## Automated Optimization under Dynamic Flow Conditions <br> Supporting Information

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## Analytical Methods:

## HPLC Analysis:

Samples were collected at fixed time intervals throughout the preliminary and optimization experiments for offline HPLC analysis in order to obtain reaction understanding and generate an FTIR chemo-metric model. During preliminary experiments, samples were collected every minute, leveraging a sampling interval protocol requiring 30 seconds of collection, followed by 30 seconds of waste. For the optimizationexperiments, the sampling frequency was reduced as these samples were to be leveraged to verify the optimization-driving FTIR results. Additionally, given the extended duration of the optimization experiments ( $\sim 1000$ minutes), an every minute sampling protocol was deemed excessive. For optimization experiment \#1, samples were collected every three minutes (intervals: collection $=30$ seconds, waste $=150$ seconds) whereas, for optimization experiment \#2, samples were collected every twelve minutes (intervals: collection $=60$ seconds, waste $=660$ seconds).

HPLC analysis was leveraged during initial experimentation in an effort to establish a feasible operating region from a reaction kinetics perspective as well as a reference result for the calibration and generation of an FTIR predictive model. After FTIR model validation, HPLC was leveraged sparingly as an orthogonal analytical tool to confirm predicted concentration results and trends during the automated optimization runs. All HPLC results were obtained using an Agilent 1290 Infinity II instrument (and associated software) equipped with an Agilent ZORBAX Eclipse Plus C18 ( $50 \mathrm{~mm} \times 4.6 \mathrm{~mm}, 1.8 \mu \mathrm{~m}$ particle size) column. A fit-for-purpose gradient HPLC method was adapted from a method reported previously [1] for this particular chemistry application. Method details are provided in the table below.

Table S1. Chromatographic Conditions for all HPLC analyses

| Column: | Agilent ZORBAX Eclipse Plus C18 ( $50 \mathrm{~mm} \times 4.6 \mathrm{~mm}, 1.8 \mu \mathrm{~m}$ particle size) |  |
| :---: | :---: | :---: |
| Detector: | UV at 220 nm ; Response: 40 Hz |  |
| Temperature: | $20^{\circ} \mathrm{C}$ |  |
| Flow rate: | $1.0 \mathrm{~mL} / \mathrm{min}$ |  |
| Injection Volume: Needle Wash: | $2 \mu \mathrm{~L}$ <br> Acetonitrile |  |
| Sample Tray Temperature: | $20^{\circ} \mathrm{C}$ |  |
| Mobile Phase: | A: $0.1 \% \mathrm{H}_{3} \mathrm{PO}_{4}$ in Water |  |
|  | B: Acetonitrile |  |
| Mobile Phase Program: | Time (min) | \% B |
|  | 0.0 | 50 |
|  | 2.0 | 50 |
|  | 3.5 | 95 |
|  | 4.4 | 95 |
|  | 4.5 | 50 |
|  | 6.5 | 50 (built in post-time) |


| Name | RT (min) |
| :---: | :---: |
| 4,4'-(4-nitro-1,3-phenylene)dimorpholine | 1.14 |
| 4-(3-fluoro-4nitrophenyl)morpholine | 1.38 |
| 2,4-Difluoronitrobenzene | 1.57 |
| 4-(5-fluoro-2-nitrophenyl)morpholine | 1.69 |
| Biphenyl | 4.29 |
| Bibenzyl | 4.70 |

## Preparation of Solutions

In preparation for analysis, $4 \mu \mathrm{~L}$ of reaction sample from each fraction is diluted with 1 mL pure acetonitrile.

Prior to each run, $2 \mu \mathrm{~L}$ of the starting material solution ( $\sim 1.25 \mathrm{M} 2,4$-difluoronitrobenzene in ethanol) was diluted with 1 mL pure acetonitrile to confirm concentration. Additionally, to allow for quantification of the flow ratio (and, by association, the realized equivalents) of the morpholine (2) relative to the 2,4difluoronitrobenzene (1) streams, a solution of 0.5 mL (1) +0.5 mL (2) was mixed, a 4 uL aliquot taken and diluted in 1 mL acetonitrile, and analyzed to quantify the ratio between internal standards knowing the concentration of each stream.

## Analysis

Calibration curves were generated using known standards to extract response factors for each of the species within the linear region. To enable calibration, 2,4-difluoronitrobenzene was obtained from Aldrich, 4,4'-(4-nitro-1,3-phenylene)dimorpholine and 4-(5-fluoro-2-nitrophenyl)morpholine were obtained from AmBeed, and 4-(3-fluoro-4-nitrophenyl)morpholine was procured from Accela, and were all used asreceived. Biphenyl and Bibenzyl were procured from Sigma-Aldrich and Aldrich, respectively, and were used as-received.

Results were analyzed in bulk with a few key data transformations. To account for error associated with sample preparation (pipetting, dilution) and sample injection volumes, all species area counts were normalized leveraging the biphenyl internal standard area counts to ensure consistency across the dataset. To predict the experienced morpholine equivalents for a given sample fraction, $i$, the ratio of biphenyl to bibenzyl area counts was quantified and compared to a known value based on concentrations (see Equation S1).

$$
\begin{equation*}
\text { Equiv }_{\text {Morpholine, } \mathrm{i}}=\left[\frac{C_{\text {Morpholine }}}{C_{2,4-\text { difluoronitrobenzene }}}\right]\left[\left(\frac{\left(\frac{\mathrm{A}_{\text {Bibenzyl }}}{\mathrm{A}_{\text {Biphenyl }}}\right) \text { i, Experimental }}{\left(\frac{\mathrm{A}_{\text {Bibenzyl }}}{\mathrm{A}_{\text {Biphenyl }}}\right) \text { Calibration }}\right]\right. \tag{Eq.S1}
\end{equation*}
$$

All area count responses were converted to solution concentrations leveraging the above stated response factors to enable evaluation of mass balance closure, calibration of the FTIR for predictive quantification, and facilitate kinetic modeling efforts.

A few example chromatograms are provided in Figure $\mathbf{S} 1$ below.


Figure S1. HPLC traces of pure solvent along with reaction samples showing the four reaction species and two internal standards. The bottom figure portrays the HPLC spectra as a close-up.

## FTIR Analysis:

Principal to success of the as-designed automated optimization platform was an accurate, predictive in-line analytics model. The platform leveraged an in-line MettlerToledo ReactIR 702L equipped with a D-Sub Micro Flow Cell DiComp to record the IR spectra at 1-minute sampling intervals. The impact of scan rate was initially explored during model generation bearing in mind a desire to balance reliable model predictabilty and rapid reaction analysis. One-minute scan times were deemed sufficient for this specific chemistry application given the observed accuracy of the predictive model. Analyses of the spectra (and predictive model outputs) were performed within the Mettler Toledo iC IR software environment (version 7.1). A background spectrum was taken in air prior to each experiment and was regularly to previously obtained spectra to ensure consistency across experiments.

The iC Quant portion of the iC IR software was leveraged for the generation of a predictive multivariate model. No mathematical manipulations (i.e., $1^{\text {st }}$ or $2^{\text {nd }}$ derivative) were performed on the spectral set prior to use in iC Quant. The useful spectral information was restricted to two regions, $3800-2500 \mathrm{~cm}^{-1}$ and $1800-750 \mathrm{~cm}^{-1}$. Approximately 370 data points, from initial experiments investigating the reaction performance using one-dimensional dynamic experiments as well as initial tests of the dynamic design of experiments (DDoEs), were utilized to build the multivariate model. To facilitate the model calibration, the program was allowed to leave 1 sample out in its analysis and auto-select the number of factors chosen for the fit. The final model results are shown below in Table S2. Parity plots comparing experimental versus predicted results are shown in Figure S2.

Table S2. FTIR model results and information

| Species | \# of <br> Factors | RMSEC | RMSECV | RMSEP | $\mathrm{R}^{2}$ cumulative <br> (training) | $\mathrm{R}^{2}$ cumulative <br> (test) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2,4-difluoronitrobenzene | 12 | 0.00401 | 0.00434 | 0.0155 | 0.999 | 0.988 |
| 4,4'-(4-nitro-1,3- | 12 | 0.000731 | 0.00811 | 0.00289 | 0.997 | 0.987 |


| phenylene)dimorpholine |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4-(3-fluoro- <br> 4nitrophenyl)morpholine | 12 | 0.000584 | 0.000617 | 0.00221 | 0.998 | 0.999 |  |
| 4-(5-fluoro-2- <br> nitrophenyl)morpholine | 12 | 0.00481 | 0.00516 | 0.00996 | 0.998 | 0.999 |  |
| Biphenyl | $\mathrm{N} / \mathrm{A}^{\#}$ |  |  |  |  |  |  |$|$

\# As the 2,4-difluoronitrobenzene represents the stream basis by which other stream flowrates are set, the biphenyl (which is an inert species in this reaction) concentration was unvaried throughout the set of experiments.


Figure S2. Parity plots comparing the actual and predicted concentration results for the four reaction species of interest.

## Materials:

2,4-difluoronitrobenzene ( $99 \%$ ) and bibenzyl (ReagentPlus, $99 \%$ ) were ordered from Aldrich and used asreceived. Morpholine (ACS Reagent, $\geq 99.0 \%$ ), biphenyl (ReagentPlus, $99.5 \%$ ), ethanol (Pure 200 proof, for molecular biology), and trifluoroacetic acid (HPLC grade, $\geq 99.0 \%$ ) were obtained from Sigma-Aldrich and used as-is. For HPLC analyses, acetonitrile (Optima ${ }^{\circledR}$ LC/MS grade, 0.1 micron filtered) and water (UHPLC-MS grade) were purchased from Fisher and phosphoric acid was obtained from Aldrich ( $85 \mathrm{wt} \%$ in $\mathrm{H} 2 \mathrm{O}, 99.99 \%$ ).

Reaction streams were generated as-needed for each experiment. To generate the 1.25 M 2,4difluoronitrobenzene solution with biphenyl, approximately 198.86 g of 2,4 -difluoronitrobenzene was combined with approximately 46.3 g biphenyl in a 1 L volumetric flask and diluted with ethanol to achieve a total volume of 1 L . To prepare the 3.5 M morpholine solution with bibenzyl, approximately 304.85 g of morpholine was combined with approximately 72.9 g bibenzyl in a 1 L volumetric flask. This mixture was diluted to 1 L with ethanol as well. Solutions were sonicated as needed to dissolve all solids and allowed to
re-equilibrate prior to topping the solution up to the desired volume. To ensure the accuracy of off-line HPLC analyses, the reaction mixture was quenched in-line with a 3.5M trifluoroacetic acid (TFA) solution, immediately following passing through the FTIR flow cell. To prepare the TFA solution, approximately 399.08 g trifluoroacetic acid is added to a 1 L volumetric flask and diluted to 1 L with ethanol. All solutions were transferred to Wheaton bottles for use. Acetonitrile solvent ( $\sim 1 \mathrm{~L}$ ) was added to a Wheaton bottle for use as-is.

## Reactor Automation:

Complete automation of the flow reactor platform was enabled using LabVIEW (National Instruments, 2017 SPI Full Developer Suite), a state machine architecture from JKI (https://www.jki.net), and OPC UA capabilities within the Mettler Toledo framework. The custom LabVIEW code integrates equipment control and real-time data processing and analysis to enable the automated optimization. Each subgroup of commands (e.g., equipment inputs/outputs, data recording and write-to-file, optimization regressions/termination criteria checks) are formatted into individual states within the case structure that are queued either by asynchronous loop triggers or user-events, typically triggered when a variable reaches a threshold value or a Boolean value changes throughout the algorithmic workflow. After starting the initial circular dynamic design of experiments, the reactor platform is fully automated through termination and no user input is requested. A depiction of the LabVIEW generated user interface and code are provided in Figure S3.


Figure S3. Illustration of the LabVIEW (A) user interface and (B) block diagram detailing the state machine architecture and command states

## Correcting Reaction Conditions - Matlab:

Correction of conditions was performed using a custom Matlab code in which the time-dependent flowrate profile integrated over a variable time duration is compared to the measured reactor volume. Working backwards from the point of measurement, a minimizer is used to equate the calculated and measured reaction volume in order to calculate the associated initial and final reaction times, from which residence time and equivalents can be extracted. At a high level, the algorithm is based on the following:

$$
\begin{gather*}
\min \left[\left(V_{\text {reactor }+ \text { dead }}-\int_{t_{f}}^{t_{\text {meas }}} Q_{\text {reactor }+ \text { dead }}(t) d t\right)^{2}\right]  \tag{Eq.S2}\\
\min \left[\left(V_{\text {reactor }}-\int_{t_{i}}^{t f} Q_{\text {reactor }}(t) d t\right)^{2}\right] \tag{Eq.S3}
\end{gather*}
$$

Knowing the flowrate profile, Eq. S2 is solved to calculate a final reaction time, $t_{f}$, for each measurement time, $t_{\text {meas. }}$. Subsequently, Eq. S3 is then used to calculate the initial reaction time, $t_{i}$. With these variables calculated, characterization of the reaction conditions becomes facile.

