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Supplementary Material for

Experimental and theoretical studies on the dynamic landing of water striders on water

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S1. Geometrical shape and relevant parameters of the bionic water strider

Fig. S1 gives the geometrical shape of the bionic strider fabricated in our experiments. The parameters such as the leg length *l* and the mass *m* of the entire body are marked.

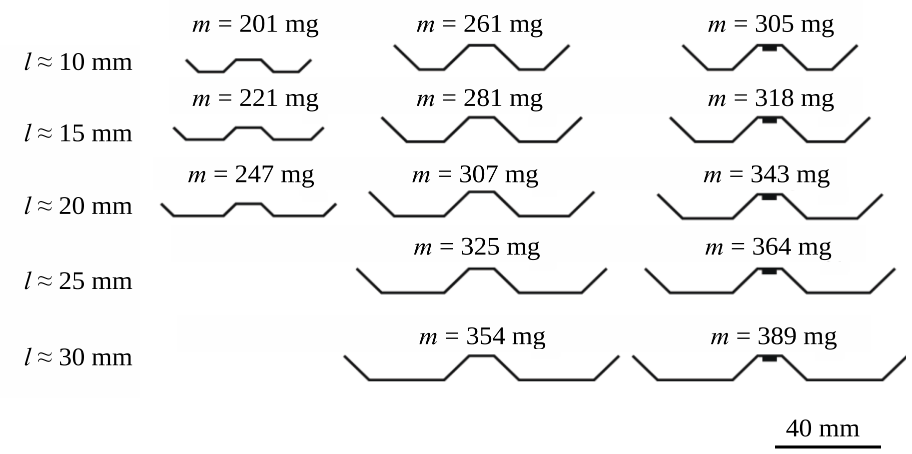

Fig. S1 Geometrical shape and mass of the bionic strider. *l* denotes the length of a single leg and *m* is its mass.

In our experiments, we fabricated thirteen bionic water striders in total. The leg length *l*, and the mass of the entire body *m*, as well as the mass per length of the leg m_p (i.e., $m_p = m/L$ with $L = 4l$), are listed in Table S1. The mass *m* of a bionic strider is measured by a precision electronic balance (LiChen FA2004) with a resolution of 0.1 mg. The length of each leg is measured using a ruler, and then *l* is obtained by averaging the four leg lengths.

Table S1 Parameters of the bionic water striders. l, m and m_p denote the average value of four leg lengths, the mass of the entire body and the mass per length of the leg, respectively.

No.	l (mm)	m (mg)	m_p (g/m)
1	9.48 ± 0.24	201	5.3 ± 0.13
2	9.70 ± 0.28	268	6.9 ± 0.20
3	9.70 ± 0.28	305	6.9 ± 0.20
4	14.81 ± 0.43	221	3.7 ± 0.11
5	14.49 ± 0.15	281	4.8 ± 0.05
6	14.49 ± 0.15	318	4.8 ± 0.06
7	19.58 ± 0.64	247	3.2 ± 0.10
8	19.50 ± 0.76	307	3.9 ± 0.15
9	19.50 ± 0.76	343	4.4 ± 0.17
10	24.96 ± 0.46	325	3.3 ± 0.06
11	24.96 ± 0.46	364	3.6 ± 0.07
12	29.83 ± 0.69	354	3.0 ± 0.07
13	29.83 ± 0.69	389	3.3 ± 0.08

S2. Influence of the contact angle on the buoyancy force

In our experiments, the contact angle of the leg of the bionic water strider (θ_Y = 102.1 ± 3.7°) is much smaller than the real case (167.6 ± 4.4°).[1] We investigate the influence of the contact angle on the buoyancy force for a two-dimensional problem. As shown in Fig. S2, based on the theory developed previously,[2,3] we obtain the relation between the buoyancy force F_b and the depth of the solid-liquid-vapor three-phase contact line h in the dimensionless form. Here, the diameter of the cylinder is d = 0.4 mm, and the black, green, blue and red curves correspond to θ_Y = 90°, 110°, 135°, 180°. The results in Fig. S2 suggests that when the diameter of the cylinder is very small, the contact angle does not remarkably influence the buoyancy force, and the contact angle of our bionic water strider is quite fine.

