Supporting Information: Coherent Ring-Current Dynamics of Mg-phthalocyanine probed by Time-Resolved Circular Dichroism

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S1 Derivation of Time-Resolved Circular Dichroism

The electronic dynamics driven by the pump pulse is simulated by the evolution of wave function. Here we derive the TRCD signal induced by the probe pulse.

A Gaussian enveloped wave with amplitude Am , central frequency ω_c and duration σ , $W(t) = Am e^{-\frac{(t-T)^2}{2\sigma^2}}(e^{-i\omega_c t} + e^{i\omega_c t})$, can be expanded by frequency components $W(t)$ = $\frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \left[e^{i\omega t} \tilde{W}_+(\omega) + e^{-i\omega t} \tilde{W}_-(\omega) \right]$, where

$$
\tilde{W}_{+}(\omega) = Am \int_{-\infty}^{\infty} dt e^{-i\omega t} e^{-\frac{(t-T)^{2}}{2\sigma^{2}}} e^{i\omega_{c}t} = \sqrt{2\pi}\sigma \left[e^{-i(\omega - \omega_{c})T} e^{-\frac{\sigma^{2}}{2}(\omega - \omega_{c})^{2}} \right]
$$
(S1)

$$
\tilde{W}_{-}(\omega) = Am \int_{-\infty}^{\infty} dt e^{i\omega t} e^{-\frac{(t-T)^2}{2\sigma^2}} e^{-i\omega_c t} = \sqrt{2\pi} \sigma \left[e^{i(\omega - \omega_c)T} e^{-\frac{\sigma^2}{2}(\omega - \omega_c)^2} \right]
$$
(S2)

The vector potential of Gaussian enveloped probe pulse with unit vector of polarization vector $\pmb{\epsilon}$ can be written as

$$
\mathbf{A}(\mathbf{r},t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega (\tilde{W}_{-}(\omega)\boldsymbol{\epsilon}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + \tilde{W}_{+}(\omega)\boldsymbol{\epsilon}^{*}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)})
$$

=
$$
\int_{-\infty}^{\infty} d\omega (\mathbf{A}^{-}(\omega,\mathbf{r},t) + \mathbf{A}^{+}(\omega,\mathbf{r},t))
$$
(S3)

where $\mathbf{A}^-(\omega,\mathbf{r},t) = \frac{1}{2\pi}\tilde{W}_-(\omega)\boldsymbol{\epsilon}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ and $\mathbf{A}^+(\omega,\mathbf{r},t) = \frac{1}{2\pi}\tilde{W}_+(\omega)\boldsymbol{\epsilon}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$.

With rotating wave approximation, the signal at photon frequency ω_1 is

$$
S(\omega_1) = -\frac{2}{\hbar} Im \int d\mathbf{r} dt \langle \mathbf{j}^-(\mathbf{r}, t) \cdot \mathbf{A}^+(\omega_1, \mathbf{r}, t) \rangle
$$
 (S4)

To first order, from Fig. S1, with rotating wave approximation, we have first order ex-

Figure S1. Ladder diagram of X-ray TRXCD. We only consider the excited state absorption contribution.

pression

$$
S^{(1)}(\omega_1) = -\frac{2}{\hbar} Im \int d\mathbf{r} dt \int d\mathbf{r}_1 dt_1 d\omega_2 \left(-\frac{i}{\hbar} \left\langle \mathbf{j}^-(\mathbf{r},t) \cdot \mathbf{A}^+(\omega_1,\mathbf{r},t) \mathbf{j}^+(\mathbf{r}_1,t_1) \cdot \mathbf{A}^-(\omega_2,\mathbf{r}_1,t_1) \right\rangle \right)
$$
(S5)

change from Heisenberg picture to Schrödinger picture,

$$
S^{(1)}(\omega_1) = -\frac{2}{\hbar}Im \int d\tau d\tau' \int d\mathbf{r} d\mathbf{r}_1 \int d\omega_2 \left(-\frac{i}{\hbar}\right)
$$
 (S6)