## Residence Time Distributions:

The ability to accurately correct/predict reaction conditions when a reactor is operated under dynamic flow mode is predicated on the reactor behaving as an ideal plug-flow reactor, exhibiting minimal axial dispersion throughout. The work presented herein leverages the same reactor as previously reported work [B.M. Wyvratt, J.P. McMullen and S.T. Grosser, React. Chem. Eng., 2019, 4, 1637-1645]. In that work, residence time distribution studies aimed at evaluating axial dispersion were performed; the raw data was fit to well-known dispersion models [O. Levenspiel. Chemical Reaction Engineering, John Wiley \& Sons, Inc., New York, ed. 3, 1999] to obtain estimates on the dispersion number, $D / u L$, at 0.5 - and 10 minute residence times. Both condition sets yielded dispersion numbers significantly less than 0.01 , the threshold below which a reactor exhibits plug flow behavior per Levenspiel. Given that the exact same reactor (e.g., tubing, length) was leveraged for this work and similar conditions ( 0.5 to 10 min vs. 1 to 20 min residence times) were employed, additional residence time distribution studies were not required.

## Penalty Function Definition:

As presented in the main text (Eq. 5), the objective function is defined by four key components, a productive term as well as three separate penalty terms. The productive term is simply the quantification of the yield of the desired product relative to the starting material concentration, defined as the concentration of 4-(5-fluoro-2-nitrophenyl)morpholine divided by the initial concentration of 2,4-difluoronitrobenzene at the point of mixing. The three penalty terms, $P_{1}, P_{2}$, and $P_{3}$, help to drive conditions towards an optimal solution satisfying reaction yield needs while controlling excessive double addition impurity growth, morpholine equivalent usage or long residence times. These terms are described as piecewise functions below in Eq. S4-S6.

$$
\begin{align*}
& 0 \quad \frac{100 \cdot[4]}{[1]_{0}} \leq 2 \\
& P_{1}=0.1 *\left(\frac{100 \cdot[4]}{[1]_{0}}-2\right)^{2} \quad 2<\frac{100 \cdot[4]}{[1]_{0}}<5.2 \\
& 1 \quad \frac{100 \cdot[4]}{[1]_{0}} \geq 5.2  \tag{Eq.S4}\\
& P_{2}=\left\{\begin{array}{cc}
0 & \text { Equiv }_{\text {Morpholine }} \leq 2 \\
0.4 *\left(\frac{\text { Equiv }_{\text {Morpholine }}}{2}-1\right)^{2} & 2 \leq \text { Equiv }_{\text {Morpholine }} \leq 4
\end{array}\right.  \tag{Eq.S5}\\
& P_{3}=\left\{\begin{array}{cc}
0 & \tau_{\text {Reaction }} \leq 10 \\
0.4 *\left(\frac{\tau_{\text {Reaction }}}{10}-1\right)^{2} & 10 \leq \tau_{\text {Reaction }} \leq 20
\end{array}\right. \tag{Eq.S6}
\end{align*}
$$

## Dynamic Design of Experiments:

DOE designs
Numerous DOE designs amenable to dynamic conditions were considered and investigated through simulations. Figure S4 shows simulations of circular- and cardioid-based DOE designs under consistent conditions ( 5 minute residence time, 75 and 300 minute dynamic time). The instantaneous input conditions and corrected reaction conditions are shown in black and red lines, respectively. While the cardioid provides numerous benefits in the form of a balanced dataset across the design space, numerous reproducibility points at the center, and the ability to traverse the design and obtain centerpoint data without requiring a step change, the corrected conditions are quite convoluted due to the speed at which the system has to move through the design within the allotted dynamic time. Conversely, while the centerpoint requires a step change to obtain centerpoint data, the alignment between input and corrected conditions is significantly improved. Cardioid-based designs would require lengthy dynamic times ( $300+$ minutes at $\tau=5 \mathrm{~min}$ ) to obtain acceptable agreement between input and corrected conditions. As such, a conscious choice was made to accept a required step change to obtain the centerpoint to enable a more rapid DOE, thereby reducing both experimental time but also resource/material burden.


Figure S4. Comparison of circular (A,B) and cardioid (C,D) DOE designs from both an instantaneous input and corrected conditions perspective at differing dynamic times ( 75 or 300 minutes). The DOE centerpoint and variation for all simulations was [ $5 \pm 1.25 \mathrm{~min}, 2.0 \pm 0.5$ equiv].

## Simulations to optimize circular DOE input-corrected translatability

As highlighted previous in literature [2,3], due to the finite holdup volumes associated with the flow reactor itself, instantaneous changes made at the reactor inlet are not immediately realized at the outlet. As such, under dynamic or transient flow conditions, care must be taken to ensure that the conditions represent those actually experienced by a specific fluid element, corrected for any instantaneous changes in flowrate leveraging Equations (1-4) in the main text, included again below for convenience.

$$
\begin{gather*}
V_{\text {Reactor }}=\int_{t_{i}}^{t_{f}} Q(t) d t  \tag{Eq.S7}\\
\tau=t_{f}-t_{i}  \tag{Eq.S8}\\
V_{d}=\int_{t_{f}}^{t_{\text {meas }}}\left[Q(t)+Q_{\text {quench }}(t)\right] d t  \tag{Eq.S9}\\
\text { Base } E q\left(t_{\text {meas }}\right)=\frac{C \square_{\text {Base }} Q_{\text {Base }}\left(t_{i}\right)}{C \square_{S M} Q_{S M}\left(t_{i}\right)} \tag{Eq.S10}
\end{gather*}
$$

An appropriate understanding of the deviation of the corrected conditions from those target conditions established by the instantaneous flowrates is critical to obtaining accurate, predictive response surface models intended to guide the optimization path. Ideally, "experiments" within the circular DOE would exhibit comparable weighting to all variables with minimal skew/bias in any one direction over others.

Recognizing the delay between setting conditions and realizing the change due to the finite holdup volumes, numerous simulations were performed in an attempt to standardize the dynamic DOE and resulting design space studied following correction of conditions. Results from a subset of the simulations are shown in Figure S5. On a high level, it is quite evident from the deviation of the corrected conditions from the instantaneous conditions that longer residence times (e.g., 15 min ) coupled with shorter dynamic times (e.g., 30 min ) lead to a poor decision space coverage. Admittedly, this is expected as the system is undergoing large input changes before fluid elements are able to make their way through the reactor at these longer residence times. Conversely, at a dynamic time of 300 min , the corrected conditions closely match the instantaneous input conditions, though a trend still exists showcasing better agreement for the shorter residence times than the longer residence times. Enlightening about this dataset though were the similarities in response identified for specific combinations of residence time and dynamic time. That is, the shape of design space coverage for the corrected conditions were identical for equivalent $\mathrm{t}_{\text {dynamic }} / \tau_{\text {reaction }}$ ratios as seen in a few highlighted comparisons (red and blue boxes) within Figure S5. This relationship was leveraged to ensure that the circular dynamic DOE was consistent both within each experiment and from optimization run to run. $\mathrm{At}_{\text {dynamic }} / \tau_{\text {reaction }}$ ratio of 15 (simulations highlighted in red) was selected as an appropriate tradeoff between DOE time requirements and minimal bias/data weighting concerns. In practice, the dynamic time is calculated for each DOE based on the residence time of the new DOE centerpoint and the response surface regression/analysis is universal to any centerpoint in the design space.


Figure S5. Simulations performed comparing the instantaneous input conditions to the corrected conditions from the circular dynamic DOE at various residence time and dynamic time conditions.

As part of the holistic algorithm development, focused experiments were performed to evaluate the execution of a dynamic circular design of experiments and regression sequence within the LabVIEW code. A dynamic DOE, centered at $[\tau, \text { Equiv }]_{0}=[2 \mathrm{~min}, 1.5$ equiv], was performed with a total dynamic time of 30 minutes, consistent with a $t_{\text {dynamic }} / \tau_{\text {reaction }}$ ratio of 15 and the reaction performance analyzed via HPLC and FTIR. Concentration results from this experiment as a function of time on stream and design space conditions are presented in Figure S6A and B. FTIR and HPLC alignment was superb for all species. Trends in the conversion are clearly evident in Figure S6B with improved conversion occurring at higher morpholine equivalents and longer residence times, as expected. The concentration results and reaction conditions are together converted to a scalar objective function, shown in Figure S6C and D, which as expected, show excellent alignment between the FTIR and HPLC datasets. Within the sequential algorithm, the FTIR-based objective function reaction data was regressed against the reaction conditions (also corrected within the algorithm sequence), fitting to a quadratic function. The regression results are presented in figure (Figure S7) and tabulated form (Table S3). Overall, the model fit was excellent, exhibiting sum of squared errors (SSE), Akaike Information Criterion (AIC) and R-squared values of $0.0002567,-612.3$, and 0.9994, respectively. In Figure S7, the raw data is presented against a contour mesh as predicted by the model fit from multiple views, highlighting a clear direction towards improved objective function values. Further, the contour plot resulting from the response surface regression along with a parity plot comparing the actual-to-predicted objective function values is provided in Figure S8. The model fit results are presented in Table S3, listing the model coefficients along with the calculated gradient and search vectors.

Results indicate that the algorithm responsible for the experimental design, results manipulation/transformation, and regression to generate a model performed adequately. Worth noting, Figure S6A shows excellent agreement between the HPLC and FTIR results, showcasing the predictive power of the PLS model trained by the one-dimensional data sets. This experimental dataset was leveraged in the "test" dataset shown in Figure S2. Additionally, Figure S6B and Figure S6D offer confidence in the accuracy of the response surface model generation process, indicating minimal error between the experimental data and values predicted by the obtained quadtratic model describing the space.


Figure S6. Results from the dynamic design of experiments centered around [2 min, 1.5 equiv]. (A) Concentration data presented as a function of time on stream for both the HPLC and FTIR analytical techniques for the species: (1) 2,4-difluoronitrobenzene, (2) 4-(5-fluoro-2-nitrophenyl)morpholine), (3) 4-(3-fluoro-4-nitrophenyl)morpholine), (4) 4,4'-(4-nitro-1,3-phenylene)dimorpholine. (B) Reaction concentration data as a function of the corrected reaction conditions, presented against the design space (x-y plane). (C,D) The resulting objective function values (from both HPLC $[\circ, \square, \Delta, \diamond]$ and FTIR (-) datasets) plotted as a function of reaction conditions both in 3-dimensional and heatmap form.




Figure S7. 3-dimensional plots of the objective function raw data plotted again the response surface model prediction in various views.


Figure S8. Results from the dynamic design of experiment centered at $\left[\mathrm{x}_{1}, \mathrm{x}_{2}\right]=[2 \mathrm{~min}, 1.5$ equiv]. (A) Contour plot of the response surface model obtained. (B) Parity plot of the experimental data against predicted objective function values based on the regressed model.

Table S3. Table summarizing the RSM results and calculated gradient/search vector obtained from the dynamic design of experiments starting at $[\tau \text {, Equiv }]_{0}=[2 \mathrm{~min}, 1.5]$.

| Tau $_{\text {CP }}$ <br> (min) | Equiv. <br> CP | $\beta_{0}$ | $\beta_{1}$ | $\beta_{2}$ | $\beta_{12}$ | $\beta_{11}$ | $\beta_{22}$ | ${\mathbf{g}\left(\mathbf{x}_{\text {Coded }}\right)}^{\mathbf{p}\left(\mathbf{x}_{\text {Coded }}\right)}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 1.5 | -0.3565 | -0.0496 | -0.1025 | -0.0117 | 0.0058 | 0.0044 | $[-0.0496,-0.1025]$ | $[0.0496,0.1025]$ |

## Experiment centered at $[\tau, E q]_{0}=[15 \mathrm{~min}, 3.0 \mathrm{eq}]$

Similarly, a dynamic DOE, centered at $[\tau, \text { Equiv }]_{0}=[15 \mathrm{~min}, 3.0 \mathrm{eq}]$, was performed with the intent to understand the reaction performance under conditions more prone to overreaction in order to assess the impact of penalty functions on the resulting objective function values and response surface model directionality. The total dynamic time for this study was 225 minutes, again consistent with a $\mathrm{t}_{\text {dynamic }} / \tau_{\text {reaction }}$ ratio of 15. Concentration results, shown in Figure S9A and B, convey the relative insensitivity of the desired product (green) levels to the reaction conditions in this design subspace. Instead, the overreaction impurity to 4,4'-(4-nitro-1,3-phenylene)dimorpholine (red) exhibits greater sensitivity, largely coupled to the side reaction levels. Given the weighty penalty on excessive levels of the overreaction impurity as expressed by Eq. S4 above, objective function values achieve elevated levels and appear to exhibit a discontinuity when compared against the design space, likely due to the discrete nature of the penalty function on the overreaction impurity. This stark change in experimental objective function values is evident in Figure S9C as well as the images in Figure S10A. Despite the discontinuity, the RSM fit to the experimental data is still quite reasonable as illustrated by parity plot (Figure S10C) as well as the SSE, AIC, and $\mathrm{R}^{2}$ values from the regression ( $0.682,-779.1,0.984$, respectively). The gradient directionality and search direction from the resulting model fit appear appropriate for the dataset (Figure S10B and Table S4).


