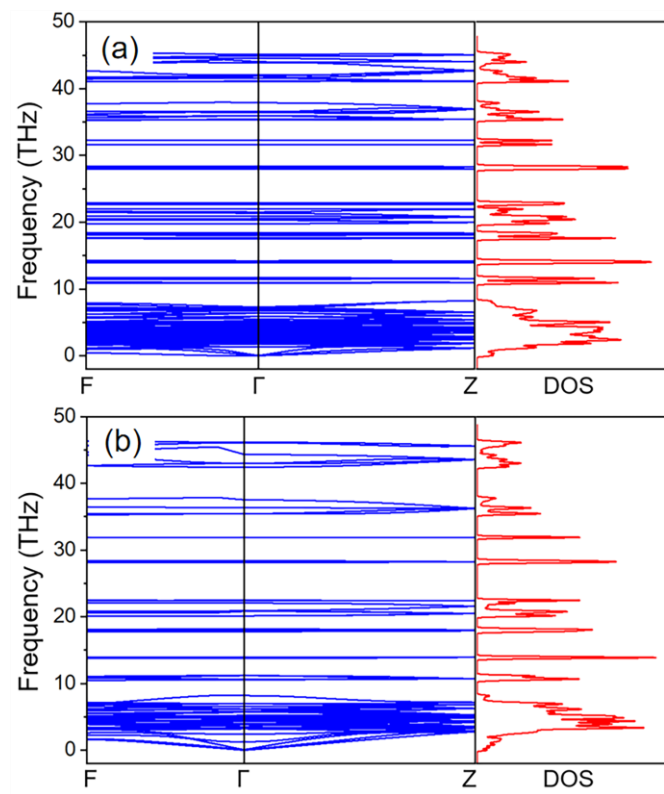


Fig. S4 Computational phonon dispersions (blue lines) and densities of states (red lines) for (a) β -BaB₂O₄ and (b) NaBO₂.

Table. S2 Primitive cell parameters of β -BaB₂O₄ and NaBO₂ at different volume expansion rates

$\Delta V/V$ (%)	β -BaB ₂ O ₄ ^a			NaBO ₂ ^a		
	a (Å)	α (°)	V (Å ³)	a (Å)	α (°)	V (Å ³)
0	8.5062	96.03	604.45	7.2623	111.25	273.68
2	8.5554	95.62	616.54	7.2909	111.08	279.15
4	8.6040	95.18	628.63	7.3174	110.89	284.62
6	8.6524	94.74	640.72	7.3410	110.68	290.10
8	8.7006	94.29	652.81	7.3644	110.47	295.57
10	8.7477	93.75	664.90	7.3865	110.24	301.05

^a β -BaB₂O₄ and NaBO₂ crystallize in the trigonal crystal system; therefore, $a = b = c$, $\alpha = \beta = \gamma$.

Table. S3 Computational valence-band-edge (E_{VB}) and conduction-band-edge (E_{CB}) positions of β -BaB₂O₄ and NaBO₂ with different expansion ratios and the corresponding atomic chemical potentials

$\Delta V/V$ (%)	β -BaB ₂ O ₄			NaBO ₂		
	E_{CB} (eV)	E_{VB} (eV)	ACP (eV)	E_{CB} (eV)	E_{VB} (eV)	ACP (eV)
0	5.728	-0.671	2.528	5.587	-0.0017	2.793
2	5.544	-0.776	2.384	5.387	-0.121	2.633
4	5.367	-0.881	2.243	5.195	-0.234	2.481
6	5.197	-0.981	2.108	5.011	-0.339	2.336
8	5.034	-1.077	1.978	4.834	-0.440	2.197
10	4.873	-1.178	1.847	4.666	-0.535	2.065

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