Electronic Supplementary Material (ESI) for Soft Matter.

Supplementary Material for: Breaking Action-Reaction with Active Apolar Colloids: Emergent Transport and Velocity Inversion

Joan Codina Sala, Helena Massana-Cid, Pietro Tierno, and Ignacio Pagonabarraga

THEORETICAL MODEL

We consider a system composed by two spheres with different chemical surface properties, in a chemical solution in a viscous fluid. The chemical activity of the colloids generates a chemical imbalance that produces a slip velocity on the surface of each particle. We present a minimal model that accounts for the effects of the chemical products, as a function of the pair's size ratio and diffusiophoretic mobilities, and disregards the hydrodynamic interactions between the spheres. We can then estimate the velocity of each particle by averaging the slip velocity field on their surface.

To this end, we introduce an analytical solution for concentration field, which obeys the Laplace equation $\nabla^2 c = 0$, in a two-sphere geometry. The corresponding chemical field profiles are, then, invariant under rotations with respect to the axis defined by the axis joining the centres of the two spheres. We consider two spheres of radii $R_\pm$, with uniform but distinct surface properties, located along the z axis with centres at $z_+ > 0$, and $z_- < 0$. The centre-centre distance is $z_+ - z_+ = d > R_+ + R_-$, ensuring that spheres do not overlap. Given the symmetry of the problem we consider the bispherical coordinate system [1] as has been previously presented for similar problems concerning pairs of catalytic colloids [2–7].

In bispherical coordinates, see Fig. 1, we relate the 3d cylindrical coordinates, $\{\rho,\ z,\ \phi\}$, to $\{\tau,\ \mu,\ \phi\}$ as

$$\rho = a\frac{\sqrt{1-\mu^2}}{\cosh\tau - \mu}, \quad z = a\frac{\sinh\tau}{\cosh\tau-\mu}, \tag{1}$$

with Lamé coefficients of the coordinate change given by $h_\tau = a(\cosh\tau-\mu)^{-1}$, $h_\mu = a(\cosh\tau-\mu)^{-1}/\sqrt{1-\mu^2}$, and $h_\phi = a\sqrt{1-\mu^2}(\cosh\tau-\mu)^{-1}$. The value of a is a geometry-dependent parameter defined both by the positions and radii of the spheres $R_\pm = a/|\sinh\tau_\pm|$, $z_\pm = a\coth\tau_\pm$. Once a, and $\tau_\pm$ are defined, the separation distance can be rewritten as $d = a(\coth\tau_+ - \coth\tau_-)$.

The boundaries of the two spheres are defined by surfaces of constant $\tau = \tau_\pm$ and the region of the space outside the spheres is defined by $\tau_+ > \tau > \tau_-$. Given the symmetry in ϕ, we further exclude any ϕ dependence in the solution $c(\tau,\ \mu,\ \phi) = c(\tau,\ \mu)$,

$$c(\tau,\mu) = \sqrt{\cosh\tau-\mu}\sum_{n=0}^{\infty} s_n(\tau)P_n(\mu)\ , \tag{2}$$

where $P_n(\mu)$ are the Legendre polynomials, and the coefficients that determine the chemical field are given by

$$s_n(\tau) = a_n\cosh(n+1/2)\tau + b_n\sinh(n+1/2)\tau\ . \tag{3}$$

The chemical nature of the particles' surfaces define the boundary value problem for the chemical field. The details of the boundary conditions vary from works where the concentration gradient of chemicals is set on surface [3, 6, 8–10] or the production/consumption is tied to the local concentration chemical field [4, 5]. The active surface, located at τ_-, triggers a chemical reaction and chemicals are consumed, the details of the chemical reaction may vary. In this work we model the consumption of chemical products is introduced as a steady gradient of the chemical field perpendicular to the surface of the particle, $\hat{\boldsymbol{n}}\cdot\nabla c(\tau,\mu)|_{\tau=\tau_-} = \alpha$. The passive particle, unable to trigger chemical reactions, imposes a boundary condition without chemical consumption nor generation $\hat{\boldsymbol{n}}\cdot\nabla c(\tau,\mu)|_{\tau=\tau_+} = 0$. As illustrated in Fig.1, the perpendicular direction to the sphere surface is parallel to $\hat{\boldsymbol{e}}_\tau(\tau_\pm,\mu)$. Accordingly, the gradient perpendicular to the surface is easily computed as $\nabla_\perp c = h_\tau^{-1}\partial_\tau c(\tau,\mu)$

$$\frac{\sqrt{\cosh\tau-\mu}}{a}\sum_n\left[\frac{\sinh\tau}{2}s_n(\tau) + (\cosh\tau-\mu)s_n'(\tau)\right]P_n(\mu) \tag{4}$$