Figure S9. Results from the dynamic design of experiments centered around [15 min, 3.0 equiv]. (A) Concentration data presented as a function of time on stream for both the HPLC and FTIR analytical techniques for the species: (1) 2,4-difluoronitrobenzene, (2) 4-(5-fluoro-2-nitrophenyl)morpholine), (3) 4-
(3-fluoro-4-nitrophenyl)morpholine), (4) 4,4'-(4-nitro-1,3-phenylene)dimorpholine. (B) Reaction concentration data as a function of the corrected reaction conditions, presented against the design space (x-y plane). (C,D) The resulting objective function values (from both HPLC [ $\circ$ ] and FTIR (-) datasets) plotted as a function of reaction conditions both in 3-dimensional and heatmap form.


Figure S10. (A) 3-dimensional plots of the objective function raw data plotted again the response surface model prediction in various views from the dynamic design of experiments starting at $[\tau, \text { Equiv }]_{0}=[15$ $\mathrm{min}, 3.0]$. (B) Contour plot of the RSM over the design space. (C) Parity plot of the predicted objective function generated via the RSM against the actual objective function.

Table S4. Table summarizing the RSM results and calculated gradient/search vector obtained from the dynamic design of experiments starting at $[\tau \text {, Equiv }]_{0}=[15 \mathrm{~min}, 3.0]$.

| $\mathbf{T a u}_{\mathbf{C P}}$ <br> (min) | Equiv. <br> CP | $\beta_{0}$ | $\beta_{1}$ | $\beta_{2}$ | $\beta_{12}$ | $\beta_{11}$ | $\beta_{22}$ | ${\mathbf{g}\left(\mathbf{x}_{\text {Coded }}\right)}^{\mathbf{p}\left(\mathbf{x}_{\text {Coded }}\right)}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15 | 3.0 | 0.3920 | 0.3496 | 0.3985 | -0.1939 | -0.1312 | -0.2895 | $[0.3496,0.3985]$ | $[-0.3496,-0.3985]$ |

## Automated Optimization Runs:

Optimization Experiment Starting at $[\tau, E q]_{0}=[2 \mathrm{~min}, 1.5 \mathrm{eq}]$
See main text for the bulk of results from the optimization run starting at [ $2 \mathrm{~min}, 2.0 \mathrm{eq}$ ]. That said, Table S 5 provides greater details into the response surface model, gradient and search directions.

Table S5. Table summarizing the series of RSMs and calculated gradient/search vectors obtained during the progression through the automated optimization run starting at $[\tau \text {, Equiv }]_{0}=[15 \mathrm{~min}, 3.5]$.

| RSM \# | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Tau}_{\text {CP }}$ (min) | 2.0 | 2.62 | 5.52 | 7.61 | 10.23 |
| Equiv $_{\text {CP }}$ | 1.5 | 2.577 | 2.948 | 2.381 | 2.372 |
| $\beta_{0}$ | -0.361 | -0.591 | -0.703 | -0.740 | -0.769 |
| $\beta_{1}$ | -0.054 | -0.077 | -0.028 | -0.029 | -0.001 |
| $\beta_{2}$ | -0.098 | -0.004 | 0.058 | -0.024 | -0.004 |
| $\beta_{12}$ | -0.005 | 0.010 | 0.020 | 0.013 | 0.033 |
| $\beta_{11}$ | 0.000 | 0.017 | 0.016 | 0.006 | 0.027 |
| $\beta_{22}$ | 0.000 | 0.030 | 0.039 | 0.044 | 0.061 |
| $\mathbf{g}\left(\mathbf{x}_{\text {Coded }}\right)$ | $\begin{aligned} & {[-0.054,} \\ & -0.098] \end{aligned}$ | $\begin{aligned} & {[-0.077} \\ & -0.0043] \end{aligned}$ | $\begin{gathered} {[-0.028,} \\ 0.058] \end{gathered}$ | $\begin{aligned} & {[-0.029,} \\ & -0.024] \end{aligned}$ | $\begin{aligned} & {[-0.001,} \\ & -0.004] \end{aligned}$ |
| $\mathbf{p}\left(\mathbf{x}_{\text {Coded }}\right)$ | $\begin{gathered} {[0.054,} \\ 0.098] \end{gathered}$ | $\begin{aligned} & {[0.083,} \\ & 0.015] \end{aligned}$ | $\begin{gathered} {[0.060,} \\ -0.052] \end{gathered}$ | $\begin{aligned} & {[0.057,} \\ & -0.001] \end{aligned}$ | - ${ }^{+}$ |
| g*g | 0.0125 | 0.0060 | 0.0042 | 0.0014 | $2 \times 10^{-5}$ |

\# Algorithm terminated due to optimum internal to RSM as confirmed by the Hessian of the RSM.

Optimization Experiment Starting at $[\tau, \text { Equiv }]_{0}=[15 \mathrm{~min}, 3.5$ equiv $]$
To evaluate the reproducibility of an discovered optimal result and ensure that a local optima was not discovered, the optimization was also performed from a different starting point, [ $15 \mathrm{~min}, 3.5$ equiv]. The experiment consisted of one search sequence and 3 total design of experiments with the discovered optimum located within the final DOE as shown by the Hessian of the RSM. Figure S11 provides details into the reaction conditions employed (Figures A-C) as well as the concentration and objective function results (D, E) as a function of time on stream. Corrected conditions again matched very closely to the instantaneous input conditions, owing to appropriate choices for the dynamic times for the search and DOE operations. Overall, the optimization progresses rapidly from an area with excessive penalization due to elevated overreaction levels and high residence time/equivalents values to an area very close to the optimum showing a decrease in objective function from $\sim 0.5$ to -0.6 after a single search (see Figure S11E and Figure S12). There is a clear discontinuity in the search data results, owing to the discrete changes in the overreaction penalty function as illustrated in Eq. S4. Response surface model results are shown in Figure S13 in various forms: (top row) experimental objective function results superimposed against a contour mesh generated from the RSM prediction, (middle row) 2 -dimensional contour plots showing the directionality of the objective function values across the design space studied in each design of experiments, (bottom row) parity plot comparing the actual objective function values to those predicted by the RSM. Additional "views" of the top row of plots are provided in Figure S14 to better illustrate the curvature and model fit for the various DOEs/RSMs. There are two key aspects to note from Figure S13 and Figure S14. First, It is quite evident from the curvature associated with each subsequent RSM and the associated contour plot that the system progresses from an area characterized by a steep gradient to one consisting with concavity and a minimum value. RSM \#2 predicted an optimum in the lower right quadrant of the design subspace ( $\sim 10 \mathrm{~min}, 2.23$ equiv) and the subsequent circular dynamic DOE centered at this predicted optimum confirmed the result (terminating with a predicted optimum of $10.0 \mathrm{~min}, 2.26$ equiv). The second point to be made from these results stems from the stark differences in response surface model accuracy between the optimization experiment initiated at [ $2 \mathrm{~min}, 2.0$ equiv] and that initiated at [ $15 \mathrm{~min}, 3.5$ equiv]. The model fits exhibit a significantly lower sum of squared error for most of the RSMs from the
optimization from [ $2 \mathrm{~min}, 2.0$ equiv] as seen in Figure 11 of the main text relative to those in [ $15 \mathrm{~min}, 3.5$ equiv] (see Figure S13). While this is potentially due to experimental error associated with the FTIR data, it is more likely due to the larger implementation of penalty functions in these areas of interest that generate some discontinuities/atypical curvature in the responses. Response surface model results are summarized in Table S6 as well.


Figure S11. Conditions and experimental results from the automated optimization run starting from [15 $\mathrm{min}, 3.5$ equiv] presented as a function of experimental time on stream. (A) Flowrates for the 3 streams combined to drive the reaction and control conditions: 2,4-difluoronitrobenzene (SM), Morpholine (Base) and Ethanol (Solvent). (B,C) Instantaneous/input conditions and subsequently corrected conditions as a function of time on stream. (D) HPLC and FTIR data results for the 4 species: (1) 2,4-difluoronitrobenzene, (2) 4-(5-fluoro-2-nitrophenyl)morpholine), (3) 4-(3-fluoro-4-nitrophenyl)morpholine), (4) 4,4'-(4-nitro-1,3-phenylene)dimorpholine. (E) Calculated objective function based on the FTIR results and corrected conditions as a function of experiment time.


Figure S12. Search progression results for the first (and only) search in the optimization experiment starting at $[\tau \text {, Equiv }]_{0}=[15 \mathrm{~min}, 3.5$ equiv $]$


Figure S13. Response surface model results for the various DOEs performed during the automated optimization run starting from [ $15 \mathrm{~min}, 3.5 \mathrm{eq}$ ]. Each DOE and regression is represented within each column. (Top Row) Comparison of the experimental objective function results and the response surface model mesh against the design space region. (Middle Row) Contour plot describing the response surface model and directionality. (Bottom Row) Parity plots of the experimental data against the predicted values based on the RSM.


Figure S14. Various views showing the comparison of the experimental objective function results and the response surface model mesh against the design space region for the automated optimization run starting from [ $15 \mathrm{~min}, 3.5$ equiv]. Different views of the data are provided in the various rows for a given RSM.

Table S6. Table summarizing the series of RSMs and calculated gradient/search vectors obtained during the progression through the automated optimization run starting at $[\tau \text {, Equiv }]_{0}=[15 \mathrm{~min}, 3.5]$.

| $\mathbf{R S M} \#$ | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: |
| $\mathbf{T a u}_{\mathbf{C P}}(\mathbf{m i n})$ | 15.00 | 8.22 | 10.00 |
| $\mathbf{E q u i v}_{\mathbf{C P}}$ | 3.50 | 2.57 | 2.23 |
| $\beta_{0}$ | 0.4410 | -0.7816 | -0.7841 |
| $\beta_{1}$ | 0.2283 | 0.0090 | -0.0026 |
| $\beta_{2}$ | 0.2475 | 0.0706 | -0.0110 |
| $\beta_{12}$ | -0.1877 | 0.0838 | 0.0581 |
| $\beta_{11}$ | 0.0882 | 0.0276 | 0.0249 |
| $\beta_{22}$ | 0.0000 | 0.1052 | 0.0791 |
|  | $[0.2283$, | $[0.0090$, | $[-0.0026$, |
| $\mathbf{g}\left(\mathbf{x}_{\text {Coded }}\right)$ | $0.2475]$ | $0.0706]$ | $-0.0110]$ |
|  |  |  |  |
| $\mathbf{p ( \mathbf { x } _ { \text { Coded } } )}$ | $[-0.2283$, | $[0.0201$, | $-{ }^{\#}$ |
|  | $-0.2475]$ | $-0.0390]$ |  |
| $\mathbf{g \bullet g}$ | 0.1134 | 0.0051 | 0.00013 |

[^0]A separate experiment was performed following the two automated optimization experiments in an effort to evaluate the accuracy of the autonomous system relative to traditional steady state experiments. In that experiment, reactions were performed under steady state at the two optimum conditions as determined by the algorithm. Overall, the automated optimization and steady state results algin well.

