Edwards volume ensemble in cyclically sheared granular experiments: Supplementary Material

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March 16, 2022

The Supplementary Materials below contain additional details on: (a)the correlation functions of particle's free volume and the particle's Voronoi cell orientation over shear cycles, respectively, (b) the self intermediate scattering functions, (c) the local and global anisotropies in the system, (d) the volume fluctuations with respect to different shear amplitudes, and (e) the packing fraction within one shear cycle.

1. The correlation functions of particle's free volume

The structure evolution depends on the shear amplitude γ_m, which provides an estimation of the decorrelation time and hence an estimation of the independent samples of particle configurations at the given γ_m. The Voronoi cell of a particle has at least two attributions of volume and orientation. First, we define the time autocorrelation function of free volume $C_{v_f}(t)$ as follows, where t refers to the number of shear cycles:

$$C_{v_f}(t) = \frac{\left\langle \frac{1}{N}\sum_i (v_{f,i}(0) - \overline{v_{f,i}(0)})(v_{f,i}(t) - \overline{v_{f,i}(t)}) \right\rangle}{\left\langle \frac{1}{N}\sum_i (v_{f,i}(0) - \overline{v_{f,i}(0)})^2 \right\rangle}. \tag{1}$$

Here N is the total number of particles, $v_{f,i}$ is the free volume of the disk i, the overline averages over different disk i at a given strain for a fixed shear amplitude, and the angular bracket $\langle\cdot\rangle$ averages over different shear cycles t. The autocorrelation functions for different γ_m are shown in Fig.S1 (a) and (c). All the data are calculated at $\varepsilon = 0$ for all γ_m.

2. The correlation functions of particle's Voronoi cell orientation

Here the orientation of a particle's Voronoi cell is determined from the major principal direction θ of the Minkowski tensor $W_1^{0,2}$ (See the main text for detail). we define the autocorrelation function of the cell orientation:

$$C_\theta(t) = \left\langle \frac{1}{N}\sum_i cos(2(\theta_i(t) - \theta_i(0))) \right\rangle \tag{2}$$

Here θ_i is the cell orientation angle of the disk i, and other symbols

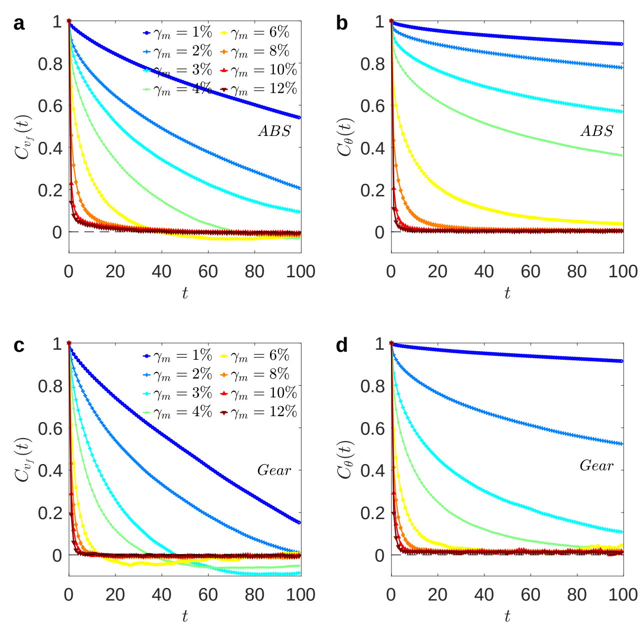

Fig. S1 The autocorrelation functions of particle's free volume v_f as a function of shear cycle for ABS disks (a) and gears (c) at a range of different γ_m. The autocorrelation functions of the major principal direction θ of $W_1^{0,2}$ for ABS disks (b) and gears (d) at a range of different γ_m.

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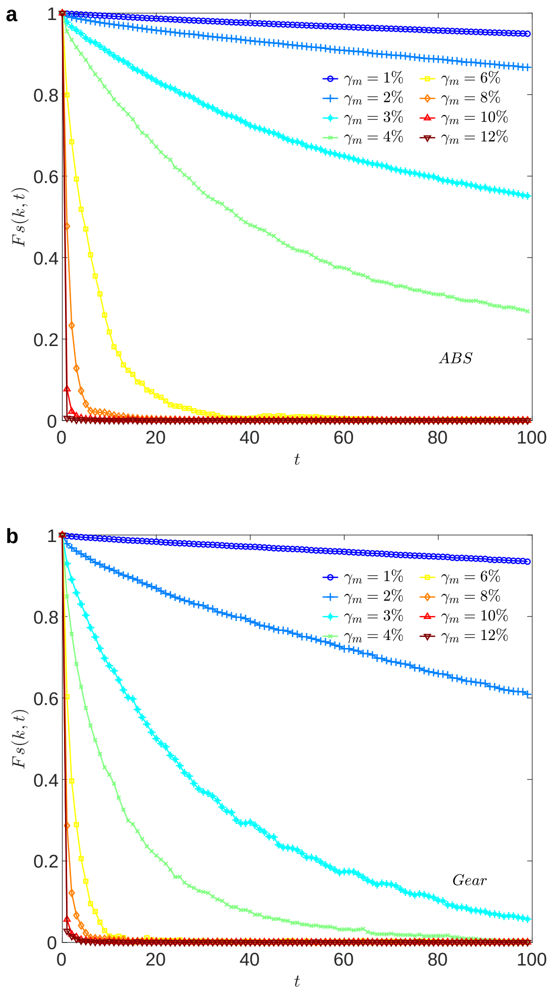