To ease the computation of a_n, and b_n we expand $1/\sqrt{\cosh\tau_\pm - \mu} = \sqrt{2}\sum_k \exp[-(k+1/2)|\tau_\pm|]P_k(\mu)$ in terms of Legendre polynomials and exploit their orthogonality to extract the relations

$$\mp\sqrt{2}a\alpha_\pm\sqrt{h_\pm}h_\pm^n = \frac{1}{2}\sinh(\tau_\pm) - \frac{n\ s'_{n-1}(\tau_\pm)}{2n-1} + \cosh(\tau_\pm)s'_n(\tau_\pm) - \frac{n+1}{2n+3}s'_{n+1}(\tau_\pm). \tag{5}$$

with $h_\pm = \exp(-|\tau_\pm|)$. By introducing the surface value of the chemical field (Eq. 3) in the boundary coefficients (Eq. 5) we arrive at a set of equations for a_n, and b_n.

Once $\{a_n,\ b_n\}$ are determined by either numerical methods or analytical calculation, we can determine the diffusiophoretic velocity on the particle surface with magnitude proportional to the chemical field gradient $\boldsymbol{v}_{s,(k)} = M_k\nabla_\parallel c$ [11]. The tangential direction to the surface of the particles, $\hat{\boldsymbol{t}}$, can be expressed as $\hat{\boldsymbol{e}}_\mu(\tau_\pm,\ \mu) = -\hat{\boldsymbol{t}}$, and the resulting slip velocity is then $\boldsymbol{v}_s(\tau_\pm,\mu) = M_\pm/h_\mu^{-1}\partial_\mu c(\tau_\pm,\mu)\ \hat{\boldsymbol{e}}_\mu$, can be expressed as

$$\boldsymbol{v}_{s,\pm} = v_\mu\,\hat{\boldsymbol{e}}_\mu = \hat{\boldsymbol{e}}_\mu\,\frac{M_\pm}{a}(1-\mu^2)^{1/2}\sqrt{\cosh\tau_\pm-\mu}\ \times \times\sum_{n=0}\left(-\frac{1}{2}P_n(\mu) + (\cosh\tau_\pm-\mu)P'_n(\mu)\right)s_n(\tau_\pm). \tag{6}$$

When a complete expansion of the fluid slip velocity, $\boldsymbol{v}_s$, on the colloid surface is determined, one can extract the asymmetry of the surface slip velocity, related to the propulsion velocity of the sphere, using the squirmer model [4, 12]. The squirmer model is the low Reynolds number solution of the fluid flow generated by a general slip velocity tangential to the surface of a sphere, and enables the physical interpretation of the surface velocity in terms of global quantities, *i.e.,* centre of mass velocity, induced stress, etc.. In a spherical coordinate system with axial symmetry a generic tangential slip velocity has the form, $\boldsymbol{v}_s = v_\theta(R,\ \theta)\ \hat{\boldsymbol{\theta}}$, with $v_\theta(x) = 2\sum_{n=1}\sin\theta/[n(n+1)]B_nP'_n(\cos\theta)$ expressed as the linear combination of the squirming modes, with B_n the squirmer coefficients, $P'_n(x)$ the derivative of the nth-Legendre polynomials, $P'_n(x) = \partial_x P_n(x)$, and θ the polar angle, measured from $\hat{\boldsymbol{z}}$. To extract the squirmer modes, we project the tangential slip velocity on the squirmer basis, and compute $B_n^{(\pm)}$ for each sphere as,

$$B_n^{(\pm)} = (2n+1)\int_{-1}^{1}dx\sqrt{1-x^2}P'_n(x)v_\theta(R_\pm,x)\ . \tag{7}$$

where $x=\cos\theta$ is expressed in terms of the spherical coordinates with origin on each sphere, identified by $\mu=1$ in Fig.(6). Using the coordinate definitions (Eq. 36) we relate the azimuthal angle $x=\cos\theta$, in spherical coordinates, to its bispherical coordinate counterpart μ on the surface of each sphere by the relation $x(\mu) = \sinh^2\tau_\pm/(\cosh\tau_\pm-\mu) - \cosh\tau_\pm$, where we have $\theta=0$ at $\mu=1$, see Fig. 1. With the relation $x(\mu)$ and the coincidence in directions, $\hat{\boldsymbol{e}}_\mu = -\hat{\boldsymbol{\theta}}$ – only on the spheres, we can introduce the slip velocity from (Eq. 6) in the integrals for the squirmer components (Eq. 7). The interpretation of the first squirming mode, B_1, corresponds to the total swimming velocity of the sphere, $\boldsymbol{V} = 2/3B_1\hat{\boldsymbol{z}}$.