Table S7. Comparison of the results at or near the determined optimum obtained either during the automated optimization experiment or subsequent, confirmatory steady state experiments.

|  | Conditions |  |  |  | Concentration (M) |  |  |  | Fobj |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Tau } \\ (\mathrm{min}) \\ \hline \end{gathered}$ | Equiv | Method | Analytical | $2,4-$ <br> difluoronitrobenzene | 4-(5-fluoro-2- <br> nitrophenyl)morpholine | 4-(3-fluoro-4nitrophenyl)morpholine | 4,4'-(4-nitro-1,3phenylene)dimorpholine |  |
| Optimization <br> Run \#1 | 10.2 | 2.37 | Auto- | FTIR | 0.04263 | 0.39042 | 0.05074 | 0.00599 | -0.767 |
|  | 10.2 | 2.37 | Opt | HPLC | 0.02590 | 0.38930 | 0.04414 | 0.00892 | -0.765 |
|  | 10.2 | 239 | Steady | FTIR | $0.03125 \pm 0.00063$ | $0.39450 \pm 0.00039$ | $0.05071 \pm 0.00007$ | $0.00727 \pm 0.00019$ | -0.774 |
|  | 10.2 | 2.39 | State | HPLC | $0.02334 \pm 0.00071$ | $0.39757 \pm 0.00165$ | $0.05088 \pm 0.00047$ | $0.01026 \pm 0.00007$ | -0.779 |
| Optimization Run \#2 | 10.0 | 2.2269 | AutoOpt | FTIR <br> HPLC | $0.0331$ | 0.39275 | $0.04957$ | $0.00841$ | $-0.780$ |
|  |  |  | Steady | FTIR | $0.04118 \pm 0.00060$ | $0.38581 \pm 0.00011$ | $0.05004 \pm 0.00000$ | $0.00604 \pm 0.00019$ | -0.765 |
|  | 10.0 | 2.26 | State | HPLC | $0.03374 \pm 0.00101$ | $0.39152 \pm 0.00153$ | $0.04946 \pm 0.00194$ | $0.00874 \pm 0.00018$ | -0.776 |

## Comparison of response surface model structure on fits and predictions

As is typical practice for response surface modelling, a quadratic function was leveraged for describing the various RSMs generated during the optimization runs. That said, as part of platform development/validation and refinement, the dynamic design of experiments data results from the optimization run starting at [ $15 \mathrm{~min}, 3.5$ equiv] were fit to linear, quadratic, and cubic functions (detailed in Table S8) in Matlab using the stepwiselm function post-run to validate the regression model selection; the predicted values based on the model fit were compared to the raw data for the various RSMs, both as a parity plot and by superimposing the raw data against a contour mesh of the regressed model. To summarize the datasets, as expected, the linear fits inadequately describe the response surface given the inability to fit an inherently convoluted, curved response surface associated with reaction kinetics and the applied penalty functions. While the cubic function provides the best model fit based on the parity plots, the models appear to be overfit, exhibiting features with little physical meaning, that can ultimately skew the gradient values used to inform the next search. Given this data, the use of quadratic functions to describe the RSMs in these optimization experiments is considered justified.

Table S8. Response surface model structures considered for regression against the circular dynamic DOE data

| Model Type | Function Structure |
| :--- | :--- |
| Linear | $F(x)=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}$ |
| Quadratic | $F(x)=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\beta_{12} x_{1} x_{2}+\beta_{11} x_{1}^{2}+\beta_{22} x_{2}^{2}$ |
| Cubic | $F(x)=\beta_{0}+\beta_{1} x_{1}+\beta_{2} x_{2}+\beta_{12} x_{1} x_{2}+\beta_{11} x_{1}^{2}+\beta_{22} x_{2}^{2}+\beta_{111} x_{1}^{3}+\beta_{112} x_{1}^{2} x_{2}+\beta_{122} x_{1} x_{2}^{2}+\beta_{2}$ |



Figure S15. Parity plots of the experimental versus predicted objective function values for the three RSMs from the optimization run starting at [ $15 \mathrm{~min}, 3.5$ equiv], but regressed using different functions


Figure S16. Experimental objective function results versus RSMs regressed to different functions for the first DOE/RSM from the optimization run starting at [ $15 \mathrm{~min}, 3.5$ equiv]. Objective function improves from yellow to blue.


Figure S17. Experimental objective function results versus RSMs regressed to different functions for the second DOE/RSM from the optimization run starting at [ $15 \mathrm{~min}, 3.5$ equiv]. Objective function improves from yellow to blue.


Figure S18. Experimental objective function results versus RSMs regressed to different functions for the third (final) DOE/RSM from the optimization run starting at [ $15 \mathrm{~min}, 3.5$ equiv]. Objective function improves from yellow to blue.

## Kinetic Modeling:

Kinetic modeling and parameter regression were performed in Matlab 2012b (Mathworks ${ }^{\circledR}$ ). The reaction network given in Scheme S1 was described by the reaction rates and differential equations given .



Scheme S1: Reaction network for case study $\mathrm{S}_{\mathrm{N}} \mathrm{Ar}$.

$$
r_{1}=-k_{1}[1][5]
$$

$$
r_{2}=-k_{2}[2][5]
$$

$$
r_{3}=-k_{3}[1][5]
$$

$$
r_{4}=-k_{4}[3][5]
$$

$$
r_{5}=-k_{5}\left([6][5]-\frac{1}{K_{e q}}[7]\right)
$$

$$
\frac{d[1]}{d t}=r_{1}+r_{3} \quad \frac{d[2]}{d t}=-r_{1}+r_{2}
$$

$$
\frac{d[3]}{d t}=-r_{1}+r_{4}
$$

$$
\frac{d[4]}{d t}=-r_{3}-r_{4}
$$

$$
\frac{d[5]}{d t}=r_{1}+r_{2}+r_{3}+r_{4}+r_{5}
$$

$$
\frac{d[6]}{d t}=-r_{1}-r_{2}-r_{3}-r_{4}-r_{5}
$$

$$
\frac{d[7]}{d t}=-r_{5}
$$

Figure S19: Reaction rate expression and differential equations used in the kinetic modeling of $\mathrm{S}_{\mathrm{N}} \mathrm{Ar}$ case study.

For a given set of corrected experimental conditions (residence time and initial morpholine equivalents), the solution to the network of differential equations was calculated using the Matlab function ode15s. The modeled concentration compounds $\mathbf{1 , 2 , 3}$, and $\mathbf{4}$ were compared to corresponding FTIR concentration data from both dynamic optimization investigations. At points in the dynamic experiments, negative FTIR concentrations were observed and attributed to spectroscopic noise associated with the limit of detection of the FTIR or the resolution of the FTIR PLS model at low concentrations. To maintain physical constraints, these negative values were converted to " 0 " for the parameter regression. The acid-base neutralization reaction rate was considered rapid and the associated rate constant $\left(k_{5}\right)$ was arbitrarily selected to be $10 \mathrm{M}^{-}$ ${ }^{1} \mathrm{~min}^{-1}$. The equilibrium constant, Keq, was estimated from the pKa values for morpholine $(\mathrm{pKa}=8.36)$ and $\mathrm{HF}(\mathrm{pKa}=3.8)$. These parameters remained fixed during regression. The Matlab function Isqnonlin was used to fit the rate constants $k_{l}-k_{4}$ to minimize the weighted sum of squared error term given in .

$$
\begin{equation*}
w S S E=\sum_{c=1}^{C} \sum_{i=1}^{N} \frac{\left(y_{i, c}-\hat{y}_{i, c}\right)^{2}}{\max \left(y_{c}\right)} \tag{Eq.S11}
\end{equation*}
$$

Where $y_{i, c}$ is the FTIR concentration for species $c$ ( $C$ is total number of species used in regression, 4) measured at corrected conditions for experiment $i$ ( $N$ is total number of dynamic experiments, 1389), and $\hat{y}_{i c}$ is the model predictions for compound $c$ for experiment $i$.

## Optimization Experiment Data in Tabulated Format:

## Optimization Run \#1

Table S9. Tabulated data from the first optimization run starting at $[\tau \text {, Equiv }]_{0}=[2 \mathrm{~min}, 1.5]$

| Tau (min) | Eq (-) | Obj Func (-) | 2.468 | 2.311 | -0.542 | 3.122 | 2.629 | -0.623 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2.003 | 2.000 | -0.465 | 2.484 | 2.340 | -0.547 | 3.201 | 2.640 | -0.629 |
| 2.045 | 2.000 | -0.466 | 2.501 | 2.370 | -0.552 | 3.281 | 2.650 | -0.634 |
| 2.130 | 1.999 | -0.473 | 2.517 | 2.399 | -0.556 | 3.361 | 2.661 | -0.639 |
| 2.222 | 1.986 | -0.480 | 2.534 | 2.429 | -0.560 | 3.441 | 2.671 | -0.641 |
| 2.306 | 1.955 | -0.483 | 2.550 | 2.458 | -0.564 | 3.521 | 2.682 | -0.645 |
| 2.378 | 1.906 | -0.481 | 2.567 | 2.488 | -0.564 | 3.601 | 2.693 | -0.649 |
| 2.434 | 1.841 | -0.476 | 2.583 | 2.518 | -0.567 | 3.681 | 2.703 | -0.657 |
| 2.473 | 1.761 | -0.465 | 2.599 | 2.547 | -0.561 | 3.761 | 2.714 | -0.660 |
| 2.493 | 1.669 | -0.449 | 2.616 | 2.577 | -0.577 | 3.841 | 2.725 | -0.661 |
| 2.490 | 1.567 | -0.429 | 2.632 | 2.606 | -0.575 | 3.921 | 2.735 | -0.666 |
| 2.466 | 1.460 | -0.404 | 2.649 | 2.636 | -0.575 | 4.001 | 2.746 | -0.669 |
| 2.420 | 1.353 | -0.377 | 2.665 | 2.665 | -0.575 | 4.081 | 2.757 | -0.671 |
| 2.355 | 1.251 | -0.349 | 2.682 | 2.695 | -0.576 | 4.161 | 2.767 | -0.673 |
| 2.272 | 1.160 | -0.321 | 2.624 | 3.077 | -0.580 | 4.240 | 2.778 | -0.677 |
| 2.175 | 1.085 | -0.296 | 2.668 | 3.077 | -0.578 | 4.320 | 2.788 | -0.678 |
| 2.069 | 1.032 | -0.274 | 2.745 | 3.077 | -0.579 | 4.400 | 2.799 | -0.679 |
| 1.960 | 1.004 | -0.257 | 2.840 | 3.072 | -0.583 | 4.480 | 2.810 | -0.681 |
| 1.853 | 1.003 | -0.248 | 2.930 | 3.058 | -0.591 | 4.560 | 2.820 | -0.684 |
| 1.753 | 1.028 | -0.243 | 3.013 | 3.033 | -0.601 | 4.640 | 2.831 | -0.684 |
| 1.665 | 1.077 | -0.245 | 3.087 | 2.997 | -0.610 | 4.720 | 2.842 | -0.686 |
| 1.594 | 1.147 | -0.251 | 3.150 | 2.952 | -0.621 | 4.800 | 2.852 | -0.687 |
| 1.542 | 1.233 | -0.263 | 3.200 | 2.898 | -0.632 | 4.880 | 2.863 | -0.687 |
| 1.511 | 1.329 | -0.277 | 3.236 | 2.835 | -0.639 | 4.960 | 2.874 | -0.689 |
| 1.502 | 1.432 | -0.295 | 3.257 | 2.765 | -0.646 | 5.040 | 2.884 | -0.689 |
| 1.515 | 1.535 | -0.313 | 3.262 | 2.689 | -0.650 | 5.120 | 2.895 | -0.690 |
| 1.548 | 1.634 | -0.335 | 3.250 | 2.609 | -0.651 | 5.200 | 2.906 | -0.691 |
| 1.600 | 1.727 | -0.358 | 3.221 | 2.527 | -0.650 | 5.279 | 2.916 | -0.692 |
| 1.667 | 1.809 | -0.381 | 3.176 | 2.445 | -0.646 | 5.359 | 2.927 | -0.694 |
| 1.748 | 1.878 | -0.404 | 3.116 | 2.365 | -0.638 | 5.439 | 2.937 | -0.693 |
| 1.838 | 1.933 | -0.425 | 3.041 | 2.291 | -0.625 | 5.519 | 2.948 | -0.695 |
| 1.931 | 1.973 | -0.445 | 2.954 | 2.224 | -0.612 | 5.599 | 2.959 | -0.694 |
| 1.989 | 1.995 | -0.459 | 2.856 | 2.167 | -0.595 | 5.679 | 2.969 | -0.693 |
| 2.000 | 1.500 | -0.364 | 2.752 | 2.123 | -0.578 | 5.527 | 3.448 | -0.605 |
| 2.009 | 1.500 | -0.363 | 2.644 | 2.093 | -0.561 | 5.554 | 3.448 | -0.605 |
| 2.025 | 1.514 | -0.364 | 2.534 | 2.078 | -0.546 | 5.597 | 3.448 | -0.603 |
| 2.041 | 1.544 | -0.363 | 2.427 | 2.079 | -0.531 | 5.658 | 3.448 | -0.606 |
| 2.057 | 1.573 | -0.368 | 2.326 | 2.095 | -0.517 | 5.734 | 3.448 | -0.607 |
| 2.074 | 1.603 | -0.376 | 2.233 | 2.126 | -0.509 | 5.825 | 3.448 | -0.606 |
| 2.090 | 1.632 | -0.384 | 2.151 | 2.170 | -0.502 | 5.920 | 3.447 | -0.608 |
| 2.107 | 1.662 | -0.390 | 2.082 | 2.225 | -0.496 | 6.013 | 3.443 | -0.608 |
| 2.123 | 1.691 | -0.397 | 2.028 | 2.290 | -0.493 | 6.104 | 3.437 | -0.610 |
| 2.140 | 1.721 | -0.407 | 1.991 | 2.362 | -0.491 | 6.192 | 3.428 | -0.611 |
| 2.156 | 1.750 | -0.414 | 1.969 | 2.438 | -0.493 | 6.277 | 3.417 | -0.610 |
| 2.172 | 1.780 | -0.421 | 1.965 | 2.517 | -0.494 | 6.358 | 3.404 | -0.617 |
| 2.189 | 1.809 | -0.429 | 1.977 | 2.596 | -0.498 | 6.434 | 3.389 | -0.620 |
| 2.205 | 1.839 | -0.437 | 2.005 | 2.673 | -0.503 | 6.506 | 3.371 | -0.621 |
| 2.222 | 1.868 | -0.444 | 2.048 | 2.747 | -0.509 | 6.572 | 3.351 | -0.628 |
| 2.238 | 1.898 | -0.452 | 2.104 | 2.816 | -0.519 | 6.633 | 3.329 | -0.635 |
| 2.255 | 1.927 | -0.460 | 2.172 | 2.878 | -0.527 | 6.688 | 3.305 | -0.642 |
| 2.271 | 1.957 | -0.467 | 2.250 | 2.934 | -0.534 | 6.737 | 3.278 | -0.647 |
| 2.287 | 1.986 | -0.474 | 2.336 | 2.981 | -0.541 | 6.780 | 3.250 | -0.659 |
| 2.304 | 2.016 | -0.480 | 2.429 | 3.019 | -0.551 | 6.815 | 3.220 | -0.661 |
| 2.320 | 2.045 | -0.489 | 2.522 | 3.048 | -0.559 | 6.843 | 3.189 | -0.672 |
| 2.337 | 2.075 | -0.496 | 2.587 | 3.068 | -0.565 | 6.864 | 3.155 | -0.682 |
| 2.353 | 2.104 | -0.502 | 2.616 | 2.577 | -0.591 | 6.878 | 3.121 | -0.688 |
| 2.370 | 2.134 | -0.508 | 2.658 | 2.577 | -0.600 | 6.883 | 3.085 | -0.694 |
| 2.386 | 2.163 | -0.516 | 2.722 | 2.577 | -0.606 | 6.881 | 3.048 | -0.700 |
| 2.402 | 2.193 | -0.520 | 2.802 | 2.586 | -0.607 | 6.871 | 3.010 | -0.707 |
| 2.419 | 2.222 | -0.527 | 2.882 | 2.597 | -0.607 | 6.853 | 2.972 | -0.713 |
| 2.435 | 2.252 | -0.531 | 2.962 | 2.608 | -0.612 | 6.826 | 2.933 | -0.719 |
| 2.452 | 2.281 | -0.537 | 3.042 | 2.618 | -0.617 | 6.792 | 2.894 | -0.724 |