$$
\langle U^{\dagger}(\tau')\mathbf{j}^-(\mathbf{r})\cdot\mathbf{A}^+(\omega_1,\mathbf{r},\tau+\tau')U(\tau)\mathbf{j}^+(\mathbf{r}_1)\cdot\mathbf{A}^-(\omega_2,\mathbf{r}_1,\tau)\rangle\tag{S7}
$$

expand with electronic states, the bracket expression can be expressed as

$$
S^{(1)}(\omega_{1}) = -\frac{2}{\hbar}Im \int d\tau d\tau' \int d\mathbf{r} d\mathbf{r}_{1} \int d\omega_{2} \left(-\frac{i}{\hbar} \right) \sum_{ACD} a_{D}^{*}(\tau) a_{A}(\tau) \langle D|U^{\dagger}(\tau')|D \rangle
$$

\n
$$
\langle D|\mathbf{j}(\mathbf{r}) \cdot \boldsymbol{\epsilon}^{\dagger}|C\rangle \tilde{W}_{+}(\omega_{1}) e^{i\omega_{1}(\tau+\tau')} e^{-i\mathbf{k}\cdot\mathbf{r}} \langle C|U(\tau')|C\rangle \langle C|\mathbf{j}^{\dagger}(\mathbf{r}_{1}) \cdot \boldsymbol{\epsilon}|A\rangle \tilde{W}_{-}(\omega_{2}) e^{-i\omega_{2}\tau} e^{i\mathbf{k}\cdot\mathbf{r}_{1}}
$$

\n
$$
= -\frac{2}{\hbar}Im \int d\tau d\tau' \int d\omega_{2} \left(-\frac{i}{\hbar} \right) \tilde{W}_{+}(\omega_{1}) \tilde{W}_{-}(\omega_{2})
$$

\n
$$
\times \sum_{ACD} a_{D}^{*}(\tau) a_{A}(\tau) \mathbf{j}_{DC}(\mathbf{k}) \cdot \boldsymbol{\epsilon}^{\dagger} \mathbf{j}_{CA}^{\dagger}(-\mathbf{k}) \cdot \boldsymbol{\epsilon} e^{i(E_{D}-E_{C}+\omega_{1})\tau'} e^{-i(\omega_{2}-\omega_{1})\tau}
$$
(S8)

where $\mathbf{k} = \frac{\omega_1}{c}$ $c_c^{\frac{v_1}{c}}\hat{\mathbf{e}}_k$. Multiplying by damping factor to account for core states (index C) decay, and integrate τ , τ' ,

$$
S^{(1)}(\omega_1) = -\frac{2}{\hbar} Im \left(-\frac{i}{\hbar} \right) \sum_{ACD} \mathbf{j}_{DC}(\mathbf{k}) \cdot \boldsymbol{\epsilon}^{\dagger} \mathbf{j}_{CA}^{\dagger}(-\mathbf{k}) \cdot \boldsymbol{\epsilon} \int d\omega_2 \int_0^{+\infty} d\tau \int_0^{+\infty} d\tau'
$$

$$
\tilde{W}_+(\omega_1) \tilde{W}_-(\omega_2) a_D^*(\tau) a_A(\tau) e^{-i(\omega_2 - \omega_1)\tau} e^{i(E_D - E_C + \omega_1)\tau'} e^{-\Gamma \tau'}
$$
(S9)

To account for dephasing of the excited state coherences, we multiply pure dephasing factor $\exp[-\Gamma_{1,DA}\tau]$ for the coherence of excited state $A-D$, and factor $\exp[-\Gamma_{2,DC}\tau']$ for the valence-core state dephasing,

$$
S^{(1)}(\omega_{1}) = \frac{2}{\hbar^{2}} Im \, i \sum_{ACD} \mathbf{j}_{DC}(\mathbf{k}) \cdot \boldsymbol{\epsilon}^{\dagger} \mathbf{j}_{CA}^{\dagger}(-\mathbf{k}) \cdot \boldsymbol{\epsilon} \int_{0}^{+\infty} d\tau \int_{0}^{+\infty} d\tau' Am
$$