$$B_1^{(\pm)} = \frac{M_\pm\sinh^3|\tau_\pm|}{a}\sum_{n=0}\left[\int_{-1}^{1}d\mu\frac{(1-\mu^2)/2}{(\cosh\tau_\pm-\mu)^{5/2}} - \int_{-1}^{1}d\mu\frac{(1-\mu^2)P'_n(\mu)}{(\cosh\tau_\pm-\mu)^{3/2}}.\right] \tag{8}$$

The integration, given in the Supporting Information, results in a simple expression in terms of the coefficients of the chemical field on the surface of each particle:

$$\hat{\boldsymbol{e}}_\mu\cdot\boldsymbol{V}_\pm = -\frac{4\sqrt{2}}{3}\frac{M_\pm}{R_\pm}\frac{1-h_\pm^2}{h_\pm^{3/2}}\sum_{n=0}\left[n-(n+1)h_\pm^2\right]h_\pm^n s_n(\tau_\pm)\ . \tag{9}$$

The second squirming mode, B_2, is related to the nature of the induced hydrodynamic field, either extensile or contractile, with the symmetric induced stress given by $S = 4\pi\mu R^2B_2$ [13].

$$B_2^{(\pm)} = -3\sqrt{2}\frac{M_\pm}{R_\pm}\frac{1-h_\pm^2}{h_\pm^{5/2}}\sum_{n=0}H_2h_\pm^n s_n(\tau_\pm)\ , \tag{10}$$

with $H_2 = (n-1)n - 2n(n+1)h_\pm^2 + (n+1)(n+2)h_\pm^4$ and the integrals computed following the procedures detailed in the Supporting Information.

Once the analytical framework is introduced we present a solution of the experimental situation of a pair of active an passive pair for low values of the separation distance. Within the considered geometry, Fig. 1, we place the passive cargo, of diameter $d_p = 2R_+$, at $z > 0$, and the active particle, of diameter $d_a = 2R_-$, at $z < 0$. The chemical activity $\alpha_\pm$, is substituted by $\alpha_+ = \alpha_p = 0$, for the passive particle, and by $\alpha_- = \alpha_a = \alpha$. And apply the boundary conditions (Eq. 5) to solve $s_n(\tau)$ as a power series in δ in the limit of low δ. We project then the resulting slip velocity $\boldsymbol{v}_{s,(k)}$ on the first squirming mode (Eq. 8) and obtain the propulsion velocity of each sphere, and the centre-of-mass velocity $v_{\boldsymbol{u}} = (V_a - V_p)/2$ of the dimer, disregarding the contribution stemming from the hydrodynamic interactions due to the induced flows. This procedure is detailed in the Supporting Information. Here we substitute the the values $d_a = 2R_-$, and $d_p = 2R_+$ and obtain:

$$v_{\boldsymbol{u}} = \frac{M_a d_a \alpha}{d_a + d_p}\Big[\frac{M_p}{M_a}\Big(4 - 2\gamma - \ln\frac{4\delta/d_a}{d_p/d_a(1+d_p/d_a)}\Big) - \frac{d_p}{d_a}\Big(4 - 2\gamma - \ln\frac{4\delta d_p/d_a^2}{(1+d_p/d_a)}\Big)\Big]. \tag{11}$$

The velocity of the pair reverses direction for a size ratio $(d_p/d_a)_v$ obtained by solving $v_{\boldsymbol{u}} = 0$. A simple solution for low values of M_p/M_a is obtained expanding d_p/d_a around the value M_p/M_a, then the solution $(d_p/d_a)_v$ is approximated by

$$\left(\frac{d_p}{d_a}\right)_v \approx \frac{M_p}{M_a}\frac{2 - \gamma + \ln\frac{4\delta}{d_a} + \ln\left[\frac{M_p}{M_a}\left(\frac{M_p}{M_a}+1\right)\right]}{2 - \gamma + \ln\frac{4\delta}{d_a} + \ln\left[\frac{M_a}{M_p}\left(\frac{M_p}{M_a}+1\right)\right]} \tag{12}$$