| 6.750 | 2.855 | -0.728 | 6.250 | 2.797 | -0.719 | 8.906 | 2.114 | -0.735 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6.701 | 2.816 | -0.732 | 6.330 | 2.772 | -0.722 | 8.826 | 2.089 | -0.732 |
| 6.644 | 2.778 | -0.734 | 6.410 | 2.748 | -0.725 | 8.741 | 2.065 | -0.726 |
| 6.580 | 2.740 | -0.737 | 6.490 | 2.723 | -0.727 | 8.653 | 2.043 | -0.721 |
| 6.509 | 2.704 | -0.741 | 6.570 | 2.699 | -0.729 | 8.561 | 2.021 | -0.717 |
| 6.432 | 2.669 | -0.741 | 6.650 | 2.674 | -0.732 | 8.466 | 2.001 | -0.712 |
| 6.349 | 2.636 | -0.745 | 6.730 | 2.650 | -0.734 | 8.367 | 1.982 | -0.707 |
| 6.261 | 2.605 | -0.742 | 6.810 | 2.625 | -0.738 | 8.266 | 1.964 | -0.701 |
| 6.167 | 2.576 | -0.740 | 6.890 | 2.601 | -0.739 | 8.163 | 1.948 | -0.696 |
| 6.069 | 2.549 | -0.739 | 6.969 | 2.576 | -0.740 | 8.058 | 1.934 | -0.691 |
| 5.968 | 2.526 | -0.736 | 7.049 | 2.552 | -0.742 | 7.951 | 1.921 | -0.689 |
| 5.864 | 2.505 | -0.733 | 7.129 | 2.527 | -0.743 | 7.843 | 1.910 | -0.681 |
| 5.757 | 2.487 | -0.729 | 7.209 | 2.503 | -0.743 | 7.734 | 1.900 | -0.679 |
| 5.648 | 2.472 | -0.725 | 7.289 | 2.479 | -0.744 | 7.624 | 1.893 | -0.674 |
| 5.539 | 2.461 | -0.722 | 7.369 | 2.454 | -0.744 | 7.515 | 1.887 | -0.671 |
| 5.430 | 2.453 | -0.719 | 7.449 | 2.430 | -0.745 | 7.406 | 1.883 | -0.668 |
| 5.321 | 2.449 | -0.715 | 7.529 | 2.405 | -0.745 | 7.297 | 1.881 | -0.665 |
| 5.213 | 2.448 | -0.711 | 7.609 | 2.381 | -0.747 | 7.190 | 1.881 | -0.662 |
| 5.107 | 2.451 | -0.705 | 7.689 | 2.356 | -0.746 | 7.084 | 1.882 | -0.660 |
| 5.005 | 2.457 | -0.703 | 7.769 | 2.332 | -0.747 | 6.980 | 1.886 | -0.660 |
| 4.905 | 2.467 | -0.699 | 7.849 | 2.307 | -0.746 | 6.878 | 1.891 | -0.658 |
| 4.810 | 2.480 | -0.697 | 7.929 | 2.283 | -0.744 | 6.778 | 1.898 | -0.657 |
| 4.719 | 2.496 | -0.693 | 7.609 | 2.881 | -0.722 | 6.682 | 1.907 | -0.656 |
| 4.633 | 2.515 | -0.690 | 7.612 | 2.881 | -0.722 | 6.588 | 1.917 | -0.658 |
| 4.553 | 2.537 | -0.687 | 7.628 | 2.881 | -0.722 | 6.498 | 1.929 | -0.657 |
| 4.480 | 2.561 | -0.685 | 7.656 | 2.881 | -0.722 | 6.412 | 1.943 | -0.660 |
| 4.413 | 2.588 | -0.682 | 7.698 | 2.881 | -0.722 | 6.330 | 1.958 | -0.660 |
| 4.352 | 2.617 | -0.679 | 7.752 | 2.881 | -0.723 | 6.253 | 1.975 | -0.660 |
| 4.300 | 2.648 | -0.676 | 7.817 | 2.881 | -0.724 | 6.180 | 1.993 | -0.663 |
| 4.254 | 2.681 | -0.674 | 7.894 | 2.881 | -0.723 | 6.111 | 2.012 | -0.665 |
| 4.217 | 2.715 | -0.672 | 7.982 | 2.881 | -0.724 | 6.048 | 2.033 | -0.669 |
| 4.187 | 2.750 | -0.670 | 8.077 | 2.880 | -0.724 | 5.990 | 2.054 | -0.672 |
| 4.165 | 2.787 | -0.668 | 8.172 | 2.879 | -0.724 | 5.937 | 2.077 | -0.673 |
| 4.151 | 2.823 | -0.667 | 8.266 | 2.876 | -0.725 | 5.890 | 2.100 | -0.675 |
| 4.145 | 2.861 | -0.664 | 8.358 | 2.872 | -0.727 | 5.848 | 2.124 | -0.681 |
| 4.147 | 2.898 | -0.661 | 8.447 | 2.867 | -0.728 | 5.811 | 2.149 | -0.683 |
| 4.157 | 2.936 | -0.658 | 8.535 | 2.861 | -0.729 | 5.781 | 2.174 | -0.685 |
| 4.175 | 2.973 | -0.656 | 8.620 | 2.853 | -0.731 | 5.756 | 2.201 | -0.687 |
| 4.200 | 3.010 | -0.654 | 8.703 | 2.844 | -0.732 | 5.737 | 2.227 | -0.690 |
| 4.232 | 3.046 | -0.651 | 8.782 | 2.834 | -0.734 | 5.724 | 2.254 | -0.693 |
| 4.271 | 3.082 | -0.649 | 8.858 | 2.823 | -0.736 | 5.716 | 2.281 | -0.696 |
| 4.317 | 3.116 | -0.647 | 8.931 | 2.811 | -0.738 | 5.715 | 2.308 | -0.698 |
| 4.369 | 3.149 | -0.645 | 9.001 | 2.797 | -0.739 | 5.719 | 2.335 | -0.701 |
| 4.428 | 3.182 | -0.641 | 9.066 | 2.782 | -0.740 | 5.728 | 2.362 | -0.703 |
| 4.492 | 3.212 | -0.639 | 9.127 | 2.766 | -0.744 | 5.744 | 2.389 | -0.706 |
| 4.561 | 3.242 | -0.636 | 9.185 | 2.749 | -0.744 | 5.764 | 2.416 | -0.708 |
| 4.636 | 3.269 | -0.635 | 9.238 | 2.731 | -0.748 | 5.790 | 2.443 | -0.710 |
| 4.715 | 3.295 | -0.632 | 9.286 | 2.712 | -0.749 | 5.822 | 2.469 | -0.712 |
| 4.797 | 3.320 | -0.629 | 9.329 | 2.692 | -0.750 | 5.858 | 2.495 | -0.712 |
| 4.884 | 3.342 | -0.624 | 9.368 | 2.671 | -0.753 | 5.900 | 2.521 | -0.715 |
| 4.974 | 3.362 | -0.620 | 9.401 | 2.649 | -0.756 | 5.946 | 2.546 | -0.717 |
| 5.066 | 3.380 | -0.618 | 9.430 | 2.626 | -0.757 | 5.997 | 2.570 | -0.718 |
| 5.160 | 3.397 | -0.618 | 9.453 | 2.602 | -0.758 | 6.052 | 2.594 | -0.720 |
| 5.257 | 3.411 | -0.616 | 9.470 | 2.577 | -0.761 | 6.112 | 2.617 | -0.722 |
| 5.343 | 3.423 | -0.614 | 9.482 | 2.552 | -0.762 | 6.176 | 2.639 | -0.722 |
| 5.413 | 3.433 | -0.611 | 9.488 | 2.526 | -0.762 | 6.244 | 2.661 | -0.722 |
| 5.466 | 3.440 | -0.610 | 9.489 | 2.499 | -0.761 | 6.315 | 2.682 | -0.721 |
| 5.501 | 3.445 | -0.610 | 9.484 | 2.472 | -0.764 | 6.390 | 2.702 | -0.724 |
| 5.518 | 3.448 | -0.609 | 9.473 | 2.445 | -0.763 | 6.468 | 2.721 | -0.722 |
| 5.519 | 2.948 | -0.703 | 9.456 | 2.417 | -0.763 | 6.550 | 2.739 | -0.721 |
| 5.547 | 2.948 | -0.692 | 9.434 | 2.389 | -0.761 | 6.634 | 2.756 | -0.724 |
| 5.583 | 2.948 | -0.692 | 9.406 | 2.361 | -0.761 | 6.720 | 2.772 | -0.722 |
| 5.632 | 2.948 | -0.690 | 9.372 | 2.332 | -0.761 | 6.809 | 2.787 | -0.722 |
| 5.695 | 2.948 | -0.692 | 9.332 | 2.304 | -0.758 | 6.900 | 2.801 | -0.721 |
| 5.771 | 2.943 | -0.692 | 9.287 | 2.276 | -0.755 | 6.993 | 2.814 | -0.721 |
| 5.850 | 2.919 | -0.698 | 9.236 | 2.248 | -0.755 | 7.088 | 2.826 | -0.721 |
| 5.930 | 2.895 | -0.705 | 9.180 | 2.220 | -0.752 | 7.183 | 2.837 | -0.719 |
| 6.010 | 2.870 | -0.709 | 9.119 | 2.192 | -0.747 | 7.279 | 2.846 | -0.721 |
| 6.090 | 2.846 | -0.713 | 9.053 | 2.166 | -0.744 | 7.363 | 2.855 | -0.722 |
| 6.170 | 2.821 | -0.715 | 8.982 | 2.139 | -0.741 | 7.436 | 2.862 | -0.721 |


