

Supplementary Material for "Anisotropic Dependence of Radiation from Exciton in Ga₂O₃/MoS₂ Heterostructure"

Zexiang Deng^{*a}

^aSchool of Science, Guilin University of Aerospace Technology, Guilin 541004, People's Republic of China

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S1 Computational Details

In this paper, our calculations are based on density functional theory (DFT) and many-body perturbation theory, including the GW and Bethe-Salpeter equation (BSE) method, implemented in BerkeleyGW package.[1, 2, 3] We first calculated the mean-field wavefunctions and eigenvalues based on the density functional theory (DFT), implemented in the Quantum Espresso package, within the local density approximation (LDA) functional with the plane wave basis.[4, 5] As the Ga₂O₃ monolayer in this paper is ferroelectric, to better understand the interactions of the adjacent layers in the heterostructure, the van der Waals correction is included with the type of DFT-D2[6] in the functionals. The pseudopotential is used in a norm-conserving type with a energy cutoff 60 Ry. The force and electronic convergence tolerance is set to 0.001 Ry/Bohr and 10⁻⁸ Ry. A Monkhorst-Pack- Γ -centered k grid is set to 11×11×1 in the structure optimization, while it is set to 15×15×1 in the self-consistent calculation. As the ferroelectric properties of Ga₂O₃ monolayer, the vacuum slab is set to 30 Å. The exciton wave function is visualized with a supercell of 10 × 10.

To verify the thermal stability of the heterostructure, by setting the threshold of 10⁻¹⁴, the force sets can be calculated by Quantum Espresso, with the displacement method in a supercell with the k grid of 15×15×1. In the GW and BSE calculations, the mean-field calculations are performed by using Quantum Espresso package. Then the self-energy in GW calculation is calculated with k grid of 21×21×1. In the BSE calculations, the k -point is doubled by interpolating with a finer k -mesh of 41×41×1. To make sure the convergence, the number of empty band is set to 500, which is 10 times larger than the band number of occupied states and makes sure the

convergences of band gap below 0.05eV. The convergence of k-points and numbers of bands are shown in table 1 and figure 1. From table 1 and figure 1, we notice that the GW calculations with 21x21 converges to 0.05eV, and the BSE calculations with 21x21 (interpolated with a finer k-mesh of 41x41) converges to 0.05 eV.

Table 1: Convergence of k-points and band numbers in GW calculations of FE-ZB/MoS₂ heterostructures. In the calculation of the convergence of band numbers, the k-point is set to 21x21.

k-points	$E_{gw}(eV)$
11x11	2.68
15x15	2.33
21x21	2.18
35x35	2.15
number of bands	$E_{gw}(eV)$
200	2.49
500	2.18
2000	2.16

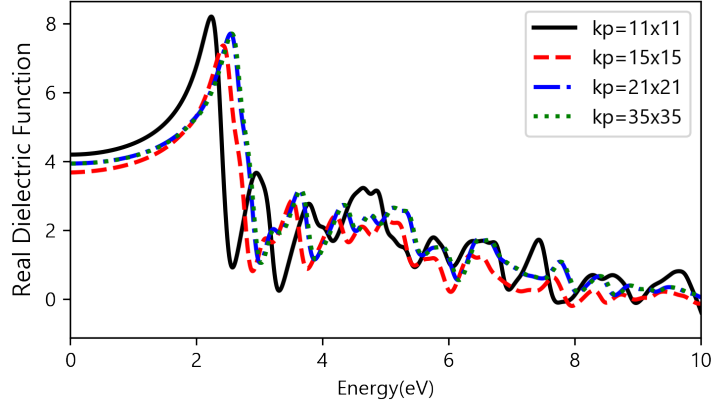


Figure 1: The real dielectric function of FE-ZB/MoS₂ heterostructure in the BSE calculations.

S2 Intensity of Radiation

Our approach to computing the lifetimes of radiation is based on the Fermi's golden rule formula,

$$\gamma = \frac{2\pi}{\hbar} \left| \langle G | H_{\text{int}} | S \rangle \right|^2 \delta(E_S - \hbar\omega_k) \quad (1)$$

where γ represents the rate of emission of a photon with angular frequency ω_k , and E_s is the exciton energy. The Hamiltonian describing the electron-photon interaction can be expressed as,[7]

$$H_{\text{int}} = -\frac{e}{m} \int d\mathbf{r} \psi^\dagger(\mathbf{r}) \mathbf{A}(\mathbf{r}) \cdot \mathbf{p} \psi(\mathbf{r}) \quad (2)$$

where \mathbf{p} is the electron momentum operator and the vector-potential operator $\mathbf{A}(\mathbf{r})$ is,[8]

$$\mathbf{A}(\mathbf{r}) = \sum_{\mathbf{k}} \sum_{\lambda=1,2} \sqrt{\frac{\hbar}{2\epsilon_0 V \omega_k}} \mathbf{e}_{\mathbf{k}\lambda} [a_{\mathbf{k}\lambda}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}} + h.c.]. \quad (3)$$