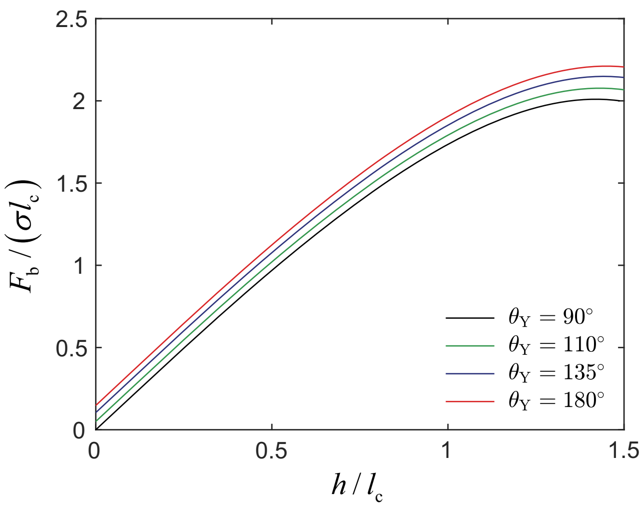

Fig. S2 Relation between the normalized buoyancy force F_b and the depth h/l_c of the solid-liquid-vapor three-phase contact line. The black, green, blue and red curves correspond to $\theta_Y = 90°$, 110°, 135°, 180°. The diameter of the cylinder is $d = 0.4$ mm.

S3. Theory of surface waves

To figure out the drag force which suppresses the oscillations of the strider, we quantitatively investigate the influences of surface waves.

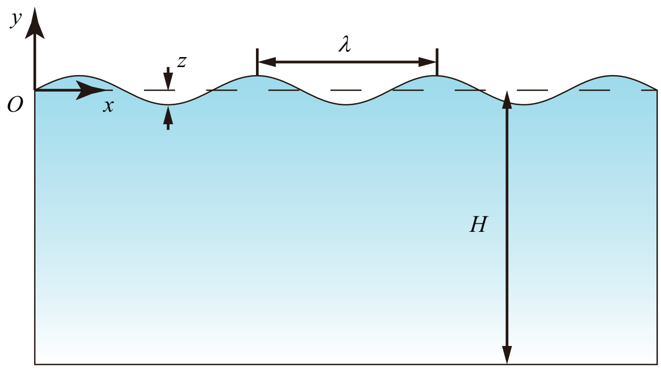

Fig. S3 Surface waves triggered by the landing of the strider. A semi-infinite space is given, and the relevant geometrical parameters are defined. λ represent the wave length, and z represent the position of the water surface corresponding to certain values of x and t. H represent the depth of water, and $H >> l_c$.

As shown in Fig. S3, we consider a surface wave. Classical theory[4] gives the governing equations along with the boundary conditions as

$$\nabla^2\phi = 0, \tag{S1}$$

$$\frac{\partial\phi}{\partial y} = \frac{\partial z}{\partial t}, \quad y = 0, \tag{S2}$$

$$\frac{\partial\phi}{\partial t} + gh = \frac{\sigma}{\rho}\frac{\partial^2 z}{\partial x^2}, \quad y = 0, \tag{S3}$$

where $\phi = \phi(x, y)$ represents the velocity potential and we have the velocity of the fluid $\mathbf{U} = \nabla\phi$. $\nabla^2 = \nabla \cdot \nabla$ represents the Laplace operator. $z = z(x, t)$ is the height of the water surface corresponding to a certain location x and a certain time t. g represents the acceleration of gravity. σ and ρ represent the surface tension and mass density of water, respectively.

Since the depth of water $H >> l_c$, we assume that the depth of the tank is infinite. Considering the flow velocity $U_z = \partial\phi/\partial y$ at the bottom of the tank must be zero, we have the boundary condition

$$\frac{\partial\phi}{\partial y} = 0, \quad (y = -H). \tag{S4}$$

From the above equations, we obtain the solution of the monochromatic wave

$$z(x,t) = A\sin(kx - \omega t + \varphi) + Be^{-x/l_c}, \tag{S5}$$

$$\phi(x,y) = -\frac{A\omega}{k}e^{ky}\cos(kx - \omega t + \varphi), \tag{S6}$$

where A is the amplitude of the wave and B is a coefficient. k is the wave number and $k = 2\pi/\lambda$ with λ being the wave length. ω is the angular frequency and $\omega = 2\pi/T$ with T being the period. t and φ represent time and phase, respectively.

Furthermore, the relation between ω and k can be derived from eqn (S3) as

$$\omega^2 = gk\left(1 + k^2 l_c^2\right), \tag{S7}$$

where $l_c = [\sigma/(\rho g)]^{1/2}$ represents the capillary length. Since the wave velocity $c = \omega/k$, we have

$$c^2 = \frac{g}{k}\left(1 + k^2 l_c^2\right). \tag{S8}$$

From eqn (S8), we can see that c has a minimum, i.e., $dc^2/dk = 0$ leads to $c_{min} = (2gl_c)^{1/2}$, corresponding to $k = 1/l_c$ and $\lambda = 2\pi/k = 2\pi l_c$.

