Electronic supplementary information for

Arbitrary Jones matrix on-demand design in metasurfaces using multiple meta-atoms

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Section 1 Decomposition of Jones matrix in CPS and ROA

Mathematically, a real symmetric matrix can be diagonalized as

$$J = S^{-1}\Lambda S. \tag{S1}$$

The above transformation has following properties:

1. S is a real orthogonal matrix;
2. Λ is a diagonal real matrix; and
3. Λ and J have equal trace, $tr(J) = tr(\Lambda)$.

For CPS in the main text, the corresponding matrix is

$$J_{CPS} = \frac{1}{2}\begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix} \tag{S2}$$

which can be converted to two symmetric matrices adding,

$$J_{CPS} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + \frac{i}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{1}{2}P + Q = \frac{1}{2}P + \frac{i}{2}R^{-1}(\theta)XR(\theta) \tag{S3}$$

where

$$R(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}. \tag{S4}$$

Since $tr(Q) = tr(X)$, a tentative solution of matrix X can be expressed as

$$X = \begin{bmatrix} x & 0 \\ 0 & -x \end{bmatrix}. \tag{S5}$$

Substitute Equation (S5) into Equation (S3), and compare each element in the matrix, one can get the following solution,

$$x = 1 \;\&\; \theta = -\pi/4. \tag{S6}$$

So J_{CPS} can be decomposed as

$$J_{CPS} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} + R^{-1}\left(-\frac{\pi}{4}\right)\frac{1}{2}\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}R\left(-\frac{\pi}{4}\right). \tag{S7}$$

For ROA, the corresponding Jones matrix can be described by

$$J_{ROA} = \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix} \tag{S8}$$

which can be converted to two symmetric matrices multiplying,

$$J_{ROA} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} cos\alpha & -sin\alpha \\ -sin\alpha & -cos\alpha \end{bmatrix} = UV = UR^{-1}(\theta)YR(\theta). \tag{S9}$$

Also, $tr(V) = tr(Y)$, a tentative solution of matrix Y can be expressed as

$$Y = \begin{bmatrix} y & 0 \\ 0 & -y \end{bmatrix}. \tag{S10}$$

Use the same strategy as CPS, one can get the following solution,

$$y = 1 \ \& \ \theta = \alpha/2. \tag{S11}$$

So J_{ROA} can be decomposed as

$$J_{ROA} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot R^{-1}\left(\frac{\alpha}{2}\right) \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} R\left(\frac{\alpha}{2}\right). \tag{S12}$$

For CPS, if the output state is set to be linear polarization, the overall Jones matrix and combined form method will be different. In this case, we have

$$J_{CPS}^{L} \cdot \frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ -i \end{bmatrix} = exp\{j\psi\} \begin{bmatrix} cos\delta \\ sin\delta \end{bmatrix} \tag{S13}$$

and

$$J_{CPS}^{L} \cdot \frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ i \end{bmatrix} = 0, \tag{S14}$$

where δ is the polarized angle of the output linear polarization. By solving Equation (S13) and (S14), one solution is

$$J_{CPS}^{L} = \frac{1}{2} \begin{bmatrix} i & -1 \\ i & -1 \end{bmatrix}. \tag{S15}$$

With $\psi = 0, \delta = \pi / 4,$ the transmission under circular polarization is

$$t_{LCP} = norm\left\{ J_{CPS}^{L} \cdot \frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ -i \end{bmatrix} \right\} = 1 \tag{S16}$$

and

$$t_{RCP} = norm\left\{ J_{CPS}^{L} \cdot \frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ i \end{bmatrix} \right\} = 0, \tag{S17}$$

where J_{CPS}^{L} *and* J_{CPS} have the same transmittance, however, to construct J_{CPS}^{L}, we need three meta-atoms,

$$\begin{aligned} J_{CPS}^{L} &= \frac{1}{2} \begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix} \left(\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} -1 & 0 \\ 0 & i \end{bmatrix} \left(\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} + R^{-1}\left(\frac{\pi}{4}\right) \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} R\left(\frac{\pi}{4}\right) \right). \end{aligned} \tag{S18}$$

Section 2 Simulation results of nanobricks

Our simulation is carried out in terahertz (THz) band, the materials of substrate and nanobricks are silicon, with refractive index set as 3.4. As shown in Fig. S1, the unit cell size is 160 × 160 µm and nanobrick's height 450 µm. The model is simulated by CST Studio Suite, unit cell boundary conditions are used in both x and y directions. In propagation direction-z, port1 in the bottom face of substrate (with no added space) is used to provide plane wave illumination, this equals to placing the light source in the substate and helps to avoid FP resonance. Port2 on the other side is set to open. To guarantee enough accuracy, 18 modes are excited in both ports. The *S*-parameter of TE and TM modes are calculated, amplitude and phase at 0.6 THz are displayed in Fig. S1(b)-(e).