| 7.497 | 2.869 | -0.721 | 12.127 | 2.785 | -0.683 | 8.701 | 1.925 | -0.694 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7.545 | 2.874 | -0.721 | 12.192 | 2.774 | -0.686 | 8.615 | 1.936 | -0.695 |
| 7.579 | 2.877 | -0.720 | 12.254 | 2.762 | -0.688 | 8.533 | 1.947 | -0.697 |
| 7.609 | 2.381 | -0.754 | 12.313 | 2.750 | -0.690 | 8.454 | 1.959 | -0.699 |
| 7.624 | 2.381 | -0.745 | 12.368 | 2.737 | -0.699 | 8.378 | 1.972 | -0.702 |
| 7.647 | 2.381 | -0.743 | 12.421 | 2.723 | -0.697 | 8.306 | 1.985 | -0.705 |
| 7.681 | 2.381 | -0.743 | 12.470 | 2.709 | -0.701 | 8.237 | 2.000 | -0.703 |
| 7.724 | 2.381 | -0.744 | 12.515 | 2.694 | -0.706 | 8.172 | 2.015 | -0.704 |
| 7.778 | 2.381 | -0.745 | 12.557 | 2.679 | -0.715 | 8.111 | 2.030 | -0.706 |
| 7.841 | 2.381 | -0.745 | 12.595 | 2.663 | -0.717 | 8.054 | 2.046 | -0.706 |
| 7.914 | 2.381 | -0.746 | 12.629 | 2.647 | -0.720 | 8.000 | 2.063 | -0.707 |
| 7.994 | 2.380 | -0.748 | 12.660 | 2.630 | -0.728 | 7.951 | 2.080 | -0.709 |
| 8.074 | 2.380 | -0.748 | 12.686 | 2.613 | -0.734 | 7.905 | 2.098 | -0.709 |
| 8.154 | 2.380 | -0.750 | 12.709 | 2.595 | -0.733 | 7.864 | 2.116 | -0.713 |
| 8.234 | 2.380 | -0.751 | 12.728 | 2.577 | -0.736 | 7.827 | 2.134 | -0.716 |
| 8.313 | 2.379 | -0.753 | 12.742 | 2.558 | -0.739 | 7.794 | 2.153 | -0.719 |
| 8.393 | 2.379 | -0.753 | 12.752 | 2.539 | -0.743 | 7.765 | 2.172 | -0.720 |
| 8.473 | 2.379 | -0.753 | 12.759 | 2.520 | -0.743 | 7.741 | 2.191 | -0.720 |
| 8.553 | 2.378 | -0.755 | 12.761 | 2.501 | -0.744 | 7.721 | 2.211 | -0.723 |
| 8.633 | 2.378 | -0.756 | 12.758 | 2.481 | -0.747 | 7.706 | 2.231 | -0.724 |
| 8.713 | 2.378 | -0.758 | 12.752 | 2.460 | -0.747 | 7.694 | 2.251 | -0.726 |
| 8.793 | 2.378 | -0.758 | 12.741 | 2.440 | -0.748 | 7.687 | 2.271 | -0.728 |
| 8.873 | 2.377 | -0.759 | 12.725 | 2.419 | -0.749 | 7.684 | 2.291 | -0.730 |
| 8.953 | 2.377 | -0.760 | 12.706 | 2.398 | -0.750 | 7.686 | 2.311 | -0.733 |
| 9.033 | 2.377 | -0.761 | 12.682 | 2.378 | -0.751 | 7.692 | 2.331 | -0.735 |
| 9.113 | 2.376 | -0.761 | 12.654 | 2.357 | -0.751 | 7.701 | 2.352 | -0.736 |
| 9.193 | 2.376 | -0.762 | 12.622 | 2.335 | -0.752 | 7.715 | 2.372 | -0.737 |
| 9.273 | 2.376 | -0.763 | 12.585 | 2.314 | -0.751 | 7.734 | 2.392 | -0.738 |
| 9.352 | 2.376 | -0.763 | 12.544 | 2.293 | -0.753 | 7.756 | 2.412 | -0.741 |
| 9.432 | 2.375 | -0.764 | 12.499 | 2.272 | -0.752 | 7.782 | 2.432 | -0.741 |
| 9.512 | 2.375 | -0.766 | 12.450 | 2.251 | -0.752 | 7.812 | 2.451 | -0.742 |
| 9.592 | 2.375 | -0.765 | 12.397 | 2.230 | -0.751 | 7.846 | 2.471 | -0.745 |
| 9.672 | 2.374 | -0.766 | 12.340 | 2.210 | -0.751 | 7.884 | 2.490 | -0.745 |
| 9.752 | 2.374 | -0.767 | 12.279 | 2.189 | -0.750 | 7.925 | 2.509 | -0.746 |
| 9.832 | 2.374 | -0.768 | 12.214 | 2.169 | -0.750 | 7.970 | 2.528 | -0.746 |
| 9.912 | 2.374 | -0.768 | 12.146 | 2.150 | -0.749 | 8.019 | 2.546 | -0.747 |
| 9.992 | 2.373 | -0.769 | 12.074 | 2.130 | -0.748 | 8.071 | 2.564 | -0.747 |
| 10.072 | 2.373 | -0.770 | 11.999 | 2.111 | -0.746 | 8.126 | 2.582 | -0.747 |
| 10.152 | 2.373 | -0.771 | 11.920 | 2.093 | -0.744 | 8.185 | 2.599 | -0.748 |
| 10.232 | 2.372 | -0.771 | 11.838 | 2.074 | -0.742 | 8.246 | 2.616 | -0.748 |
| 10.312 | 2.372 | -0.771 | 11.753 | 2.057 | -0.740 | 8.311 | 2.633 | -0.748 |
| 10.392 | 2.372 | -0.771 | 11.665 | 2.040 | -0.736 | 8.379 | 2.649 | -0.749 |
| 10.471 | 2.372 | -0.771 | 11.574 | 2.024 | -0.736 | 8.449 | 2.665 | -0.748 |
| 10.551 | 2.371 | -0.771 | 11.481 | 2.008 | -0.734 | 8.522 | 2.680 | -0.748 |
| 10.235 | 2.872 | -0.722 | 11.386 | 1.993 | -0.732 | 8.597 | 2.694 | -0.747 |
| 10.248 | 2.872 | -0.724 | 11.288 | 1.979 | -0.729 | 8.675 | 2.709 | -0.746 |
| 10.271 | 2.872 | -0.722 | 11.188 | 1.965 | -0.726 | 8.756 | 2.722 | -0.746 |
| 10.304 | 2.872 | -0.723 | 11.086 | 1.953 | -0.724 | 8.838 | 2.735 | -0.746 |
| 10.345 | 2.872 | -0.718 | 10.982 | 1.941 | -0.721 | 8.922 | 2.748 | -0.745 |
| 10.397 | 2.872 | -0.718 | 10.877 | 1.930 | -0.718 | 9.009 | 2.760 | -0.746 |
| 10.456 | 2.872 | -0.711 | 10.771 | 1.920 | -0.718 | 9.097 | 2.772 | -0.744 |
| 10.525 | 2.872 | -0.717 | 10.664 | 1.911 | -0.714 | 9.187 | 2.783 | -0.744 |
| 10.602 | 2.872 | -0.709 | 10.556 | 1.902 | -0.713 | 9.278 | 2.793 | -0.742 |
| 10.687 | 2.872 | -0.713 | 10.447 | 1.895 | -0.711 | 9.370 | 2.803 | -0.742 |
| 10.780 | 2.872 | -0.710 | 10.338 | 1.889 | -0.708 | 9.464 | 2.812 | -0.742 |
| 10.876 | 2.872 | -0.706 | 10.228 | 1.884 | -0.707 | 9.559 | 2.820 | -0.740 |
| 10.971 | 2.871 | -0.704 | 10.118 | 1.879 | -0.705 | 9.655 | 2.828 | -0.738 |
| 11.065 | 2.869 | -0.692 | 10.009 | 1.876 | -0.703 | 9.752 | 2.836 | -0.736 |
| 11.158 | 2.867 | -0.679 | 9.900 | 1.874 | -0.701 | 9.841 | 2.842 | -0.733 |
| 11.249 | 2.864 | -0.689 | 9.792 | 1.873 | -0.698 | 9.922 | 2.848 | -0.730 |
| 11.339 | 2.860 | -0.680 | 9.685 | 1.872 | -0.697 | 9.994 | 2.854 | -0.726 |
| 11.427 | 2.855 | -0.676 | 9.578 | 1.873 | -0.695 | 10.057 | 2.859 | -0.723 |
| 11.514 | 2.850 | -0.672 | 9.473 | 1.875 | -0.694 | 10.111 | 2.863 | -0.723 |
| 11.598 | 2.844 | -0.679 | 9.369 | 1.878 | -0.693 | 10.155 | 2.866 | -0.721 |
| 11.681 | 2.838 | -0.683 | 9.267 | 1.882 | -0.692 | 10.189 | 2.869 | -0.719 |
| 11.761 | 2.831 | -0.681 | 9.167 | 1.887 | -0.692 | 10.213 | 2.871 | -0.721 |
| 11.840 | 2.823 | -0.682 | 9.069 | 1.893 | -0.694 | 10.227 | 2.872 | -0.718 |
| 11.915 | 2.814 | -0.676 | 8.973 | 1.900 | -0.694 | 10.232 | 2.372 | -0.767 |
| 11.989 | 2.805 | -0.684 | 8.880 | 1.907 | -0.693 |  |  |  |
| 12.059 | 2.795 | -0.675 | 8.789 | 1.916 | -0.693 |  |  |  |

## Optimization Run \#2

Table S10. Tabulated data from the second optimization run starting at $[\tau \text {, Equiv }]_{0}=[15 \mathrm{~min}, 3.5]$

