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Electronic Supplementary Information. Optical measurement of electron spins in quantum dots: Quantum Zeno effects.

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Electronic Supplementary Information includes discussion of the hyperfine interaction, solution of the kinetic equation in the steady state, and details on the numerical calculations.

1. HYPERFINE INTERACTION

The Hamiltonian of the hyperfine interaction of the electron spin $\boldsymbol{S}$ with the spins of the host lattice nuclei $\boldsymbol{I}_k$ has the form [1]

$$\mathcal{H}_{\rm hf} = \sum_k A_k |\Psi(\boldsymbol{R}_k)|^2 \boldsymbol{I}_k \cdot \boldsymbol{S}, \tag{S1}$$

where k enumerates the nuclei at positions $\boldsymbol{R}_k$ with the hyperfine coupling constants A_k and $\Psi(\boldsymbol{r})$ is the electron envelop wave function.

Spins $\boldsymbol{S}$ and $\boldsymbol{I}_k$ for any k obey angular momentum $\boldsymbol{J}$ commutation relations

$$[J_\alpha, J_\beta] = i\varepsilon_{\alpha\beta\gamma} J_\gamma,$$

where α, β, γ denote the Cartesian components of vectors and $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Chivita symbol. So from the Heisenberg equation we obtain the equations of spin dynamics:

$$\frac{\mathrm{d}\boldsymbol{I}_k}{\mathrm{d}t} = \boldsymbol{\Omega}_K^{(k)} \times \boldsymbol{I}_k, \tag{S2a}$$

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{\Omega}_N \times \boldsymbol{S}, \tag{S2b}$$

where

$$\boldsymbol{\Omega}_K^{(k)} = \frac{A_k}{\hbar} |\Psi(\boldsymbol{R}_k)|^2 \boldsymbol{S}, \tag{S3a}$$

$$\boldsymbol{\Omega}_N = \sum_k \frac{A_k}{\hbar} |\Psi(\boldsymbol{R}_k)|^2 \boldsymbol{I}_k \tag{S3b}$$

are the nuclear and electron spin precession frequencies related to the Knight and Overhauser fields, respectively.

Provided the nuclear spin temperature is large as compared with $\hbar\Omega_K^{(k)}/k_B$, where k_B is the Boltzman constant, the nuclear spin distribution function is Gaussian:

$$\mathcal{F}(\boldsymbol{\Omega}_N) = \frac{1}{(\sqrt{\pi}\delta)^3} \exp\left(-\frac{\Omega_N^2}{\delta^2}\right), \tag{S4}$$

where

$$\delta = \frac{1}{\hbar}\sqrt{\frac{2}{3}\sum_k A_k^2 |\Psi(\boldsymbol{R}_k)|^2 I_k(I_k+1)} \tag{S5}$$

is a parameter which determines the dispersion, as we find from Eq. (S3b).

Since the typical number of nuclei in quantum dots is very large, of the order of 10^5—10^6, the typical nuclear spin precession frequencies $\Omega_K^{(k)}$ are 100—1000 times smaller than the typical electron spin precession frequency δ. This allows one to neglect the nuclear spin dynamics and to describe the electron spin dynamics by Eq. (S2b) with constant but random $\boldsymbol{\Omega}_N$ taken from the distribution function Eq. (S4) [2, 3]. Apart from the Knight field, the nuclear spin dynamics can be also driven by the quadrupole interaction and dipole-dipole interaction between the nuclei [1]. However, the corresponding timescales are still large, so the model of "frozen" nuclear spin fluctuations remains valid. However, the electron hopping between the quantum dots (or donors) can lead to the fast changes of the random Overhauser field experienced by electrons. This, in principle, can be taken into account as described, for example, in Refs. [2, 4, 5]. This would introduce an additional timescale, which may change the transition region between Zeno and anti-Zeno regimes.

Thus the spin dynamics in the quantum dot is described by the Hamiltonian

$$\mathcal{H}_{\rm hf} = \hbar \boldsymbol{\Omega}_N \cdot \boldsymbol{S}, \tag{S6}$$

which is equivalent to Eq. (S1). The electron spin operator can be rewritten using the creation ($a_i^\dagger$) and annihilation (a_i) operators of the states with the electron spin $i = \pm 1/2$ as

$$\boldsymbol{S} = \frac{1}{2} \sum_{i,j} \boldsymbol{\sigma}_{ij} a_i^\dagger a_j. \tag{S7}$$

Then the Hamiltonian of the hyperfine interaction takes the form

$$\mathcal{H}_{\rm hf} = \frac{\hbar}{2} \boldsymbol{\Omega}_N \cdot \sum_{i,j} \boldsymbol{\sigma}_{ij} a_i^\dagger a_j, \tag{S8}$$

which is the first term of the total system Hamiltonian, Eq. (1) in the main text.

2. SOLUTION OF THE KINETIC EQUATION

In the main text we find using the density matrix formalism that the electron spin dynamics under continuous electron spin orientation and measurement by elliptically polarized light is described by Eq. (20) in the main text:

$$\dot{\boldsymbol{S}}(t) = \boldsymbol{\Omega}_N \times \boldsymbol{S}(t) - \frac{\boldsymbol{S}(t)}{\tau_s} + g \boldsymbol{e}_z - 2\lambda (S_x(t) \boldsymbol{e}_x + S_y(t) \boldsymbol{e}_y). \tag{S9}$$

Here τ_s is the electron spin relaxation time unrelated with the hyperfine interaction and g and λ are the spin generation rate and measurement strength, respectively.