The size ratio where the doublet reverses direction increases with increasing values of the ratio between diffusophoretic mobilities M_p/M_a. A large value of M_p/M_a increases the magnitude of the induced velocity passive particle. The reversal, then takes place at larger aspect ratios where the effect of confinement of the chemical product increases. At $M_p/M_a > 4$ the most suited approximation of (d_p/d_a) is obtained as a solution of the series expansion at $d_p/d_a \gg 1$.

$$\left(\frac{d_p}{d_a}\right)_v \approx \frac{-M_p}{M_a A} W_{-1}\left[\frac{-AM_a}{M_p}\exp\left(-\frac{1}{A} + \frac{M_a}{2M_p}\right)\right] \tag{13}$$

with $A = 2 - \gamma + \ln 4\delta/d_a$, and the Lambert function W_{-1} approximated by $W_{-1}(-x) \approx \ln x - \ln[-\ln(x)]$. At large mobility ratios the reversal $d_p/d_a \approx M_p/M_a(1 + A\ln(M_p/M_a))$.

The effective stresslet that the dimer exerts to the fluid is obtained adding the surface average contribution coming from each colloid $S_{\boldsymbol{u}} = \pi\eta\left(d_a^2 B_2^{(a)} - d_p^2 B_2^{(p)}\right)$, leading to

$$S_{\boldsymbol{u}} = 3\sqrt{2}\pi\eta\frac{M_a d_a \alpha}{d_a + d_p}\frac{d_p}{d_a}\Big[\frac{M_p d_p}{M_a d_a}\Big(4 - 2\gamma - \ln\frac{4\delta/d_a}{d_p/d_a(1+d_p/d_a)}\Big) - \Big(4 - 2\gamma - \ln\frac{4\delta d_p/d_a^2}{1+d_p/d_a}\Big)\Big]. \tag{14}$$

The dimer vanished for a dimer ratio $(d_p/d_a)_S$. An expansion of (Eq. 14) at low values of (d_p/d_a) leads to an approximate solution for $(d_p/d_a)_S$ which reads

$$\left(\frac{d_p}{d_a}\right)_S = \frac{M_a}{M_p}\frac{D + \ln(M_p/M_a + 1)}{D + \ln(M_p/M_a + 1) - 2\ln M_p/M_a}, \tag{15}$$

with $D = 6 - 2\gamma - \ln 4\delta/d_a$. This asymptotic expression has a relative error, when compared to the numerical solution to $S_u = 0$, below 3% in the range $10^{-2} < M < 10^2$. Altogether, we have a reversal in both the velocity, $(d_p/d_a)_v \sim M_p/M_a$, and the stress generated in the surrounding fluid $(d_p/d_a)_v \sim (M_p/M_a)^{-1}$. This indicates that the change in the orientation of the velocity does not modify the hydrodynamic signature of the doublet. Only for $M_p \approx M_a$ the system changes both the propulsion direction and its hydrodynamic signature at the same size ratio $d_p \approx d_a$.

SECTION A. USEFUL INTEGRALS

In the paper we use several integrals that involve several forms of the Legendre polynomials and multiple powers of $1/\sqrt{\cosh\tau-\mu}$, evaluated at $\tau=\tau_\pm$. We introduce the integrals with the substitution $h_\pm=\exp(-|\tau_\pm|)$,

$$G_1=\int_{-1}^{1}\frac{P_n(\mu)}{(\cosh\tau_\pm-\mu)^{1/2}}d\mu=\frac{2\sqrt{2}}{2n+1}h_\pm^{n+1/2}, \tag{16}$$

$$G_2=\int_{-1}^{1}\frac{P_n(\mu)}{(\cosh\tau-\mu)^{3/2}}d\mu=\frac{2\sqrt{2}}{\sinh|\tau_\pm|}h_\pm^{n+1/2}, \tag{17}$$

$$G_3=\int_{-1}^{1}\frac{\mu P_n(\mu)}{(\cosh\tau_\pm-\mu)^{1/2}}d\mu=\frac{2\sqrt{2}}{2n+1}h_\pm^{n+1/2}\left(\frac{n+1}{2n+3}h_\pm+\frac{n}{2n-1}h_\pm^{-1}\right), \tag{18}$$

