# Electronic Supplementary Material (ESI) for Soft Matter. This journal is © The Royal Society of Chemistry 2022

# Supplemental Material for "Universal non-Debye low-frequency vibrations in sheared amorphous solids"

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In this document, we provide additional figures and details related to the results presented in the main text.

## CONTENTS

- Sec. A: The frequency distribution of the strains needed to achieve shear-stabilization.
- Sec. **B**: Per-bond stresses across the three types of configurations.
- Sec. C: Minimum eigenvalue distributions of large system sizes.
- Sec. D: Minimum eigenvalue distributions upon approaching a plastic-event.

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## A. Strain required to achieve Shear-stabilization



FIG. S1. Frequency distribution of strains required to attain shear-stability, in the Zero-shear-stress protocol, in systems of size N = 256. The orange bars indicate the number of plastic events encountered by samples that undergo strains in the corresponding intervals. They account for ~ 6.2% of all samples. In the Zero-shear-stress protocol, we encounter plastic events during AQS. Here we show that the proportion of straining trajectories that encounter them, while small, is not insignificant.

## B. Internal Bond-Stresses

#### 1. Distributions



FIG. S2. Distributions of bond-stresses, from configurations of systems of size N = 256. The Unstrained, Zero-shear-stress and Shear-strain-energy-minimized configurations all exhibit identical distributions.

## 2. Visualization



FIG. S3. Components of the stress tensor at each 'bond' in a two-dimensional configuration of a system of size N = 256. (a) an unstrained configuration, (b) The same configuration strained to achieve zero-shear-stress. (c) The same configuration as in (a), energy minimized with a strain degree of freedom. (d) The difference in bond stresses between the *Unstrained* and *Zero-shear-stress* configurations is an order of magnitude smaller than the original stress. (e) The difference in bond stresses between the *Zero-shear-stress* and *Shear-strain-energy-minimized* configurations is zero up to numerical precision.

## C. Large Systems



FIG. S4. Minimum eigenvalue distributions of the Unstrained and shear-stabilized configurations in 2D, corresponding to larger system sizes: (a) N = 1024 and (b) 4096. The lines indicate pure power-laws. The deviation from  $\omega_{\min}^4$  persists as larger system sizes are probed, and is not diminished with the reduced stress fluctuations shown in Fig. 1 of the main text.

#### D. Approach to Universal Distribution



FIG. S5. Minimum eigenvalue distributions of a system of size N = 1024, upon approaching a plastic event. The (blue) circles display the distribution of eigenvalues in the initial, unstrained state. The (pink) hexagons are distributions of eigenvalues measured at closest approach to the plastic strain  $\gamma_P$ . The other distributions belong to configurations at specific strains away from  $\gamma_P$ .