\n
$$
\tilde{W}_{+}(\omega_{1}) e^{-\frac{\tau^{2} + T^{2} - 2\tau (T + i\sigma^{2}(\omega_{1} - \omega_{c}))}{2\sigma^{2}}} a_{D}^{*}(\tau) a_{A}(\tau) e^{-\Gamma_{1,DA}\tau} e^{i(E_{D} - E_{C} + \omega_{1})\tau'} e^{-(\Gamma + \Gamma_{2,DC})\tau'}
$$

\n
$$
= \frac{2}{\hbar^{2}} Im \, i \sum_{ACD} \mathbf{j}_{DC}(\mathbf{k}) \cdot \boldsymbol{\epsilon}^{\dagger} \mathbf{j}_{CA}^{\dagger}(-\mathbf{k}) \cdot \boldsymbol{\epsilon} \frac{i}{E_{D} - E_{C} + \omega_{1} + i(\Gamma + \Gamma_{2,DC})} \int_{0}^{+\infty} d\tau Am
$$

\n
$$
\tilde{W}_{+}(\omega_{1}) e^{-\frac{\tau^{2} + T^{2} - 2\tau (T + i\sigma^{2}(\omega_{1} - \omega_{c}))}{2\sigma^{2}}} e^{-\Gamma_{1,DA}\tau} a_{D}^{*}(\tau) a_{A}(\tau)
$$

\n
$$
= -\frac{2}{\hbar^{2}} Im \left\{ \sum_{ACD} \frac{\mathbf{j}_{DC}(\mathbf{k}) \cdot \boldsymbol{\epsilon}^{\dagger} \mathbf{j}_{CA}^{\dagger}(-\mathbf{k}) \cdot \boldsymbol{\epsilon}}{E_{D} - E_{C} + \omega_{1} + i(\Gamma + \Gamma_{2,DC})} \int_{0}^{+\infty} d\tau Am
$$

\n
$$
\tilde{W}_{+}(\omega_{1}) e^{-\frac{\tau^{2} + T^{2} - 2\tau (T + i\sigma^{2}(\omega_{1} - \omega_{c}))}{2\sigma^{2}}} e^{-\Gamma_{1,DA}\tau} a_{D}^{*}(\tau) a_{A}(\tau) \right\}
$$
(S10)

where E_A and E_D are the energy of ground state and valence excited states, E_C is the energy of core excited states. The integral of τ is a numerical integration since a^*_{I} $_{D}^{\ast}(\tau)a_{A}(\tau)$ is given by the electronic dynamics.

The circular dichroism is defined as the difference of absorption of left and right circularly polarized light, $S_{CD} = S_ - - S_+$. Define the polarization unit vector of circularly polarized wave as $\epsilon_{\pm}(\gamma) = \frac{1}{\sqrt{2\pi}}$ $\frac{1}{2}(\epsilon_{\alpha} \pm i \epsilon_{\beta}),$ where γ is the propagation direction of the wave, α , β and γ are perpendicular to each other and follow right hand rule. Specially, a wave propagating in z direction has circular polarization $\epsilon_{\pm}(z) = \frac{1}{\sqrt{2\pi}}$ $\frac{1}{2}(\boldsymbol{\epsilon}_x \pm i \boldsymbol{\epsilon}_y)$. Here the unit vector of propagation direction vector is ϵ_{κ} , we have relation

$$
\mathbf{p} \cdot \boldsymbol{\epsilon}_{-}^{\dagger} \mathbf{q} \cdot \boldsymbol{\epsilon}_{-} - \mathbf{p} \cdot \boldsymbol{\epsilon}_{+}^{\dagger} \mathbf{q} \cdot \boldsymbol{\epsilon}_{+} = -i \mathbf{p} \times \mathbf{q} \cdot \boldsymbol{\epsilon}_{\kappa}
$$

where **p** and **q** are arbitrary vectors.