$$G_4=\int_{-1}^{1}\frac{(1-\mu^2)P_n'(\mu)}{(\cosh\tau_\pm-\mu)^{1/2}}=\frac{2\sqrt{2}n(n+1)}{2n+1}h_\pm^{n+1/2}\left[\frac{h_\pm^{-1}}{2n-1}-\frac{h_\pm}{2n+3}\right], \tag{19}$$

$$G_5=\int_{-1}^{1}\frac{\mu P_n(\mu)}{(\cosh\tau_\pm-\mu)^{3/2}}d\mu=\frac{2\sqrt{2}h_\pm^{n+1/2}}{\sinh|\tau_\pm|}\left(\frac{nh_\pm^{-1}}{2n+1}+\frac{n+1}{2n+1}h_\pm\right). \tag{20}$$

The first integral is computed using $2\cosh\tau_\pm=h_\pm+h_\pm^{-1}$,

$$\int_{-1}^{1}\frac{P_n(\mu)}{(\cosh\tau_\pm-\mu)^{1/2}}d\mu=\int_{-1}^{1}\frac{\sqrt{2}h_\pm^{1/2}P_n(\mu)}{\left(1-2h_\pm\mu+h_\pm^2\right)^{1/2}}d\mu, \tag{21}$$

then the series expansion of $1/(1-2z\mu+z^2)^{1/2}=\sum z^{m+1}P_m(\mu)$, and finally the orthogonality of $P_n(\mu)$

$$\sqrt{2}\sum_m\int_{-1}^{1}h_\pm^{n+1/2}P_n(\mu)P_m(\mu)=\frac{2\sqrt{2}}{2n+1}h_\pm^{n+1/2}. \tag{22}$$

Next the second integral is obtained by a τ derivative of the G_1 integral, for $\tau_\pm\neq 0$,

$$G_2=\int_{-1}^{1}\frac{P_n(\mu)}{(\cosh\tau-\mu)^{3/2}}d\mu=\frac{2\sqrt{2}}{\sinh|\tau_\pm|}\exp\left[-(n+1/2)|\tau_\pm|\right]=\frac{2\sqrt{2}}{\sinh|\tau_\pm|}h_\pm^{n+1/2}. \tag{23}$$

Other integrals present terms $\mu P_n(\mu)$ which are simplified with the recurrence relations of the Legendre polynomials

$$\mu P_n(\mu)=\frac{n+1}{2n+1}P_{n+1}(\mu)+\frac{n}{2n+1}P_{n-1}(\mu) \tag{24}$$

And the resulting integrals performed by conveniently using (16). For instance

$$G_3=\int_{-1}^{1}\frac{\mu P_n(\mu)}{(\cosh\tau_\pm-\mu)^{1/2}}d\mu=\frac{2\sqrt{2}}{2n+1}\left(\frac{n+1}{2n+3}h_\pm^{n+3/2}+\frac{n}{2n-1}h_\pm^{n-1/2}\right), \tag{25}$$

and after differenciating respect to $\tau_\pm$ we obtain

$$G_5=\int_{-1}^{1}\frac{\mu P_n(\mu)}{(\cosh\tau_\pm-\mu)^{3/2}}d\mu=\frac{2\sqrt{2}}{\sinh|\tau_\pm|}\left(\frac{nh_\pm^{n-1/2}}{2n+1}+\frac{n+1}{2n+1}h_\pm^{n+3/2}\right). \tag{26}$$

We use the recurrence relation of the Legendre's for the derivative as follows,

$$(1-\mu^2)P_n'(\mu)=nP_{n-1}(\mu)-n\mu P_n(\mu), \tag{27}$$

$$G_4=\int_{-1}^{1}\frac{(1-\mu^2)P_n'(\mu)}{(\cosh\tau_\pm-\mu)^{1/2}}=n\int_{-1}^{1}\frac{P_{n-1}(\mu)-\mu P_n(\mu)}{(\cosh\tau_\pm-\mu)^{1/2}}=\frac{2\sqrt{2}n(n+1)}{2n+1}\left[\frac{h_\pm^{n-1/2}}{2n-1}-\frac{h_\pm^{n+3/2}}{2n+3}\right]. \tag{28}$$