The difficulty is to evaluate $|\langle G|H_{\text{int}}|S\rangle|^2$. Inserting equation (3) into (1), with the relation between operators \mathbf{r} and \mathbf{p} ($\mathbf{p} = -im/\hbar[\mathbf{r}, H]$), γ can be written as,

$$\begin{aligned}\gamma &= \frac{e^2\pi}{m^2\varepsilon_0V} \sum_{\mathbf{k}} \sum_{\lambda=1,2} \frac{1}{\omega_k} |\langle G|\mathbf{e}_{\mathbf{k}\lambda} \cdot \mathbf{p}|S\rangle|^2 \delta(E_s - \hbar\omega_k) \\ &= \frac{e^2\pi}{m^2\varepsilon_0Vc} \sum_{\mathbf{k}} \sum_{\lambda=1,2} \frac{1}{k} |\mathbf{e}_{\mathbf{k}\lambda} \cdot \langle G|\mathbf{p}|S\rangle|^2 \delta(E_s - \hbar ck) \\ &= \frac{E_s^2 e^2 \pi}{\hbar^2 \varepsilon_0 V c} \sum_{\mathbf{k}} \sum_{\lambda=1,2} \frac{1}{k} |\mathbf{e}_{\mathbf{k}\lambda} \cdot \boldsymbol{\mu}|^2 \delta(E_s - \hbar ck)\end{aligned}\quad (4)$$

As

$$\begin{aligned}\sum_{\mathbf{k}} &\longrightarrow \int d^3k, \\ \gamma &= \frac{E_s^2 e^2}{8\pi^2 \hbar^2 \varepsilon_0 c} \sum_{\lambda=1,2} \int d^3k \frac{1}{k} |\mathbf{e}_{\mathbf{k}\lambda} \cdot \boldsymbol{\mu}|^2 \delta(E_s - \hbar ck) \\ &= \frac{E_s^2 e^2}{8\pi^2 \hbar^3 \varepsilon_0 c^2} \sum_{\lambda=1,2} |\mathbf{e}_{\mathbf{k}\lambda} \cdot \boldsymbol{\mu}|^2 \int d^3k \frac{1}{k} \delta\left(\frac{E_s}{\hbar c} - k\right) \\ &= \frac{E_s^2 e^2}{8\pi^2 \hbar^3 \varepsilon_0 c^2} \sum_{\lambda=1,2} \int d\Omega |\mathbf{e}_{\mathbf{k}\lambda} \cdot \boldsymbol{\mu}|^2 \int dk k \delta\left(\frac{E_s}{\hbar c} - k\right) \\ &= \frac{E_s^3 e^2}{8\pi^2 \hbar^4 \varepsilon_0 c^3} \sum_{\lambda=1,2} \int d\Omega |\mathbf{e}_{\mathbf{k}\lambda} \cdot \boldsymbol{\mu}|^2 \\ &= \frac{E_s^3 e^2 \mu^2}{8\pi^2 \hbar^4 \varepsilon_0 c^3} \sum_{\lambda=1,2} \int d\Omega |\mathbf{e}_{\mathbf{k}\lambda} \cdot \mathbf{e}_{\boldsymbol{\mu}}|^2 \\ &= \gamma_0 \int d\Omega Y(\theta, \phi) \\ \gamma_0 &= \frac{E_s^3 e^2 \mu^2}{8\pi^2 \hbar^4 \varepsilon_0 c^3}\end{aligned}\quad (5)$$

where $\int d\Omega = \int \sin\theta d\theta d\phi$ is the integral over solid angle. $\gamma(\theta, \phi) = \sum_{\lambda} |\mathbf{e}_{\mathbf{k}\lambda} \cdot \mathbf{e}_{\boldsymbol{\mu}}|^2 = \gamma_{\text{IP}} + \gamma_{\text{OP}}$, $\boldsymbol{\mu} = |\langle G|\mathbf{r}|S\rangle| = (\mu_x, \mu_y, \mu_z)$.

The unit vector of light propagating direction:

$$\mathbf{e}_k = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

The unit vector of in-plane polarization:

$$\mathbf{e}_{\text{IP}} = (\sin\phi, -\cos\phi, 0)$$

The unit vector of out-of-plane polarization:

$$\mathbf{e}_{\text{OP}} = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)$$

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