

## Supplementary Information

### **$\beta$ -Ga<sub>2</sub>O<sub>3</sub>: a potential high temperature thermoelectric material**

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## Supplementary Figures

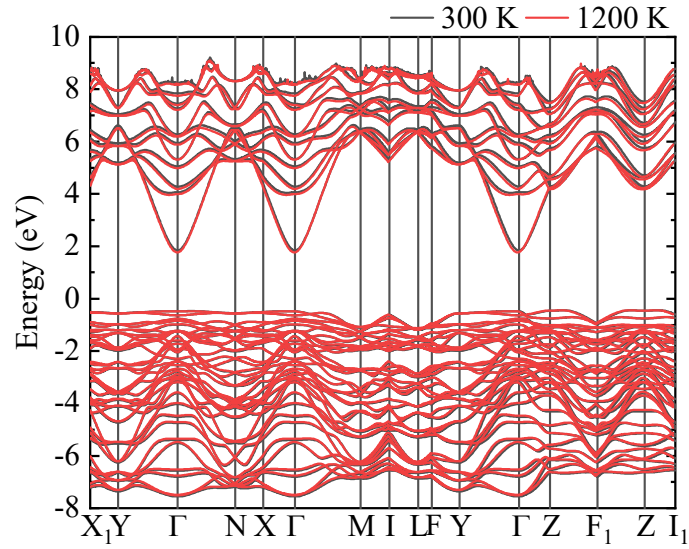


Fig. S1 The calculated energy band structure of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> applied different temperature lattice parameters considering thermal expansion effects.

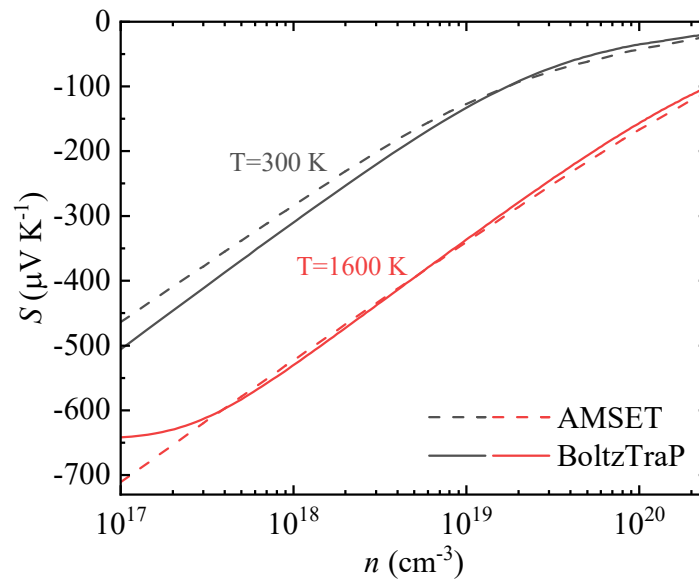


Fig. S2 The calculated Seebeck coefficient  $S$  of n-type  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> at 300 K and 1600 K applied different softwares (AMSET and BoltzTraP).

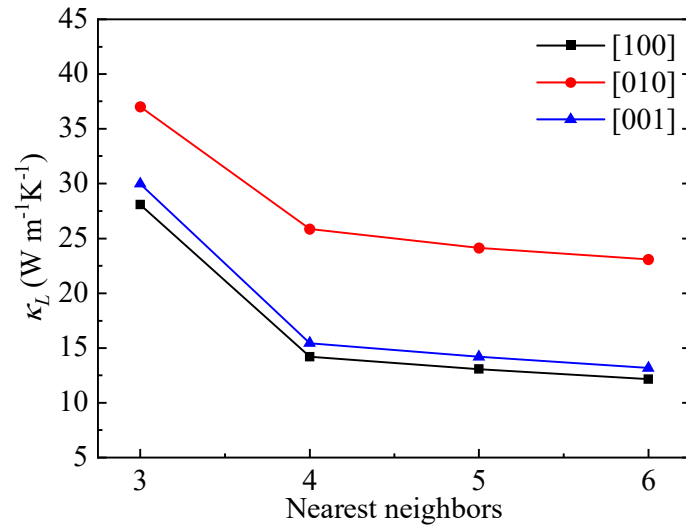


Fig. S3 The lattice thermal conductivity of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> with respect to the nearest neighbors for the anharmonic inter-atomic force constants.