| Tau (min) | Eq (-) | Obj Func (-) | 18.666 | 3.700 | 0.769 | 14.853 | 3.007 | 0.283 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15.000 | 4.000 | 0.677 | 18.680 | 3.687 | 0.765 | 14.743 | 3.005 | 0.229 |
| 15.005 | 4.000 | 0.676 | 18.691 | 3.674 | 0.762 | 14.634 | 3.003 | 0.190 |
| 15.017 | 4.000 | 0.676 | 18.699 | 3.661 | 0.760 | 14.525 | 3.002 | 0.174 |
| 15.036 | 4.000 | 0.675 | 18.705 | 3.648 | 0.755 | 14.417 | 3.001 | 0.153 |
| 15.061 | 4.000 | 0.679 | 18.707 | 3.634 | 0.751 | 14.309 | 3.000 | 0.148 |
| 15.093 | 4.000 | 0.678 | 18.707 | 3.621 | 0.746 | 14.201 | 3.000 | 0.102 |
| 15.131 | 4.000 | 0.681 | 18.704 | 3.607 | 0.740 | 14.095 | 3.001 | 0.053 |
| 15.175 | 4.000 | 0.683 | 18.697 | 3.593 | 0.736 | 13.989 | 3.001 | 0.053 |
| 15.226 | 4.000 | 0.686 | 18.688 | 3.580 | 0.731 | 13.884 | 3.003 | 0.084 |
| 15.283 | 4.000 | 0.688 | 18.676 | 3.566 | 0.725 | 13.780 | 3.005 | 0.029 |
| 15.346 | 4.000 | 0.691 | 18.661 | 3.551 | 0.719 | 13.678 | 3.007 | -0.006 |
| 15.415 | 4.000 | 0.694 | 18.643 | 3.537 | 0.712 | 13.576 | 3.010 | -0.016 |
| 15.489 | 4.000 | 0.699 | 18.623 | 3.523 | 0.706 | 13.476 | 3.013 | -0.003 |
| 15.569 | 4.000 | 0.701 | 18.599 | 3.509 | 0.700 | 13.377 | 3.017 | -0.048 |
| 15.654 | 4.000 | 0.705 | 18.572 | 3.494 | 0.693 | 13.279 | 3.021 | -0.054 |
| 15.745 | 4.000 | 0.709 | 18.543 | 3.480 | 0.687 | 13.183 | 3.026 | -0.045 |
| 15.841 | 4.000 | 0.714 | 18.510 | 3.466 | 0.679 | 13.089 | 3.031 | -0.030 |
| 15.937 | 4.000 | 0.719 | 18.475 | 3.451 | 0.673 | 12.996 | 3.036 | -0.063 |
| 16.032 | 3.999 | 0.723 | 18.436 | 3.437 | 0.666 | 12.905 | 3.042 | -0.053 |
| 16.126 | 3.998 | 0.728 | 18.395 | 3.423 | 0.659 | 12.816 | 3.048 | -0.089 |
| 16.220 | 3.997 | 0.734 | 18.351 | 3.408 | 0.650 | 12.729 | 3.055 | -0.053 |
| 16.313 | 3.995 | 0.737 | 18.304 | 3.394 | 0.643 | 12.644 | 3.062 | -0.046 |
| 16.405 | 3.993 | 0.741 | 18.255 | 3.380 | 0.637 | 12.561 | 3.070 | -0.059 |
| 16.497 | 3.991 | 0.745 | 18.203 | 3.365 | 0.628 | 12.480 | 3.078 | -0.041 |
| 16.587 | 3.989 | 0.749 | 18.147 | 3.351 | 0.621 | 12.401 | 3.086 | -0.041 |
| 16.676 | 3.986 | 0.753 | 18.090 | 3.337 | 0.611 | 12.325 | 3.095 | -0.035 |
| 16.764 | 3.983 | 0.758 | 18.029 | 3.323 | 0.603 | 12.250 | 3.104 | -0.036 |
| 16.850 | 3.979 | 0.761 | 17.966 | 3.309 | 0.597 | 12.179 | 3.113 | -0.040 |
| 16.936 | 3.975 | 0.765 | 17.901 | 3.296 | 0.589 | 12.109 | 3.122 | -0.050 |
| 17.020 | 3.971 | 0.767 | 17.833 | 3.282 | 0.580 | 12.042 | 3.132 | -0.058 |
| 17.102 | 3.967 | 0.771 | 17.763 | 3.269 | 0.572 | 11.978 | 3.143 | -0.042 |
| 17.184 | 3.962 | 0.772 | 17.690 | 3.256 | 0.563 | 11.916 | 3.153 | -0.039 |
| 17.263 | 3.957 | 0.775 | 17.614 | 3.243 | 0.555 | 11.857 | 3.164 | -0.030 |
| 17.341 | 3.952 | 0.777 | 17.537 | 3.230 | 0.546 | 11.801 | 3.175 | -0.016 |
| 17.418 | 3.946 | 0.782 | 17.457 | 3.217 | 0.539 | 11.747 | 3.186 | -0.004 |
| 17.493 | 3.940 | 0.783 | 17.375 | 3.205 | 0.530 | 11.696 | 3.198 | -0.003 |
| 17.566 | 3.934 | 0.785 | 17.291 | 3.193 | 0.522 | 11.647 | 3.210 | 0.047 |
| 17.637 | 3.927 | 0.787 | 17.205 | 3.181 | 0.515 | 11.602 | 3.222 | 0.026 |
| 17.707 | 3.920 | 0.790 | 17.117 | 3.170 | 0.506 | 11.559 | 3.234 | 0.038 |
| 17.774 | 3.913 | 0.790 | 17.027 | 3.158 | 0.498 | 11.519 | 3.246 | 0.067 |
| 17.840 | 3.906 | 0.792 | 16.936 | 3.147 | 0.491 | 11.482 | 3.259 | 0.063 |
| 17.903 | 3.898 | 0.792 | 16.842 | 3.137 | 0.483 | 11.448 | 3.272 | 0.091 |
| 17.965 | 3.890 | 0.793 | 16.747 | 3.126 | 0.473 | 11.417 | 3.285 | 0.113 |
| 18.024 | 3.881 | 0.794 | 16.650 | 3.116 | 0.466 | 11.388 | 3.298 | 0.145 |
| 18.081 | 3.873 | 0.794 | 16.552 | 3.107 | 0.459 | 11.363 | 3.311 | 0.123 |
| 18.136 | 3.864 | 0.795 | 16.453 | 3.097 | 0.451 | 11.341 | 3.324 | 0.196 |
| 18.189 | 3.854 | 0.796 | 16.352 | 3.089 | 0.444 | 11.321 | 3.337 | 0.209 |
| 18.239 | 3.845 | 0.795 | 16.250 | 3.080 | 0.437 | 11.304 | 3.351 | 0.228 |
| 18.287 | 3.835 | 0.794 | 16.146 | 3.072 | 0.429 | 11.291 | 3.365 | 0.229 |
| 18.333 | 3.825 | 0.793 | 16.042 | 3.064 | 0.423 | 11.280 | 3.378 | 0.294 |
| 18.376 | 3.815 | 0.792 | 15.937 | 3.057 | 0.416 | 11.272 | 3.392 | 0.310 |
| 18.417 | 3.805 | 0.793 | 15.831 | 3.050 | 0.410 | 11.267 | 3.406 | 0.322 |
| 18.455 | 3.794 | 0.791 | 15.724 | 3.043 | 0.403 | 11.265 | 3.419 | 0.367 |
| 18.491 | 3.783 | 0.789 | 15.616 | 3.037 | 0.395 | 11.266 | 3.433 | 0.372 |
| 18.524 | 3.772 | 0.788 | 15.508 | 3.032 | 0.390 | 11.270 | 3.447 | 0.376 |
| 18.554 | 3.760 | 0.785 | 15.399 | 3.026 | 0.384 | 11.277 | 3.461 | 0.380 |
| 18.582 | 3.748 | 0.782 | 15.290 | 3.022 | 0.364 | 11.287 | 3.475 | 0.383 |
| 18.607 | 3.736 | 0.598 | 15.181 | 3.017 | 0.338 | 11.299 | 3.488 | 0.388 |
| 18.629 | 3.724 | 0.777 | 15.072 | 3.014 | 0.297 | 11.314 | 3.502 | 0.392 |
| 18.649 | 3.712 | 0.773 | 14.962 | 3.010 | 0.256 | 11.332 | 3.516 | 0.396 |


| 11.353 | 3.529 | 0.402 | 14.793 | 3.500 | 0.486 | 9.171 | 2.713 | -0.745 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11.377 | 3.543 | 0.406 | 14.740 | 3.500 | 0.484 | 9.084 | 2.700 | -0.748 |
| 11.403 | 3.557 | 0.411 | 14.681 | 3.500 | 0.482 | 8.997 | 2.687 | -0.755 |
| 11.432 | 3.570 | 0.415 | 14.616 | 3.500 | 0.478 | 8.910 | 2.674 | -0.766 |
| 11.464 | 3.583 | 0.422 | 14.545 | 3.500 | 0.475 | 8.823 | 2.661 | -0.766 |
| 11.498 | 3.597 | 0.424 | 14.467 | 3.500 | 0.472 | 8.736 | 2.648 | -0.770 |
| 11.535 | 3.610 | 0.429 | 14.383 | 3.499 | 0.469 | 8.650 | 2.635 | -0.774 |
| 11.574 | 3.623 | 0.436 | 14.296 | 3.486 | 0.463 | 8.563 | 2.622 | -0.776 |
| 11.616 | 3.636 | 0.441 | 14.209 | 3.472 | 0.455 | 8.476 | 2.609 | -0.777 |
| 11.660 | 3.649 | 0.445 | 14.122 | 3.459 | 0.447 | 8.389 | 2.596 | -0.779 |
| 11.707 | 3.661 | 0.451 | 14.035 | 3.446 | 0.441 | 8.302 | 2.582 | -0.779 |
| 11.756 | 3.674 | 0.455 | 13.949 | 3.433 | 0.435 | 8.215 | 2.569 | -0.780 |
| 11.807 | 3.686 | 0.461 | 13.862 | 3.420 | 0.429 | 8.128 | 2.556 | -0.779 |
| 11.861 | 3.698 | 0.466 | 13.775 | 3.407 | 0.419 | 8.042 | 2.543 | -0.777 |
| 11.917 | 3.710 | 0.471 | 13.688 | 3.394 | 0.414 | 8.220 | 3.069 | -0.591 |
| 11.975 | 3.722 | 0.476 | 13.601 | 3.381 | 0.409 | 8.237 | 3.069 | -0.589 |
| 12.035 | 3.734 | 0.482 | 13.514 | 3.368 | 0.401 | 8.266 | 3.069 | -0.585 |
| 12.098 | 3.745 | 0.487 | 13.427 | 3.355 | 0.395 | 8.306 | 3.069 | -0.576 |
| 12.162 | 3.757 | 0.494 | 13.340 | 3.342 | 0.389 | 8.358 | 3.069 | -0.574 |
| 12.229 | 3.768 | 0.499 | 13.254 | 3.328 | 0.384 | 8.421 | 3.069 | -0.582 |
| 12.297 | 3.779 | 0.503 | 13.167 | 3.315 | 0.377 | 8.495 | 3.069 | -0.586 |
| 12.367 | 3.789 | 0.508 | 13.080 | 3.302 | 0.371 | 8.579 | 3.069 | -0.569 |
| 12.439 | 3.800 | 0.513 | 12.993 | 3.289 | 0.365 | 8.672 | 3.069 | -0.578 |
| 12.513 | 3.810 | 0.519 | 12.906 | 3.276 | 0.361 | 8.768 | 3.069 | -0.582 |
| 12.589 | 3.820 | 0.525 | 12.819 | 3.263 | 0.354 | 8.862 | 3.067 | -0.569 |
| 12.666 | 3.830 | 0.531 | 12.732 | 3.250 | 0.348 | 8.955 | 3.064 | -0.579 |
| 12.745 | 3.839 | 0.536 | 12.646 | 3.237 | 0.342 | 9.047 | 3.060 | -0.547 |
| 12.825 | 3.849 | 0.542 | 12.559 | 3.224 | 0.339 | 9.136 | 3.055 | -0.551 |
| 12.907 | 3.858 | 0.547 | 12.472 | 3.211 | 0.300 | 9.223 | 3.049 | -0.556 |
| 12.990 | 3.867 | 0.553 | 12.385 | 3.198 | 0.240 | 9.308 | 3.042 | -0.563 |
| 13.075 | 3.875 | 0.559 | 12.298 | 3.185 | 0.242 | 9.391 | 3.034 | -0.562 |
| 13.161 | 3.883 | 0.564 | 12.211 | 3.171 | 0.170 | 9.471 | 3.025 | -0.564 |
| 13.248 | 3.892 | 0.570 | 12.124 | 3.158 | 0.102 | 9.547 | 3.014 | -0.569 |
| 13.337 | 3.899 | 0.575 | 12.037 | 3.145 | 0.053 | 9.621 | 3.003 | -0.567 |
| 13.426 | 3.907 | 0.581 | 11.951 | 3.132 | -0.016 | 9.692 | 2.991 | -0.541 |
| 13.517 | 3.914 | 0.586 | 11.864 | 3.119 | -0.015 | 9.759 | 2.977 | -0.577 |
| 13.608 | 3.921 | 0.592 | 11.777 | 3.106 | -0.026 | 9.822 | 2.963 | -0.570 |
| 13.701 | 3.928 | 0.598 | 11.690 | 3.093 | -0.090 | 9.882 | 2.948 | -0.572 |
| 13.794 | 3.934 | 0.604 | 11.603 | 3.080 | -0.133 | 9.937 | 2.932 | -0.584 |
| 13.888 | 3.940 | 0.610 | 11.516 | 3.067 | -0.222 | 9.989 | 2.915 | -0.595 |
| 13.983 | 3.946 | 0.615 | 11.429 | 3.054 | -0.215 | 10.036 | 2.897 | -0.617 |
| 14.079 | 3.952 | 0.621 | 11.343 | 3.041 | -0.238 | 10.078 | 2.878 | -0.627 |
| 14.175 | 3.957 | 0.626 | 11.256 | 3.027 | -0.304 | 10.116 | 2.858 | -0.623 |
| 14.272 | 3.962 | 0.633 | 11.169 | 3.014 | -0.334 | 10.150 | 2.838 | -0.631 |
| 14.364 | 3.967 | 0.638 | 11.082 | 3.001 | -0.347 | 10.178 | 2.817 | -0.646 |
| 14.451 | 3.971 | 0.643 | 10.995 | 2.988 | -0.406 | 10.202 | 2.795 | -0.656 |
| 14.532 | 3.975 | 0.649 | 10.908 | 2.975 | -0.410 | 10.221 | 2.772 | -0.676 |
| 14.607 | 3.979 | 0.652 | 10.821 | 2.962 | -0.438 | 10.234 | 2.749 | -0.685 |
| 14.675 | 3.983 | 0.656 | 10.734 | 2.949 | -0.455 | 10.243 | 2.725 | -0.696 |
| 14.737 | 3.986 | 0.660 | 10.648 | 2.936 | -0.470 | 10.246 | 2.701 | -0.714 |
| 14.793 | 3.989 | 0.665 | 10.561 | 2.923 | -0.501 | 10.244 | 2.676 | -0.729 |
| 14.842 | 3.991 | 0.668 | 10.474 | 2.910 | -0.550 | 10.236 | 2.651 | -0.732 |
| 14.885 | 3.994 | 0.670 | 10.387 | 2.897 | -0.553 | 10.223 | 2.625 | -0.751 |
| 14.921 | 3.996 | 0.673 | 10.300 | 2.883 | -0.590 | 10.205 | 2.599 | -0.756 |
| 14.950 | 3.997 | 0.675 | 10.213 | 2.870 | -0.596 | 10.181 | 2.573 | -0.769 |
| 14.973 | 3.999 | 0.676 | 10.126 | 2.857 | -0.618 | 10.152 | 2.547 | -0.783 |
| 14.989 | 3.999 | 0.679 | 10.039 | 2.844 | -0.626 | 10.118 | 2.521 | -0.784 |
| 14.998 | 4.000 | 0.677 | 9.953 | 2.831 | -0.654 | 10.078 | 2.494 | -0.790 |
| 15.000 | 3.500 | 0.495 | 9.866 | 2.818 | -0.661 | 10.034 | 2.468 | -0.792 |
| 14.994 | 3.500 | 0.493 | 9.779 | 2.805 | -0.672 | 9.984 | 2.442 | -0.793 |
| 14.982 | 3.500 | 0.494 | 9.692 | 2.792 | -0.694 | 9.929 | 2.416 | -0.796 |
| 14.966 | 3.500 | 0.491 | 9.605 | 2.779 | -0.708 | 9.870 | 2.391 | -0.793 |
| 14.943 | 3.500 | 0.491 | 9.518 | 2.766 | -0.703 | 9.805 | 2.366 | -0.793 |
| 14.914 | 3.500 | 0.490 | 9.431 | 2.753 | -0.714 | 9.736 | 2.341 | -0.792 |
| 14.880 | 3.500 | 0.489 | 9.345 | 2.740 | -0.732 | 9.663 | 2.317 | -0.790 |
| 14.839 | 3.500 | 0.487 | 9.258 | 2.726 | -0.736 | 9.585 | 2.294 | -0.788 |