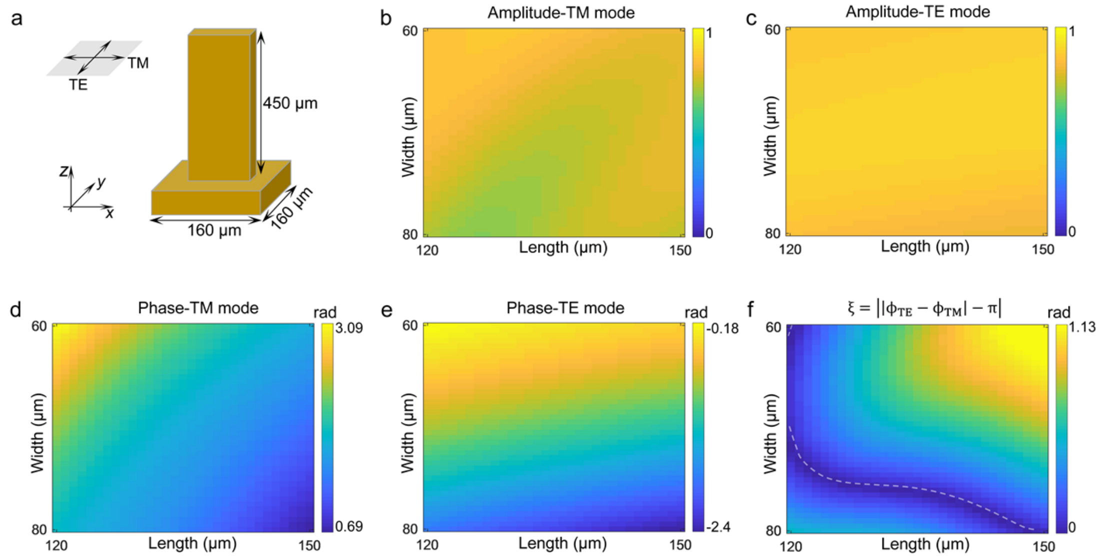

Fig. S1 Database built from a single nanobrick. (a) Configuration of unit cell. (b), (c) TM and TE mode amplitude modulation with varying length and width. (d), (e) TM and TE mode phase modulation with same parameters in (b) and (c). (f) Defined phase difference ξ between TM and TE mode.

Section 3 Amplitude and phase modulation

Using the same nanobricks in CPS but varying rotational angles, arbitrary amplitude and phase modulation can be achieved additionally. As shown in Fig. S2(a), assuming two nanobricks have independent rotation angles τ and χ, the super lattice Jones matrix is

$$J_{AM} = \frac{1}{2}R^{-1}(\tau)\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}R(\tau) + \frac{1}{2}R^{-1}(\chi)\begin{bmatrix}i & 0\\ 0 & -i\end{bmatrix}R(\chi) \tag{S19}$$

and

$$J_{AM} = \frac{1}{2}\begin{bmatrix}cos2\tau + icos2\chi & -sin2\tau - isin2\chi\\ -sin2\tau - isin2\chi & -cos2\tau - icos2\chi\end{bmatrix}. \tag{S20}$$

When it is illuminated by a LCP light, the transmitted field will be

$$E_{out} = J_{AM}\cdot\frac{\sqrt{2}}{2}\begin{bmatrix}1\\ -i\end{bmatrix} = \frac{\sqrt{2}}{2}\begin{bmatrix}cos2\tau - sin2\chi + i(sin2\tau + cos2\chi)\\ -sin2\tau - cos2\chi + i(cos2\tau - sin2\chi)\end{bmatrix}. \tag{S21}$$

When $\tau = 0$, $\chi = -\pi / 4$, it turns to the case of CPS in the main text. Here, τ and χ are variables, and the transmitted amplitude and phase can be calculated according to Equation (S21). Let $\Delta = \chi - \tau$, this item has a clear physical meaning, that is the geometric phase difference between two nanobricks. The theoretical amplitude and phase modulation under different Δ and τ are shown in Fig. S2(b) and (c). It can be found that the amplitude and phase can be modulated independently. In order to verify the accuracy of the theory, we simulated the parameters corresponding to the white curve in the Fig. S2(b) and (c), the corresponding results are shown in Fig. S2(d) and (e), where the amplitude and phase are normalized.

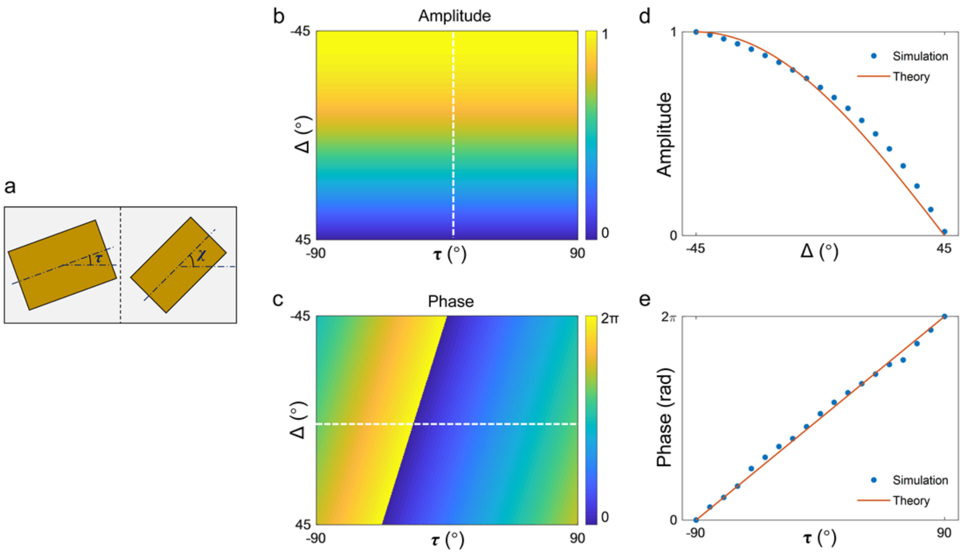

Fig. S2 Simulated and theory results of amplitude and phase modulation using two meta-atoms. (a) Illustration of the super lattice. (b), (c) Theoretical amplitude and phase modulation with varying Δ, τ. (d), (e) Transmitted amplitude and phase corresponding to the white dotted line in (b) and (c), respectively. The red curves are theory results and blue dots are simulation results.