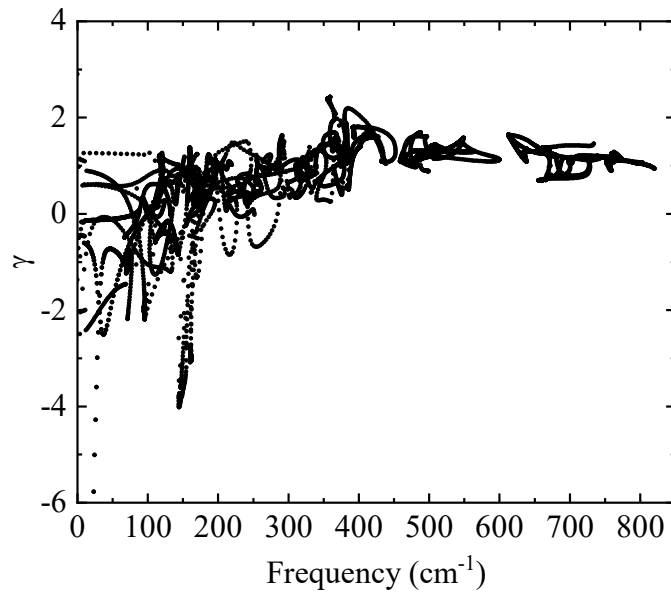


Fig. S4 The calculated mode Gruneisen parameters  $\gamma$  with respect to phonon frequency of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> at 300 K.

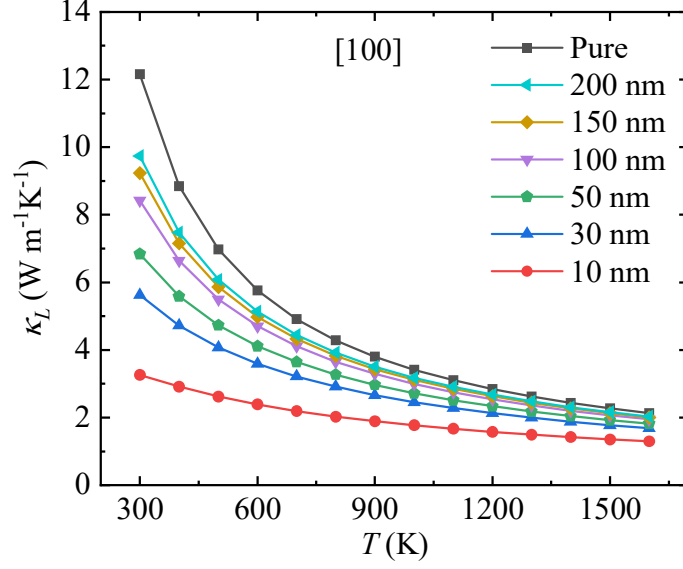


Fig. S5 The calculated lattice thermal conductivity of finite-size (10, 30, 50, 100, 150 and 200 nm)  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> as a function of temperature along [100] direction.

### Supplementary Tables

Table S1 The calculated PO phonon frequency  $\omega_{po}$  (THz), high-frequency  $\epsilon_{\infty}$  and static  $\epsilon_s$  dielectric constants of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>.

Parameter	$\omega_{po}$	$\epsilon_{\infty}$	$\epsilon_s$
Calculated Value	24.05	4.37	6.20

Table S2 The calculated elastic constants  $C_{ij}$  (GPa) of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub>, compared with the previous data.

	$C_{11}$	$C_{22}$	$C_{33}$	$C_{44}$	$C_{55}$	$C_{66}$	$C_{12}$
$\beta$ -Ga <sub>2</sub> O <sub>3</sub>	363.07	474.61	459.25	154.97	90.10	130.28	172.52
	$C_{13}$	$C_{23}$	$C_{16}$	$C_{26}$	$C_{36}$	$C_{45}$	
$\beta$ -Ga <sub>2</sub> O <sub>3</sub>	134.16	107.34	-9.90	20.76	1.03	18.90	

## Supplementary Notes

### Supplementary Notes 1. The elastic properties

The  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> compound has the base centered monoclinic structure, with 13 independent elastic constants ( $C_{11}$ ,  $C_{22}$ ,  $C_{33}$ ,  $C_{44}$ ,  $C_{55}$ ,  $C_{66}$ ,  $C_{12}$ ,  $C_{13}$ ,  $C_{23}$ ,  $C_{16}$ ,  $C_{26}$ ,  $C_{36}$  and  $C_{45}$ ).<sup>1</sup> The bulk modulus  $B$  and the shear modulus  $G$  can be given by

$$B_V = \frac{1}{9}[C_{11} + C_{22} + C_{33} + 2(C_{12} + C_{13} + C_{23})] \quad (1)$$