The CD signal can be written as

$$
S_{\text{CD}}(\omega_{1}) = -\frac{2}{\hbar^{2}} Im \left\{ \sum_{ACD} \frac{-i \mathbf{j}_{DC}(\mathbf{k}) \times \mathbf{j}_{CA}^{\dagger}(-\mathbf{k})}{E_{D} - E_{C} + \omega_{1} + i(\Gamma + \Gamma_{2,DC})} \cdot \epsilon_{\kappa} Am \right\}
$$

$$
\int_{0}^{+\infty} d\tau \tilde{W}_{+}(\omega_{1}) e^{-\frac{\tau^{2} + T^{2} - 2\tau (T + i\sigma^{2}(\omega_{1} - \omega_{c}))}{2\sigma^{2}}} e^{-\Gamma_{1,DA}\tau} a_{D}^{*}(\tau) a_{A}(\tau) \right\}
$$

$$
= \frac{2}{\hbar^{2}} Re \left\{ \tilde{W}_{+}(\omega_{1}) \sum_{ACD} \frac{\mathbf{j}_{DC}(\mathbf{k}) \times \mathbf{j}_{CA}^{\dagger}(-\mathbf{k})}{E_{D} - E_{C} + \omega_{1} + i(\Gamma + \Gamma_{2,DC})} \cdot \epsilon_{\kappa} Am \right\}
$$

$$
\int_{0}^{+\infty} d\tau e^{-\frac{\tau^{2} + T^{2} - 2\tau (T + i\sigma^{2}(\omega_{1} - \omega_{c}))}{2\sigma^{2}}} e^{-\Gamma_{1,DA}\tau} a_{D}^{*}(\tau) a_{A}(\tau) \right\}
$$

$$
= \frac{2}{\hbar^{2}} Re \left\{ \tilde{W}_{+}(\omega_{1}) \sum_{ACD} \frac{\mathbf{j}_{DC}(\mathbf{k}) \times \mathbf{j}_{CA}^{\dagger}(-\mathbf{k})}{E_{D} - E_{C} + \omega_{1} + i(\Gamma + \Gamma_{2,DC})} \cdot \epsilon_{\kappa} Am \right\}
$$

$$
\int_{0}^{+\infty} d\tau e^{-\frac{(\tau - T)^{2}}{2\sigma^{2}} e^{i\tau(\omega_{1} - \omega_{c})} e^{-\Gamma_{1,DA}\tau} a_{D}^{*}(\tau) a_{A}(\tau) \right\}
$$
(S11)

To avoid numerical instability, insert the definition of $\tilde{W}_+(\omega_1)$ and simplify, we have

$$
S_{\text{CD}}(\omega_{1}) = \frac{2(Am)^{2}}{\hbar^{2}} Re \left\{ \sqrt{2\pi} \sigma e^{-i(\omega_{1}-\omega_{c})T} e^{-\frac{\sigma^{2}(\omega_{1}-\omega_{c})^{2}}{2}} \right. \\ \left. \sum_{ACD} \frac{\mathbf{j}_{DC}(\mathbf{k}) \times \mathbf{j}_{CA}^{\dagger}(-\mathbf{k})}{E_{D} - E_{C} + \omega_{1} + i(\Gamma + \Gamma_{2,DC})} \cdot \epsilon_{\kappa} \int_{0}^{+\infty} d\tau e^{-\frac{(\tau-T)^{2}}{2\sigma^{2}}} e^{i\tau(\omega_{1}-\omega_{c})} e^{-\Gamma_{1,DA}\tau} a_{D}^{*}(\tau) a_{A}(\tau) \right\}
$$

$$
= \frac{2(Am)^{2}}{\hbar^{2}} \sqrt{2\pi} \sigma Re \left\{ \sum_{ACD} \frac{\mathbf{j}_{DC}(\mathbf{k}) \times \mathbf{j}_{CA}^{\dagger}(-\mathbf{k})}{E_{D} - E_{C} + \omega_{1} + i(\Gamma + \Gamma_{2,DC})} \cdot \epsilon_{\kappa} \right. \\ \left. \int_{0}^{+\infty} d\tau e^{i(\omega_{1}-\omega_{c})(\tau-T)} e^{-\frac{1}{2} \left[\frac{(\tau-T)^{2}}{\sigma^{2}} + \sigma^{2}(\omega_{1}-\omega_{c})^{2} \right]} e^{-\Gamma_{1,DA}\tau} a_{D}^{*}(\tau) a_{A}(\tau) \right\}
$$
(S12)

In this work, the probe wave propagate in z direction. Take zero dephasing approximation,

where $\Gamma_{2,DC} = 0$ and $\Gamma_{1,DA} = 0$, and normalize by $(Am)^2$, Eq. (S12) become Eq. (3). In section S8 we will compare the effects of different dephasing factors.