In the steady state, $\dot{\boldsymbol{S}}(t) = 0$, so the system of differential equations (S9) reduces to the system of the linear algebraic equations

$$\begin{bmatrix} \frac{1}{\tau_s} + 2\lambda & \Omega_{N,z} & -\Omega_{N,y} \\ -\Omega_{N,z} & \frac{1}{\tau_s} + 2\lambda & \Omega_{N,x} \\ \Omega_{N,y} & -\Omega_{N,x} & \frac{1}{\tau_s} \end{bmatrix} \begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}. \tag{S10}$$

The solution of this system reads

$$\frac{S_z}{S_0} = \frac{1 + 4\lambda\tau_s + \tau_s^2 (4\lambda^2 + \Omega_{N,z}^2)}{1 + 4\lambda\tau_s + (4\lambda^2 + \Omega_N^2)\tau_s^2 + 2\lambda(\Omega_{N,x}^2 + \Omega_{N,y}^2)\tau_s^3}, \tag{S11}$$

where $S_0 = g\tau_s$ is the electron steady state spin polarization in the absence of the hyperfine interaction. In the spherical coordinates with the angle θ between $\boldsymbol{\Omega}_N$ and the z-axis, this expression takes the form

$$\frac{S_z}{S_0} = \frac{1 + 4\lambda\tau_s + \left[4\lambda^2 + \Omega_N^2 \cos^2(\theta)\right] \tau_s^2}{1 + 4\lambda\tau_s + (4\lambda^2 + \Omega_N^2)\tau_s^2 + 2\lambda\Omega_N^2 \sin^2(\theta)\tau_s^3}. \tag{S12}$$

This should be averaged over the nuclear spin distribution function Eq. (S4). The result is general and describes both quantum Zeno and anti-Zeno effects.

Noteworthy, the averaging can be performed analytically in the limiting cases of strong and weak measurements.

In the limit of strong measurements when the measurement strength λ much larger than typical precession frequency δ we expand Eq. (S12) in the limit $\lambda \gg \Omega_N$ and obtain

$$\frac{S_z}{S_0} = \frac{2\lambda}{2\lambda + \Omega_N^2 \tau_s \sin^2(\theta)}. \tag{S13}$$

One can see, that the stronger the measurements, the weaker the nuclei-induced spin relaxation and the larger the spin polarization, in agreement with the quantum Zeno effect. Averaging Eq. (S13) over Eq. (S4) we obtain Eq. (25) of the main text:

$$\frac{\langle S_z \rangle}{S_0} = -\nu \mathrm{Ei}(-\nu)\exp(\nu), \tag{S14}$$

where $\nu = 2\lambda/(\tau_s\delta^2)$ and $\mathrm{Ei}(x) = -\int\limits_{-x}^{\infty} \mathrm{e}^{-t}/t\,\mathrm{d}t$ is the exponential integral function.

In the opposite limit of $\lambda \ll \delta$ the expansion of Eq. (S12) reads

$$\frac{S_z}{S_0} = \frac{\cos^2(\theta)}{1 + 2\lambda\tau_s \sin^2\theta}. \tag{S15}$$

Thus, the weak measurements accelerate the spin relaxation and suppress the spin polarization, so the quantum anti-Zeno effect takes place. Averaging over Eq. (S4) gives Eq. (27) of the main text:

$$\frac{\langle S_z \rangle}{S_0} = \frac{1}{2\lambda\tau_s}\left[\sqrt{\frac{1+2\lambda\tau_s}{2\lambda\tau_s}}\mathrm{arctanh}\left(\sqrt{\frac{2\lambda\tau_s}{1+2\lambda\tau_s}}\right) - 1\right]. \tag{S16}$$

3. NUMERICAL DETAILS

To average numerically the dimensionless quantity $\mathcal{S}(\Omega_N\tau_s, \theta) = S_z/S_0$ given by Eq. (S12) over the distribution function Eq. (S4) we use the Gauss-Laguerre quadrature to speed up the calculations. Then the average can be calculated as

$$\langle \mathcal{S} \rangle = \frac{1}{\sqrt{\pi}} \int \mathrm{d}y\, e^{-y}\sqrt{y} \int \mathrm{d}\theta \sin(\theta)\mathcal{S}(\sqrt{y}\tau_s\delta, \theta) \approx \frac{2}{\sqrt{\pi}} \sum_{i=1}^{N} w_i \overline{\mathcal{S}}(y_i\tau_s\delta)\sqrt{y_i}, \tag{S17}$$

where $y = \Omega_N^2/\delta^2$,

$$\overline{f} = \frac{1}{2}\int f \sin(\theta)\mathrm{d}\theta, \tag{S18}$$

and w_i and y_i are the weights and the roots according to the Gauss-Laguerre quadrature scheme [6]. The averaging in Eq. (S18) is performed using the Simpson's rule. In the calculations we use $N = 15$ after checking that larger N yield the same results.

REFERENCES

1. M. M. Glazov, *Electron and Nuclear Spin Dynamics in Semiconductor Nanostructures* (Oxford University Press, Oxford, 2018).
2. K. Schulten and P. G. Wolynes, "Semiclassical description of electron spin motion in radicals including the effect of electron hopping," J. Chem. Phys. **68**, 3292 (1978).
3. I. A. Merkulov, A. L. Efros, and M. Rosen, "Electron spin relaxation by nuclei in semiconductor quantum dots," Phys. Rev. B **65**, 205309 (2002).
4. M. M. Glazov, "Spin noise of localized electrons: Interplay of hopping and hyperfine interaction," Phys. Rev. B **91**, 195301 (2015).
5. D. S. Smirnov, V. N. Mantsevich, and M. M. Glazov, "Theory of optically detected spin noise in nanosystems," Phys. Usp **64**, 923 (2021).
6. H. E. Salzer and R. Zucker, "Table of the zeros and weight factors of the first fifteen Laguerre polynomials," Bull. Amer. Math. Soc. **55**, 1004 (1949).