For other integrals, once we integrate by parts, we use the eigenvalue equation,

$$\frac{d}{d\mu}\left[(1-\mu^2)\frac{d}{d\mu}P_n(\mu)\right]=-n(n+1)P_n(\mu). \tag{29}$$

A.1 Projection of the first squirmer moment

As an example we apply this to the integral of the first squirming mode obtained in the main text,

$$I^{(1)} = \int_{-1}^{1} d\mu \left[\frac{1}{2} \frac{(1-\mu^2)P_n(\mu)}{(\cosh\tau_\pm - \mu)^{5/2}} - \frac{\left(1-\mu^2\right) P_n'(\mu)}{(\cosh\tau_\pm - \mu)^{3/2}} \right]. \tag{30}$$

We first integrate by parts. To do so we write the derivative

$$\frac{d}{d\mu} \frac{(1-\mu^2)P_n(\mu)}{(\cosh\tau_\pm - \mu)^{3/2}} = \frac{3}{2} \frac{(1-\mu^2)P_n(\mu)}{(\cosh\tau_\pm - \mu)^{5/2}} + \frac{\left(1-\mu^2\right) P_n'(\mu)}{(\cosh\tau_\pm - \mu)^{3/2}} - 2 \frac{\mu P_n(\mu)}{(\cosh\tau_\pm - \mu)^{3/2}}, \tag{31}$$

and integrate both sides. Note that the left side banishes for $\tau_\pm \neq 0$ given that $P_n(\pm 1)$ are finite,

$$\int_{-1}^{1} d\mu \frac{(1-\mu^2)P_n(\mu)}{(\cosh\tau_\pm - \mu)^{5/2}} = \frac{4}{3} \int_{-1}^{1} d\mu \frac{\mu P_n(\mu)}{(\cosh\tau_\pm - \mu)^{3/2}} - \frac{2}{3} \int_{-1}^{1} d\mu \frac{\left(1-\mu^2\right) P_n'(\mu)}{(\cosh\tau_\pm - \mu)^{3/2}}. \tag{32}$$

The first resulting integral is G_5, from (20). The second integral is performed by parts, and substituting the Lengendre equation (29)

$$\frac{d}{d\mu} \frac{(1-\mu^2)P_n'(\mu)}{(\cosh\tau_\pm - \mu)^{1/2}} = \frac{1}{2} \frac{(1-\mu^2)P_n'(\mu)}{(\cosh\tau_\pm - \mu)^{3/2}} + \frac{\frac{d}{d\mu}\left[(1-\mu^2)P_n'(\mu)\right]}{(\cosh\tau_\pm - \mu)^{1/2}} \tag{33}$$

$$\int_{-1}^{1} d\mu \frac{\left(1-\mu^2\right) P_n'(\mu)}{(\cosh\tau_\pm - \mu)^{3/2}} = 2n(n+1) \int_{-1}^{1} d\mu \frac{P_n(\mu)}{(\cosh\tau_\pm - \mu)^{1/2}} \tag{34}$$

Collecting all terms we obtain

$$I^{(1)} = \frac{2}{3} G_5 - \frac{8}{3} n(n+1) G_1 = \frac{8}{3} \frac{\sqrt{2} h_\pm^{n+1/2}}{1 - h_\pm^2} \left[h_\pm^2 - n\left(1 - h_\pm^2\right) \right] \tag{35}$$

SECTION B. SHORT DISTANCE EXPANSION

We consider a stationary chemical field, $c(\boldsymbol{r})$, in the outer region of a two-sphere geometry, with radii $R_{\pm}$. This solution considers the limit where both spheres are separated a distance δ shorter than any of their sizes $R_{\pm}$.

The chemical field, in its stationary state, follows the Laplace $\nabla c(\boldsymbol{r}) = 0$ with boundary conditions in the derivatives of $c(\boldsymbol{r})$ being imposed on the surfaces of the spheres. In order to deal with the special geometry of the problem, we introduce the bispherical coordinate system $\{\eta, \mu, \varphi\}$, in which the Laplace equation is separable in 3d, via the following coordinate transformation from cylindrical coordinates

$$\rho = a\frac{\sqrt{1-\mu^2}}{\cosh\tau - \mu}, \quad z = a\frac{\sinh\tau}{\cosh\tau-\mu}, \quad \cos\phi = \cos\varphi \tag{36}$$

where a is a geometrical parameter, and $\{\rho, z, \phi\}$ are the cylindrical coordinates, with $\cos\phi = x/\rho$ and z in the direction of the axis of symmetry. Since we are introducing this coordinates with the aim of performing derivatives, and integrals, we list the Lamé coefficients associated to the coordinate change:

$$h_\tau = \frac{a}{\cosh\tau-\mu}, \quad h_\mu = \frac{a}{\cosh\tau-\mu}\frac{1}{\sqrt{1-\mu^2}}, \quad h_\varphi = \frac{a\sqrt{1-\mu^2}}{\cosh -\mu} \tag{37}$$

