## **Supporting Information For**

## Two-dimensional Be<sub>2</sub>Al and Be<sub>2</sub>Ga monolayer: an anti-van't Hoff/Le Bel planar hexacoordinate bonding and superconductivity

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Figure S1. Phonon dispersion of the Be<sub>2</sub>B, Be<sub>2</sub>In, and Be<sub>2</sub>Tl monolayers.



**Figure S2.** The crystal structure of  $Ae_2M$  (Ae = Mg, Zn; M = Al, Ga) monolayer and the corresponding phonon dispersion curves.



**Figure S3**. Top and side views of low-lying structures of a) Be<sub>2</sub>Al and b) Be<sub>2</sub>Ga 2D monolayers confirmed by particle swarm searches. The green, yellow, and blue balls represent Be, Al, and Ga atoms, respectively.



**Figure S4.** The crystal structure and phonon dispersion curves of  $Be_2M$  (M = Al, Ga) bilayer.



Figure S5. Phonon dispersion spectra of  $Be_2Al$  under strain from -0.5% to 0%.



Figure S6. Phonon dispersion spectra of  $Be_2Ga$  under strain from -2.4% to 0.6%.



Figure S7. (a) Phonon dispersion with electron-phonon coupling strength, (b) phonon density of states (PhDOS), (c) Eliashberg spectral function  $\alpha^2 F(\omega)$  and the overall electron-phonon coupling strength  $\lambda(\omega)$  of Be<sub>2</sub>Al under the strain of -0.5%.



**Figure S8.** (a) Phonon dispersion with electron-phonon coupling strength, (b) phonon density of states (PhDOS), (c) Eliashberg spectral function  $\alpha^2 F(\omega)$  and the overall electron-phonon coupling strength  $\lambda(\omega)$  of Be<sub>2</sub>Ga under the strain of -2.4%.



Figure S9. Plane-averaged charge density difference along the vertical z-direction to the  $Be_2M$  (M =Al, Ga) monolayer and the  $Ca_2N$  substrate.



Figure S10. (a) Phonon dispersion with electron-phonon coupling strength, (b) Eliashberg spectral function  $\alpha^2 F(\omega)$  and the overall electron-phonon coupling strength  $\lambda(\omega)$  of the Be<sub>2</sub>Al-Ca<sub>2</sub>N bilayer.



Figure S11. Phonon dispersion of the Be<sub>2</sub>Ga-Ca<sub>2</sub>N bilayer.