Fig. S2 The self intermediate scattering functions $F_s(k,t)$ for different shear amplitudes γ_m and for ABS disks (a) and gears (b). Here $k = 2\pi D^{-1}$, with D referring to the mean particle diameter.

are same with 1. In Fig S1(b) and (d), we show the $C_\theta(t)$ for both ABS disks and gears.

3. The self intermediate scattering function $F_s(k,t)$

As discussed in the main text, the Voronoi cell of a particle is a vectorial quantity that can not be simply characterized using just a single scalar such as volume or orientation. Therefore, its associated decorrelation is better described using the self intermediate scattering function (ISF), which is defined as Fig S2,

$$F_s(k,t) = \frac{1}{N}\left\langle \sum_j^N exp(-I\mathrm{k}\cdot(\mathrm{r}_j(t)-\mathrm{r}_j(0))\right\rangle \quad (3)$$

Here $I = \sqrt{-1}$, and $|k| = 2\pi D^{-1}$, with D being the mean particle diameter. The trend of $F_s(k,t)$ is similar to $C_{v_f}(t)$ and $C_\theta(t)$. It is clear from the figure, the structure decorrelation depends strongly on γ_m. When $\gamma_m \leq 4\%$, the structure correlation time is extremely long with more than a hundred shear cycles, we hence only present the results in the main text for $\gamma_m \geq 6\%$ for better statistics.

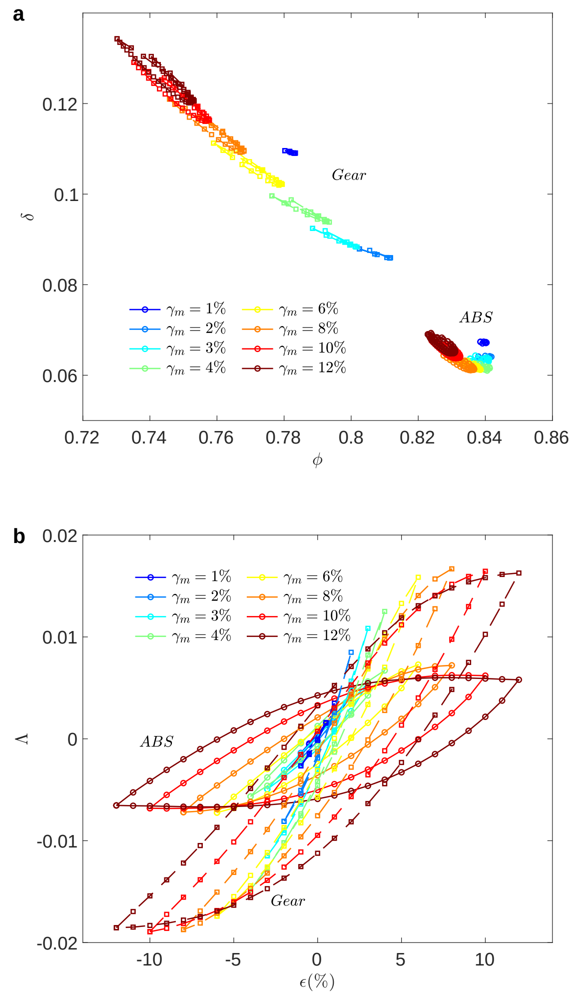

Fig. S3 (a) The mean local anisotropy $\overline{\delta}$ versus the global packing fraction ϕ for ABS disks and gears and for a range of shear amplitudes γ_m, including $\gamma_m < 6\%$. (b) The hysteresis loops of the global anisotropy of system Λ (See the main text for detail) within one shear cycle for ABS disks and gears and for a range of shear amplitudes γ_m, including $\gamma_m < 6\%$.

4. The local and global anisotropies

We plot the degrees of anisotropy for all shear amplitudes including $\gamma_m < 6\%$ that are not shown in main text.