The surface wave of water is a kind of polychromatic waves and it consists of monochromatic waves. Thus, we have

$$z(x,t) = \sum_i z_i(x,t) = \sum_i \left[A_i \sin(k_i x - \omega_i t + \varphi_i) + B_i e^{-x/l_c} \right], \tag{S9}$$

$$\phi(x,y) = \sum_i \phi_i(x,y) = -\sum_i \frac{A_i \omega_i}{k_i} e^{k_i y} \cos(k_i x - \omega_i t + \varphi_i). \tag{S10}$$

Equations (S9) and (S10) are the series solution of Eqs. (S1)-(S3) and satisfy the boundary condition eqn (S4).

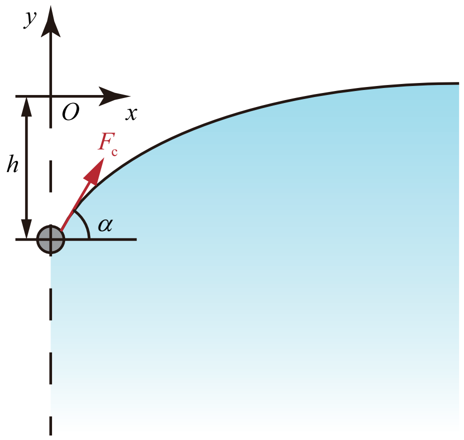

Fig. S4 The deformation of the leg-water interface (x = 0). The depth of the leg is h. α represents the slope angle of the meniscus at the solid-liquid-vapor three-phase contact line, and F_c represent the capillary force exerted on the leg.

As shown in Fig. S4, considering the boundary condition $z(x, t)|_{x=0} = h$, we have

$$z(x,t)\big|_{x=0} = \sum_i A_i \sin(-\omega_i t + \varphi_i) + \sum_i B_i = h. \tag{S11}$$

Because eqn (S11) is valid at any time t, we have $\sum_i A_i \sin(-\omega_i t + \varphi_i) = 0$ and $\sum_i B_i = h$. As a result, we rewrite eqn (S9) as

$$z(x,t)=\sum_i z_i(x,t)=\sum_i\left[A_i\sin\left(k_ix-\omega_it+\varphi_i\right)\right]+he^{-x/l_c}\,. \tag{S12}$$

The second term in the right hand of eqn (S12) represents the approximate solution of the Young-Laplace for a static state (i.e., $z(x)=he^{-x/l_c}$),[5] whereas the first term (i.e., $\sum_i\left[A_i\sin\left(k_ix-\omega_it+\varphi_i\right)\right]$) represents the oscillation around the static solution.

As shown in Fig. S4, the vertical component of the capillary force F_c that exerts on the leg is $F_c = \sigma L\sin\alpha$, with L and α being the leg length and the slope angle around the solid-liquid-vapor three-phase contact line, respectively. Let us consider a small amplitude of oscillation, which indicates $\sin\alpha \approx \tan\alpha \approx \partial z/\partial x|_{x=0}$. Then we obtain from eqn (S12)

$$\frac{F_c}{\sigma L}=\left.\frac{\partial z(x,t)}{\partial x}\right|_{x=0}=\sum_i\left[A_ik_i\cos\left(-\omega_it+\varphi_i\right)\right]-\frac{h}{l_c}\,. \tag{S13}$$

Considering

$$\dot{h}=\left.\frac{\partial z(x,t)}{\partial t}\right|_{x=0}=-\sum_i\left[A_iw_i\cos\left(-\omega_it+\varphi_i\right)\right], \tag{S14}$$

eqn (S13) can be rewritten as

$$\frac{F_c}{\sigma L}=-\frac{\sum_i\left[A_ik_i\cos\left(-\omega_it+\varphi_i\right)\right]}{\sum_i\left[A_iw_i\cos\left(-\omega_it+\varphi_i\right)\right]}\dot{h}-\frac{h}{l_c}\,. \tag{S15}$$