S2 Valence and Core Excited States

Table S1. Excited states of Mg-phthalocyanine with nonzero oscillator strength. The transition electric dipoles $\langle g | \mu | e \rangle$ are in atomic unit ea_0 .

state	$\text{energy}(\text{eV})$	$\langle g \mu_x e \rangle$	$\langle g \mu_y e \rangle$	$\langle g \mu_z e \rangle$
$1E_u$ x	2.26596903	3.2204	0.0000	0.0000
$1E_u$ y	2.26596903	0.0000	-3.2204	0.0000
$2E_u$ x	3.36094484	-0.4366	0.0000	0.0000
$2E_u$ y	3.36094484	0.0000	0.4366	0.0000
$3E_u$ x	3.65324515	0.7392	0.0000	0.0000
$3E_uy$	3.65324515	0.0000	0.7392	0.0000
$4E_u x$	3.75545851	1.6162	0.0000	0.0000
$4E_u$ y	3.75545851	0.0000	1.6162	0.0000
A_{2u}	3.88113308	0.0000	0.0000	0.2339
$5E_u$ x	3.95713036	1.0779	0.0000	0.0000
$5E_u$ y	3.95713036	0.0000	1.0779	0.0000
$6E_u$ x	4.00836971	-3.1249	0.0000	0.0000
$6E_u$ y	4.00836971	0.0000	3.1249	0.0000
$7E_u$ x	4.49228381	3.3076	0.0000	0.0000
$7E_{u}$ y	4.49228381	$\,0.000\,$	-3.3076	0.0000

Table S2. Nitrogen K-edge core excited states of Mg-phthalocyanine. Only spin singlet states are listed.

state	irreducible representation	energy (eV)
1		386.1709
2	E_g	386.1711
3	E_q	386.1711
4		386.1713
5		386.2963
6	E_g	386.2966
	E_q	386.2966
8		386.2972

Figure S2. Population dynamics in the upper panel, pulse in the lower panel.

S3 Population Dynamics

The valence excited state population contributes to the population ring currents. After the pulse is over, the population is a constant.

S4 Mg K-edge TRXCD Spectrum

The TRXCD spectrum of Mg K-edge is displayed in Fig. S3. Due to the large core state line width of 0.45 eV, different core excited states cannot be resolved in the spectrum. We thus do not use Mg K-edge for analysis.

Figure S3. Time-resolved circular dichroism (Eq. 3) of Mg K-edge.

S5 Peak assignment of the CD spectrum at 10.40 fs

The Nitrogen K-edge X-ray CD spectrum at 10.40 fs is shown in Fig. S4. We assign the peaks by evaluating $\frac{j_{DC}(\mathbf{k}) \times j_{CA}^{\dagger}(-\mathbf{k})}{E_D - E_{CA} + \omega + i\Gamma}$ $\frac{\sum_{C}(\mathbf{k})\times \mathbf{J}_{CA}(-\mathbf{k})}{E_D - E_C + \omega + i\Gamma}$ in Eq. (3).

Figure S4. Nitrogen K-edge CD spectrum at 10.40 fs.

The $7E_u - 4E_u$ coherence causes 5 peaks. The peaks at 383.6690 eV and 384.4058 eV correspond to excitation to the degenerate core state 22, 23 from $7E_u$ and $4E_u$ valence state respectively. The peaks at 383.8017 eV and 384.5385 eV correspond to excitation to core state 25 from states $7E_u$ and $4E_u$ respectively. The 382.4157 eV peak represents the excitation from $4E_u$ to core states 6, 7.