In Fig S3(a), we plot the mean local anisotropy versus global packing fraction ϕ for ABS disks and gears. We find the data of all γ_m almost fall into a master curve except for $\gamma_m = 1\%$ and 2%, which might be due to poor statistics. In Fig S3, we plot the global anisotropy of system Λ within one shear cycle for ABS disks and gears. These curves show hysteresis loops within shear cycles. It shows stronger anisotropy in Gears than in ABS disks.

5. Volume fluctuation for all shear amplitudes

Here, we show the compactivity and volume fluctuation for all shear amplitudes ranging from 1% to 12%, where the $\frac{1}{\chi} - \frac{1}{\chi_r}$ and $Var(V_f)$ both show reasonable collapsing onto a master curve. In Fig S4(b), we fit the $Var(V_f)$ versus V_f using a quadratic form of $y = kx^2$ which provides a reasonable approximation of the data points, and We get $k = 0.0342 \pm 0.0002$.

6. The packing fractions within one shear cycle

The packing fractions for ABS disks and gears change within one shear cycle as a function of the strain ε.

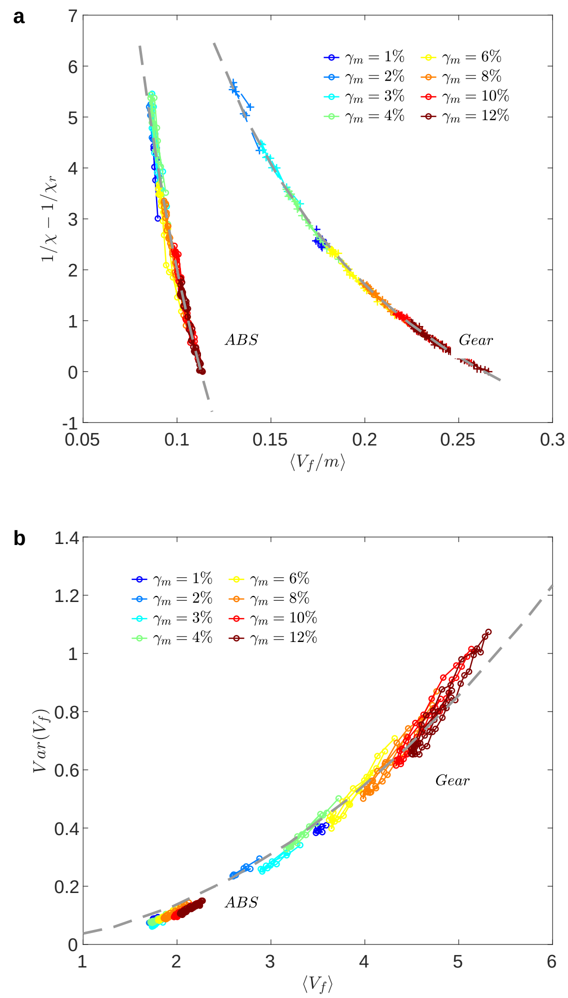

Fig. S4 The $\frac{1}{\chi} - \frac{1}{\chi_r}$ versus the V_f/m calculated from the overlapping histogram method for ABS disks and gears for a range of shear amplitudes γ_m, including $\gamma_m < 6\%$. Here χ is the compactivity and χ_r is the compactivity of the reference state. The V_f is the free volume of a cluster of $m = 20$ particles. The dash lines are fit of $y = a/x + b$ (See the main text for more details). (b)The variance $Var(V_f)$ of the fluctuations of the free volume V_f as a function of V_f for ABS disks and gears for a range of shear amplitudes γ_m, including $\gamma_m < 6\%$. The dash lines are fit of $y = kx^2$ (See the main text for more details)

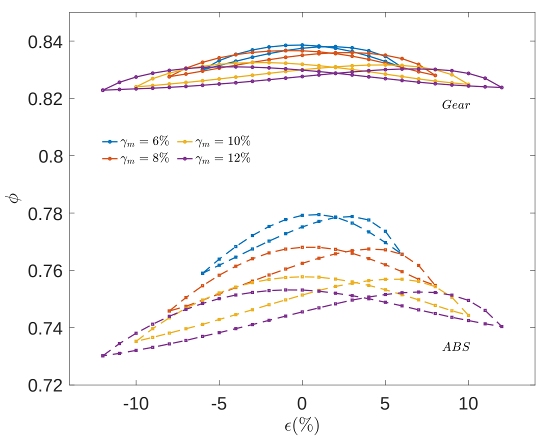

Fig. S5 The packing fractions for ABS disks and gears change as a function of the strain ε within one shear cycle of the given shear amplitude γ_m.