Within this coordinates, the surfaces of constant τ, see Fig.1, define a series of spheres with axis located at $\mu = \pm 1$. The unit vectors $\hat{\boldsymbol{e}}_\tau$, and $\hat{\boldsymbol{e}}_\mu$, on the surface of the spheres, are easily related to the spherical unit vectors $\hat{\boldsymbol{r}}$, and $\hat{\theta}$.

The symmetry of the coordinates simplifies the introduction of an asymmetric pair of spheres with surfaces located at bispherical coordinates $\tau_+ > 0$, and $\tau_- < 0$, the region of space outside the spheres is defined by values of $\tau \in (\tau_-, \tau_+)$. The size of each sphere is given by the relation $R_\pm = a/|\sinh\tau_\pm|$, and its location along the z axis is obtained as $z_+ = a\coth\tau_+$, and $z_- = a\coth\tau_- < 0$. The separation distance d is then easily defined as $d = z_+ - z_- = a\,(\coth\tau_+ - \coth\tau_-)$. These geometrical parameters, $\tau_\pm$, and a are obtained as a solution of the previous equations:

$$a = \frac{\sqrt{\left(R_+^2 - R_-^2 + d^2\right)^2 - 4d^2R_+^2}}{2d}, \tag{38}$$

$$h_+ = 2dR_+ \bigg/ \left(d^2 + R_+^2 - R_-^2 + \sqrt{\left(R_+^2 - R_-^2 + d^2\right)^2 - 4d^2R_+^2}\right), \tag{39}$$

$$h_- = \left(d^2 + R_-^2 - R_+^2 - \sqrt{\left(R_+^2 - R_-^2 + d^2\right)^2 - 4d^2R_+^2}\right) \bigg/ (2R_-d)\,, \tag{40}$$

with solution with the magnitude $h_\pm = \exp\left(-|\tau_\pm|\right)$. Within this coordinates the solution of the Laplace equation can be constructed as

$$c(\tau,\mu) = \sqrt{\cosh\tau-\mu}\sum_{n=0}^{\infty} s_n(\tau)P_n(\mu) \tag{41}$$

where the development coefficients are introduced by the following combination

$$s_n(\tau) = a_n\cosh[(n+1/2)\tau] + b_n\sinh[(n+1/2)\tau]. \tag{42}$$

The coefficients a_n, and b_n fix the solution to each boundary condition problem – gradients of chemical field perpendicular to the surfaces if the colloids are chemically active. As we have seen in the main text, it requires the calculaton of $s_n'(\tau)$:

$$s_n'(\tau) = (n+1/2)\left[a_n\sinh[(n+1/2)\tau] + b_n\cosh[(n+1/2)\tau]\right] \tag{43}$$

The gradient of the concentration field perpendicular to the surface each surface is maintained by the chemical reactions to a constant value $\nabla_\perp c(\boldsymbol{r}) = \hat{\boldsymbol{r}}\cdot(\nabla_c) = -\alpha$, with the gradient defined by

$$\nabla c(\tau,\mu) = h_\tau^{-1}\partial_\tau c(\tau,\mu)\,\hat{\boldsymbol{e}}_\tau(\tau,\mu) + h_\mu^{-1}\partial_\mu c(\tau,\mu)\,\hat{\boldsymbol{e}}_\mu(\tau,\mu). \tag{44}$$