| 9.504 | 2.271 | -0.786 | 7.392 | 2.982 | -0.707 | 12.313 | 2.172 | -0.779 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 9.418 | 2.249 | -0.783 | 7.484 | 2.995 | -0.696 | 12.273 | 2.150 | -0.777 |
| 9.329 | 2.229 | -0.780 | 7.577 | 3.006 | -0.675 | 12.229 | 2.129 | -0.775 |
| 9.237 | 2.209 | -0.776 | 7.672 | 3.017 | -0.679 | 12.180 | 2.107 | -0.775 |
| 9.142 | 2.190 | -0.774 | 7.768 | 3.027 | -0.670 | 12.127 | 2.086 | -0.773 |
| 9.044 | 2.172 | -0.771 | 7.863 | 3.036 | -0.660 | 12.070 | 2.065 | -0.770 |
| 8.943 | 2.156 | -0.766 | 7.947 | 3.044 | -0.642 | 12.009 | 2.044 | -0.767 |
| 8.840 | 2.141 | -0.764 | 8.021 | 3.051 | -0.648 | 11.945 | 2.024 | -0.763 |
| 8.735 | 2.127 | -0.760 | 8.083 | 3.056 | -0.633 | 11.876 | 2.003 | -0.763 |
| 8.629 | 2.115 | -0.758 | 8.134 | 3.061 | -0.618 | 11.804 | 1.984 | -0.759 |
| 8.521 | 2.104 | -0.754 | 8.173 | 3.065 | -0.624 | 11.729 | 1.964 | -0.756 |
| 8.413 | 2.094 | -0.751 | 8.199 | 3.068 | -0.606 | 11.650 | 1.945 | -0.752 |
| 8.304 | 2.086 | -0.748 | 8.213 | 3.069 | -0.612 | 11.567 | 1.927 | -0.750 |
| 8.194 | 2.080 | -0.744 | 8.215 | 2.569 | -0.774 | 11.482 | 1.909 | -0.747 |
| 8.085 | 2.075 | -0.741 | 10.007 | 2.727 | -0.724 | 11.394 | 1.892 | -0.743 |
| 7.975 | 2.071 | -0.740 | 10.017 | 2.727 | -0.727 | 11.303 | 1.875 | -0.738 |
| 7.867 | 2.070 | -0.737 | 10.038 | 2.727 | -0.717 | 11.209 | 1.859 | -0.736 |
| 7.760 | 2.070 | -0.734 | 10.068 | 2.727 | -0.730 | 11.113 | 1.844 | -0.733 |
| 7.654 | 2.071 | -0.733 | 10.107 | 2.727 | -0.713 | 11.014 | 1.830 | -0.729 |
| 7.549 | 2.074 | -0.730 | 10.156 | 2.727 | -0.720 | 10.914 | 1.816 | -0.724 |
| 7.447 | 2.079 | -0.731 | 10.214 | 2.727 | -0.710 | 10.812 | 1.803 | -0.722 |
| 7.347 | 2.085 | -0.728 | 10.282 | 2.727 | -0.721 | 10.708 | 1.792 | -0.716 |
| 7.250 | 2.092 | -0.730 | 10.357 | 2.727 | -0.703 | 10.602 | 1.781 | -0.713 |
| 7.155 | 2.102 | -0.729 | 10.441 | 2.727 | -0.709 | 10.495 | 1.771 | -0.711 |
| 7.063 | 2.112 | -0.727 | 10.533 | 2.727 | -0.698 | 10.388 | 1.762 | -0.707 |
| 6.975 | 2.124 | -0.728 | 10.629 | 2.727 | -0.687 | 10.279 | 1.754 | -0.703 |
| 6.891 | 2.137 | -0.727 | 10.724 | 2.726 | -0.685 | 10.170 | 1.747 | -0.701 |
| 6.810 | 2.152 | -0.727 | 10.818 | 2.724 | -0.695 | 10.061 | 1.741 | -0.697 |
| 6.734 | 2.168 | -0.728 | 10.911 | 2.721 | -0.680 | 9.951 | 1.736 | -0.695 |
| 6.662 | 2.184 | -0.728 | 11.003 | 2.718 | -0.673 | 9.842 | 1.732 | -0.691 |
| 6.594 | 2.202 | -0.730 | 11.092 | 2.714 | -0.685 | 9.733 | 1.729 | -0.690 |
| 6.531 | 2.221 | -0.731 | 11.181 | 2.710 | -0.676 | 9.624 | 1.728 | -0.687 |
| 6.472 | 2.241 | -0.733 | 11.267 | 2.704 | -0.672 | 9.516 | 1.727 | -0.684 |
| 6.419 | 2.262 | -0.733 | 11.351 | 2.698 | -0.661 | 9.410 | 1.727 | -0.683 |
| 6.370 | 2.283 | -0.734 | 11.434 | 2.692 | -0.674 | 9.304 | 1.729 | -0.682 |
| 6.327 | 2.306 | -0.736 | 11.514 | 2.684 | -0.668 | 9.200 | 1.731 | -0.681 |
| 6.289 | 2.328 | -0.737 | 11.592 | 2.676 | -0.684 | 9.097 | 1.735 | -0.681 |
| 6.256 | 2.352 | -0.737 | 11.667 | 2.667 | -0.671 | 8.996 | 1.740 | -0.680 |
| 6.228 | 2.376 | -0.740 | 11.740 | 2.658 | -0.658 | 8.898 | 1.745 | -0.679 |
| 6.206 | 2.400 | -0.741 | 11.810 | 2.647 | -0.677 | 8.801 | 1.752 | -0.678 |
| 6.189 | 2.424 | -0.741 | 11.877 | 2.637 | -0.673 | 8.707 | 1.760 | -0.678 |
| 6.177 | 2.449 | -0.742 | 11.941 | 2.625 | -0.680 | 8.615 | 1.768 | -0.680 |
| 6.171 | 2.474 | -0.742 | 12.002 | 2.613 | -0.694 | 8.527 | 1.778 | -0.681 |
| 6.170 | 2.500 | -0.744 | 12.060 | 2.600 | -0.681 | 8.441 | 1.788 | -0.681 |
| 6.175 | 2.525 | -0.744 | 12.114 | 2.586 | -0.688 | 8.358 | 1.800 | -0.683 |
| 6.184 | 2.550 | -0.746 | 12.165 | 2.572 | -0.688 | 8.278 | 1.812 | -0.683 |
| 6.199 | 2.575 | -0.746 | 12.213 | 2.558 | -0.702 | 8.202 | 1.825 | -0.685 |
| 6.219 | 2.600 | -0.746 | 12.257 | 2.542 | -0.702 | 8.130 | 1.839 | -0.688 |
| 6.244 | 2.625 | -0.746 | 12.297 | 2.527 | -0.708 | 8.060 | 1.853 | -0.688 |
| 6.274 | 2.649 | -0.746 | 12.334 | 2.510 | -0.717 | 7.995 | 1.868 | -0.691 |
| 6.308 | 2.674 | -0.745 | 12.366 | 2.493 | -0.723 | 7.934 | 1.884 | -0.693 |
| 6.348 | 2.697 | -0.746 | 12.395 | 2.476 | -0.732 | 7.876 | 1.901 | -0.695 |
| 6.391 | 2.721 | -0.746 | 12.419 | 2.458 | -0.740 | 7.823 | 1.918 | -0.699 |
| 6.440 | 2.744 | -0.745 | 12.440 | 2.440 | -0.739 | 7.773 | 1.935 | -0.701 |
| 6.492 | 2.766 | -0.745 | 12.456 | 2.421 | -0.755 | 7.728 | 1.953 | -0.704 |
| 6.549 | 2.788 | -0.744 | 12.468 | 2.402 | -0.758 | 7.687 | 1.972 | -0.707 |
| 6.609 | 2.809 | -0.743 | 12.476 | 2.382 | -0.762 | 7.651 | 1.991 | -0.710 |
| 6.673 | 2.830 | -0.742 | 12.479 | 2.362 | -0.760 | 7.618 | 2.010 | -0.713 |
| 6.741 | 2.850 | -0.739 | 12.478 | 2.342 | -0.768 | 7.590 | 2.029 | -0.717 |
| 6.813 | 2.869 | -0.738 | 12.473 | 2.321 | -0.773 | 7.567 | 2.049 | -0.721 |
| 6.887 | 2.888 | -0.736 | 12.463 | 2.300 | -0.777 | 7.548 | 2.069 | -0.724 |
| 6.965 | 2.905 | -0.731 | 12.449 | 2.279 | -0.778 | 7.533 | 2.090 | -0.726 |
| 7.045 | 2.922 | -0.729 | 12.431 | 2.258 | -0.779 | 7.523 | 2.110 | -0.730 |
| 7.129 | 2.939 | -0.724 | 12.408 | 2.237 | -0.781 | 7.517 | 2.131 | -0.733 |
| 7.214 | 2.954 | -0.718 | 12.381 | 2.215 | -0.782 | 7.515 | 2.151 | -0.735 |
| 7.302 | 2.968 | -0.715 | 12.349 | 2.194 | -0.781 | 7.518 | 2.172 | -0.739 |


| 7.525 | 2.193 | -0.742 |
| :---: | :---: | :---: |
| 7.536 | 2.213 | -0.745 |
| 7.552 | 2.234 | -0.747 |
| 7.572 | 2.255 | -0.750 |
| 7.595 | 2.275 | -0.752 |
| 7.623 | 2.295 | -0.755 |
| 7.655 | 2.315 | -0.757 |
| 7.691 | 2.335 | -0.759 |
| 7.731 | 2.354 | -0.761 |
| 7.774 | 2.374 | -0.762 |
| 7.821 | 2.393 | -0.765 |
| 7.871 | 2.411 | -0.767 |
| 7.925 | 2.430 | -0.767 |
| 7.983 | 2.447 | -0.769 |
| 8.043 | 2.465 | -0.771 |
| 8.107 | 2.482 | -0.771 |
| 8.174 | 2.499 | -0.772 |
| 8.243 | 2.515 | -0.773 |
| 8.315 | 2.530 | -0.772 |
| 8.390 | 2.545 | -0.774 |
| 8.468 | 2.560 | -0.775 |
| 8.547 | 2.574 | -0.776 |
| 8.629 | 2.588 | -0.776 |
| 8.714 | 2.601 | -0.776 |
| 8.800 | 2.613 | -0.774 |
| 8.888 | 2.625 | -0.773 |
| 8.977 | 2.636 | -0.768 |
| 9.068 | 2.647 | -0.764 |
| 9.161 | 2.657 | -0.770 |
| 9.255 | 2.666 | -0.762 |
| 9.350 | 2.675 | -0.758 |
| 9.446 | 2.683 | -0.750 |
| 9.542 | 2.690 | -0.744 |
| 9.631 | 2.697 | -0.750 |
| 9.711 | 2.703 | -0.736 |
| 9.782 | 2.709 | -0.737 |
| 9.844 | 2.714 | -0.735 |
| 9.896 | 2.718 | -0.727 |
| 9.938 | 2.721 | -0.723 |
| 9.970 | 2.724 | -0.716 |
| 9.992 | 2.726 | -0.727 |
| 10.004 | 2.727 | -0.714 |
| 10.006 | 2.227 | -0.780 |
|  |  |  |

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[^0]:    \# Algorithm terminated due to optimum internal to RSM as confirmed by the Hessian of the RSM.