For a monochromatic wave, we have the wave velocity $c = \omega/k$, whereas it is $c_i = \omega_i/k_i$ for polychromatic waves. However, the relation in eqn (S15) is quite complex. Here, we define an equivalent wave velocity $\bar{c}$ by

$$\frac{1}{\bar{c}}=\frac{\sum_i\left[A_ik_i\cos\left(-\omega_it+\varphi_i\right)\right]}{\sum_i\left[A_iw_i\cos\left(-\omega_it+\varphi_i\right)\right]}=\frac{\sum_i\left[A_ik_i\cos\left(-\omega_it+\varphi_i\right)\right]}{\sum_i\left[A_ik_i\cos\left(-\omega_it+\varphi_i\right)\right]c_i}\,. \tag{S16}$$

From Fig. 6 in the main paper (Movie S1, ESI), the wave lengths in our experiments are of the order of millimeter. Since the lowest of the monochromatic wave is $c_{\min}$ =

$(2gl_c)^{1/2}$,[4] we consider $\bar{c}$ is of order of c_{min}, i.e., $\bar{c} \sim (gl_c)^{1/2}$. Finally, we obtain

$$\frac{F_c}{\sigma L} = -\frac{\dot{h}}{\bar{c}} - \frac{h}{l_c} \,. \tag{S17}$$

Considering the dynamic equation of oscillation we have constructed in eqn (2) in the main paper, eqn (S17) suggests that the appearance of the surface waves by the landing of the strider indeed produces two parts of effect: (1) the deformation of the leg-water interface enables the surface tension to exert force on the leg along the vertical direction, which exhibit as the restoring force (capillary force); (2) the generation and the emission of the surface wave by the leg exhaust the kinetic energy of the leg, which exhibits as a drag force. Furthermore, compared with eqn (2) in the main paper, we have $k \sim \sigma L/l_c$ and $b \sim \sigma L/\bar{c}$. For convenience, we take $c = (gl_c)^{1/2}$ and $b \sim \sigma L/c$.

S4. Damped harmonic oscillator

The differential equation of a damped harmonic oscillator is given in eqn (2) in the main paper

$$m\ddot{h} + b\dot{h} + kh + F_g = 0 \,, \tag{2}$$

where h is the position of the leg along the vertical direction, m is the mass of the strider, b is the damping coefficient, k is the spring constant and F_g is the externally applied force. Here, $F_g = mg$ is the weight of the strider. In fact, eqn (2) has an analytical solution for h

$$\begin{aligned} h(t) &= h_0 + h_1 \exp(-\xi\omega_n t)\sin(\omega_d t + \varphi) \\ &= h_0 + h_1 \exp(-\beta t)\sin\left(\frac{2\pi}{T}t + \varphi\right) \end{aligned}, \tag{S18}$$

where h_0 is the final position of the leg when the oscillation stops. Furthermore, we use the following abbreviations to describe the parameters in eqn (S18)[6]

$$\beta = \frac{b}{2m} = \xi\omega_n, \quad \omega_n = \sqrt{\frac{k}{m}}, \quad \xi = \left(\frac{b}{m}\right)\frac{1}{2\omega_n} \,, \tag{S19}$$

$$\omega_d = \frac{2\pi}{T} = \sqrt{1-\xi^2}\cdot\omega_n = \sqrt{\omega_n^2 - \beta^2} = \sqrt{1 - \frac{1}{4}\frac{b^2}{mk}}\cdot\sqrt{\frac{k}{m}} \,, \tag{S20}$$

$$T=\frac{2\pi}{\sqrt{1-\frac{1}{4}\frac{b^2}{mk}}\cdot\sqrt{\frac{k}{m}}}. \tag{S21}$$

Here, we have $h_0 = h|_{t\to\infty} = -F_g/k_s = -mg/k_s$. The amplitude h_1 and the phase φ need to be determined from the initial conditions. Based on the experimental data, we have $\varphi \approx 0$. Therefore, when making derivative on both sides of eqn (S18) with respect to time and checking the result at $t = 0$, we could determine h_1 based on $\dot{h}\big|_{t=0} = 2\pi h_1/T = -v_0$ and $h_1 = -v_0T/(2\pi)$, where v_0 denotes the landing velocity. Note that the direction of the velocity is defined positive when the leg moves upwards, and *vice versa*.