The peaks at 381.6788 eV and 383.6690 eV have contributions from the excitation from $7E_u$ to degenerate core state 6, 7 and to degenerate core state 22, 23 respectively. Since they come from the $7E_u$ population, they are time-independent after the pump pulse.

The $7E_u - 6E_u$ coherence contributes to two frequencies. The peak at 381.6788 eV represents the excitation from $7E_u$ to degenerate core state 6, 7. 382.1627 eV correspond to excitation from $6E_u$ to degenerate core state 6, 7. Thus, the 382.1627 eV peak is the unique characteristic of the $7E_u - 6E_u$ coherence.

The coherence $7E_u - 1E_u$ contributes to two frequencies. Peak at 381.6788 eV excitation

from $7E_u$ to core state 6, 7. at 383.9051 eV is excitation from $1E_u$ to core state 6, 7. So, the 383.9051 is the characteristic peak of this coherence.

The coherence $7E_u - 5E_u$ corresponds to 4 peaks. 383.8017 eV is excitation from $7E_u$ to core state 25. 384.3369 eV is from $5E_u$ to core state 25. 383.6690 eV is from $7E_u$ to core state 22. 384.2041 eV is from $5E_u$ to core state 22. Thus 384.3369 and 384.2041 eV are the characteristic peaks of this coherence.

The $7E_u - 3E_u$ coherence has two peaks. 381.6788 eV is the excitation from $7E_u$ to core state 6, 7. 382.5179 eV is excitation from $3E_u$ to core state 6, 7. Thus, 382.5179 eV is the characteristic peak.

The $6E_u - 1E_u$ coherence has two peaks. 382.1627 eV, is the excitation from $6E_u$ to core state 6, 7. 383.9051 eV is from $1E_u$ to core state 6, 7. They share the same energy range as other coherence.

The $4E_u - 1E_u$ coherence has two peaks. 382.4157 eV is excitation from $4E_u$ to core state 6, 7. 383.9051 eV is excitation from $1E_u$ to core state 6, 7.

The $4E_u - 3E_u$ coherence has two peaks. 382.4157 eV is from $4E_u$ to core state 6, 7. 382.5179 eV is from $3E_u$ to core state 6, 7.

Coherence $5E_u - 1E_u$ has two peaks. 382.2140 eV is $5E_u$ to core state 6, 7. 383.9051 eV is from $1E_u$ to core state 6, 7. So 382.2140 eV is characteristic.

The peak assignment to specific coherences, the oscillation periods and the coherence frequencies are summarized in Table 2 in the main text.

S6 The Induced Magnetic Field

The induced magnetic field created by ring current can indicate the direction and strength of ring current. We use Biot-Savart law to caclculate the induced magnetic field at the center of the molecule (on Mg atom) as a function of time. Assume the molecule is in the plane of paper, z direction is pointing out of the paper to the reader, a counter-clockwise current

Figure S5. Time-resolved induced magnetic field. Panel a is the total induced magnetic field. Panel b is the contribution from the population ring current. After the pump pulse finish, their contribution becomes constant at -0.01144 T. Panel c is the contribution from the coherent ring current from the coherence between excited states.

will create induced magnetic field pointing into the paper, which is negative valued while the clockwise current create positive valued induced magnetic field pointing out of the paper.

The time evolution of induced magnetic field by ring current is displayed in Fig. S5. The total induced magnetic field include the contribution from population ring current, g-ex coherent current and ex-ex coherence between states with different energy. The fast oscillation in panel a is contributed by g-ex coherence due to their relatively large energy separation. The contribution from the population ring current is displayed in panel b. After the pulse, their contribution become a constant at -0.01144 T. The contribution from the ex-ex coherent ring current is displayed in panel c.

S7 Decomposition of Induced Magnetic Field

We decompose the ex-ex coherent ring current into individual coherence pair in Fig. S6.

Figure S6. Contribution of the excited states coherence to the induced magnetic field.