$$G_V = \frac{1}{15}[C_{11} + C_{22} + C_{33} + 3(C_{44} + C_{55} + C_{66}) - (C_{12} + C_{13} + C_{23})] \quad (2)$$

$$B_R = \Omega[a(C_{11} + C_{22} - 2C_{13}) + b(2C_{13} - 2C_{11} - C_{23}) + c(C_{16} - 2C_{36}) + d(2C_{13} + 2C_{23} - C_{12} - 2C_{33}) + 2e(C_{36} - C_{16}) + f]^{-1} \quad (3)$$

$$G_R = 15 \left\{ \frac{4[a(C_{11}+C_{33}+C_{13})+b(C_{11}-C_{13}-C_{23})+c(C_{16}+C_{36})+d(C_{33}-C_{12}-C_{23}-C_{13})+e(C_{16}-C_{36})+f]}{\Omega} + 3 \left[ \frac{g}{\Omega} + (C_{44} + C_{55}) / (C_{44}C_{55} - C_{45}^2) \right] \right\}^{-1} \quad (4)$$

Where,

$$a = C_{22}C_{66} - C_{26}^2$$

$$b = C_{23}C_{66} - C_{26}C_{36}$$

$$c = C_{12}C_{26} - C_{16}C_{22}$$

$$d = C_{12}C_{66} - C_{16}C_{26}$$

$$e = C_{12}C_{36} - C_{16}C_{23}$$

$$f = C_{11}(C_{33}C_{66} - C_{36}^2) - C_{13}(C_{13}C_{66} - C_{16}C_{36}) + C_{16}(C_{13}C_{36} - C_{16}C_{33}) + C_{36}(C_{23}C_{26} - C_{36}C_{22})$$

$$g = C_{11}C_{22}C_{33} - C_{11}C_{23}^2 - C_{22}C_{13}^2 - C_{33}C_{12}^2 + 2C_{12}C_{13}C_{23}$$

$$\Omega = 2[C_{16}C_{36}(C_{22}C_{13} - C_{12}C_{23}) + C_{16}C_{26}(C_{33}C_{12} - C_{13}C_{23}) + C_{26}C_{36}(C_{11}C_{23} - C_{12}C_{13})] - [C_{16}^2(C_{22}C_{33} - C_{23}^2) + C_{26}^2(C_{11}C_{33} - C_{13}^2) + C_{36}^2(C_{11}C_{22} - C_{12}^2)] + gC_{66}$$

In terms of the Voigt-Reuss-Hill approximations<sup>2</sup>,  $B$  and  $G$  can be obtained by

$$B = \frac{1}{2}(B_V + B_R) \quad (5)$$

$$G = \frac{1}{2}(G_V + G_R) \quad (6)$$

The longitude ( $v_l$ ) and shear ( $v_s$ ) sound velocity can be written as

$$v_l = \sqrt{\frac{B+4/3G}{\rho}} \quad (7)$$

$$v_s = \sqrt{\frac{G}{\rho}} \quad (8)$$

The averaged sound velocity  $v_a$  is obtained by

$$v_a = \left[ \frac{1}{3} \left( \frac{1}{v_l^3} + \frac{2}{v_s^3} \right) \right]^{-1/3} \quad (9)$$

The Gruneisen parameter  $\gamma$  is written by<sup>3</sup>

$$\gamma = \frac{3}{2} \left( \frac{1+v_p}{2-3v_p} \right) \quad (10)$$

Where,  $v_p = \frac{1-2(v_s/v_l)^2}{2-2(v_s/v_l)^2}$ .

The Debye temperature  $\Theta_D$  can be given by<sup>3</sup>

$$\Theta_D = \frac{h}{k_B} \left[ \frac{3N}{4\pi V} \right]^{1/3} v_a \quad (13)$$

Where,  $\rho$  is the density,  $h$  is Planck's constant,  $k_B$  is the Boltzmann constant,  $N$  is the number of atoms in the unit cell,  $V$  is the unit cell volume. According to the above formula, the elastic properties of  $\beta$ -Ga<sub>2</sub>O<sub>3</sub> is obtained.

### Supplementary References

1. Z. J. Wu, E. J. Zhao, H. P. Xiang, X. F. Hao, X. J. Liu and J. Meng, *Physical Review B*, 2007, **76**, 054115.
2. R. Hill, *Proceedings of the Physical Society*, 1952, **65**, 349-354.
3. Y. Xiao, C. Chang, Y. L. Pei, D. Wu, K. L. Peng, X. Y. Zhou, S. K. Gong, J. Q. He, Y. S. Zhang, Z. Zeng and L. D. Zhao, *Physical Review B*, 2016, **94**, 125203.