Considering $k \sim \sigma L/l_c$ and $b \sim \sigma L/c$ which we have already obtained, one has

$$\omega_n=\sqrt{\frac{k}{m}}\sim\sqrt{\frac{\sigma L}{l_c m}}\sim\sqrt{\frac{\sigma}{l_c m_p}},\quad \beta=\frac{b}{2m}\sim\frac{\sigma L}{mc}\sim\frac{\sigma}{m_p c}\sim\frac{l_c}{\tau_p^2 c}. \tag{S22}$$

S5. Energy density in a quasi-static wetting state

By considering the leg of the water strider as a two-dimensional cylinder floating on the water surface, Liu et al.[3] gave the buoyant force F_b as

$$F_b=\rho g\left[r^2(\theta-\sin\theta\cos\theta)+2\sqrt{2}l_c r\sin\theta\sqrt{1+\cos(\theta+\theta_Y)}-2l_c^2\sin(\theta+\theta_Y)\right] \tag{S23}$$

where $r = d/2$ denotes the radius of the cylinder, θ_Y denotes the Young's contact angle that characterizes the wettability of the cylinder, and θ is the sector angle that describes the position of the solid-liquid-vapor three-phase contact line.[3] In our experiments, we have $r/l_c << 1$, so eqn (S23) degrades into

$$F_b \approx -2\sigma\sin(\theta+\theta_Y). \tag{S24}$$

Since $h=\sqrt{2}l_c\sqrt{1+\cos(\theta+\theta_Y)}$,[3] we have

$$F_b=2\sigma\sqrt{\left(\frac{h}{l_c}\right)^2-\frac{1}{4}\left(\frac{h}{l_c}\right)^4}, \tag{S25}$$

where h is the depth of the three-phase contact line, i.e., the distance between the water level and the three-phase contact line in the vertical direction. Considering when

$h_{\max(p)} = \sqrt{2}l_c$, the water surface will be pierced,[3,7] we could obtain the required maximum energy density (J/m) to quasi-statically press a cylinder on water surface (before it is completely submerged in water)

$$e_{\max} = \int_0^{\sqrt{2}l_c} F_b \mathrm{d}h = \frac{2\left(4-\sqrt{2}\right)}{3}\sigma l_c \approx 1.72\sigma l_c \approx 3.34\times10^{-4}\ \text{J/m}, \qquad \text{(S26)}$$

where we take σ = 0.073 J/m and l_c = 2.73 mm. Here, it is emphasized that when we calculate eqn (S26), we still consider the condition $r/l_c << 1$.

S6. Discussion of the sinking condition

By employing a dynamic model and a scaling argument, Vella and Li[8] proposed a sinking condition accounting for the landing of a cylinder with a finite radius on water

$$\left(\frac{r}{l_c}\right)^2 \left[F_c^2 + 2^{3/2}\right] \sim \frac{7}{2\pi D}, \qquad \text{(S27)}$$

where $D = \rho_s/\rho$ is defined as the mass density ratio with ρ_s and ρ being the mass densities of the cylinder and water, respectively. r is the radius of the cylinder and $r << l_c$ with l_c being the capillary length. $F_c = v_0/(gl_c)^{1/2}$ is defined as the Froude number with v_0 and g being the landing velocity and the acceleration of gravity, respectively.

By neglecting the weight of the cylinder during landing,[8] eqn (S27) can then be reduced to

$$F_c^2 \sim \frac{1}{D\left(\frac{r}{l_c}\right)^2}. \qquad \text{(S28)}$$

Considering the relationships $m_p = \pi r^2\rho_s$ and $D = \rho_s/\rho$, we can rewrite eqn (S28) into

$$v_0^2 \sim \frac{\sigma l_c}{m_p}. \qquad \text{(S29)}$$

When the landing velocity v_0 reaches the critical value $v_{\max}$, eqn (S29) obtained by Vella and Li[8] is exactly consistent with our model, i.e., eqn (7) in the main text.

Further insights could be gained as follows. In the model of Vella and Li,[8] the authors assume that the cylinder will reach a constant depth $h_{\max(p)}$ for sinking, whereas we assume there is a maximum energy $e_{\max(s)}$ (per unit length) that the interface could store accounting for sinking. Moreover, since the energy stored at the interface could be expressed as $E_s \sim \sigma L h^2/l_c$ based on a scaling argument (see Section 3.1 in the main text), we have $e_{\max(s)} \sim \sigma h_{\max(p)}{}^2/l_c$. Basically, $h_{\max(p)}$ is on the order of l_c, therefore, we finally obtain $e_{\max(s)} \sim \sigma l_c$, which suggests that even Vella and Li[8] and the present paper propose different models to find the critical condition for sinking, the same conclusion is reached because the underlying physics does not change.

Movie legends

Supplementary Movie 1.

Movies respectively captured from the side, the front and the bottom views exhibiting the surface waves triggered by the landing of a bionic water strider. The frame rate of the high-speed camera is 4,000 frames per second, and the scale bars represent 10 mm.

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