S8 TRXCD Spectrum with Pure Dephasing

In Eq. $(S12)$, the effect of dephasing is added as pure dephasing to describe the decoherence induced by the interaction between the molecular system with bath. The factor $e^{-\Gamma_{1,DA}\tau}$ gives rise to the decoherence of valence-valence state coherence, where index D, A run over ground and valence excited states. As will be shown in the result, the ground state-valence excited state coherence does not contribute to any change, since the coherence between ground state and valence state has a shorter oscillation period than the pulse duration, thus not observable in the time-resolved spectra. The parameter $\Gamma_{2,DC}$ give rise to the decoherence of valencecore state coherence. TRXCD spectra and their Fourier transformed spectra with different pure dephasing strength are displayed in Fig. S7. For $\Gamma_{1,DA}$ and $\Gamma_{2,DC}$ equal to 1/100 fs⁻¹, the results are similar to no dephasing results. As the dephasing parameter increase, the TRXCD signal decay with time. Larger dephasing corresponds to earlier decay. As a result, in Fourier transformed spectra, as the dephasing parameters increase, the strength of the peaks decrease, so does the peak resolution in frequency axis $(h\omega')$. Aside from that, as in Eq. (S12), $\Gamma_{2,DC}$ in the denominator will broaden the peak in energy domain. Since the value of dephasing parameter is smaller than the life time decay Γ of core excited states, this broadening is not significant.

S9 Calculation of Local Ring Current

Local ring currents are calculated by numerically integrating the current density at a plane, $I = \int_A dS \,\hat{\mathbf{n}} \cdot \mathbf{j}$, where \mathbf{j} is the current density, A is the plane, $\hat{\mathbf{n}}$ is the normal vector of the plane. As depicted in Fig. S8, different plane and normal vectors are chosen for different zones. Mg atom is at the origin, with x and y axis shown in Fig. S8.

For ring α , the plane is chosen as half plane in xz plane, with boundary going through the center of benzene ring α and perpendicular to xy plane. The normal vector is the unit vector in −y direction. The integration of current density in this plane gives the current going in $-y$ direction through this plane, i.e., the current I of ring α , see top panel of Fig. 4. Similarly, the integration plane of ring γ is the half plane in xz plane, with its boundary going through the center of benzene ring γ , with normal vecor in +y. Positively valued $I(\alpha)$, $I(\gamma)$ indicates clockwise ring currents in ring α , γ .

The integration plane for ring β and δ are half plane in yz plane, with their boundary going through the center of benzene ring β and δ respectively. The normal vectors of β is in $-x$ direction while the normal vector for δ is in +x. Positively valued $I(\beta)$ and $I(\delta)$ indicates clockwise ring currents in ring β and δ respectively.

The integration plane for ring current around Mg atom (CT) is in xz plane between the Mg nucleus and the middle point of Mg and N, as the black line in Fig. S8. The normal vector is in $-y$. Positive I(CT) indicates clockwise CT ring current. Similar to CT, the integration plane for outer ring (OT) current is in xz plane between $x = 3.4$ Å and the middle point of Mg and N (the orange line in Fig. S8). The normal vector is in $-y$. Positive $I(\text{OT})$ indicates clockwise OT ring current.

Figure S7.1. Dephasing parameters $\Gamma_{1,DA}$ and $\Gamma_{2,DC}$ both set to 1/100 fs⁻¹.

Figure S7.2. Dephasing parameters $\Gamma_{1,DA}$ and $\Gamma_{2,DC}$ both set to 1/20 fs⁻¹.

Figure S7.3. Dephasing parameters $\Gamma_{1,DA}$ and $\Gamma_{2,DC}$ both set to 1/10 fs⁻¹.

Figure S7. Dephasing parameters $\Gamma_{1,DA}$ and $\Gamma_{2,DC}$ both set to $1/100$ fs⁻¹, $1/20$ fs[−]¹ and 1/10 fs[−]¹ respectively. Left panels are TRXCD spectra, right panels are the Fourier transformed spectra. Arbitrary units are used with the same scale.

Figure S8. The integration planes for calculating the local ring currents in different zones of the molecule.