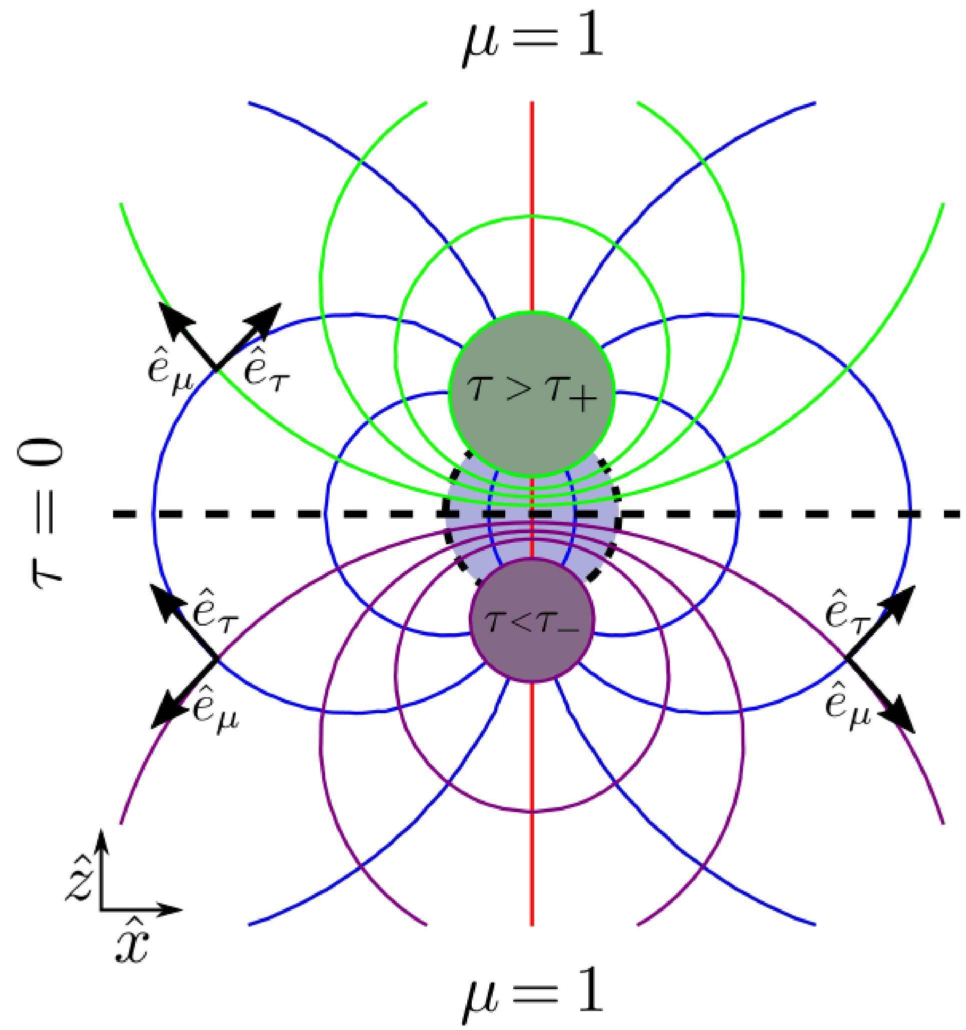

FIG. 1: Depiction of the bispherical coordinate system at $\varphi = 0$. Blue lines are sections of the surfaces at constant μ, green lines are sections of the spheres defined at constant $\tau > 0$, and purple lines are sections of the spheres adefined at constant $\tau < 0$. We shade a pair of spheres $\tau > \tau_+$, $\tau < \tau_-$, was well as the region $\mu < 0$, in blue. he solid red line denotes the symmetry axis $\mu = \pm 1$, with $\mu = -1$ in the region between spheres.

On a surface of constant τ, we have $\hat{\boldsymbol{e}}_\tau = -\hat{\boldsymbol{r}}$ for $\tau < 0$, and $\hat{\boldsymbol{e}}_\tau = \hat{\boldsymbol{r}}$ for $\tau < 0$. Since $\hat{\boldsymbol{e}}_\tau$, and $\hat{\boldsymbol{e}}_\mu$ constitute an orthogonal basis, the perpendicular gradient on each sphere is given by the τ-derivative of the concentration field $\nabla_\perp c(\boldsymbol{r})|_{\tau_\pm} = \mp h_\tau^{-1} \partial_\mu c(\tau, \mu)|_{\tau_\pm}$ evaluated on the surface of each particle.

$$\frac{1}{h_\tau} \frac{\partial c}{\partial \tau} = \frac{\sinh \tau}{2\, a} \sqrt{\cosh \tau - \mu} \sum_{n=0}^{\infty} s_n(\tau) P_n(\mu) + \frac{(\cosh \tau - \mu)^{3/2}}{a} \sum_{n=0}^{\infty} s_n'(\tau) P_n(\mu) \tag{45}$$

Applying the boundary conditions:

$$\left. \frac{\mp \alpha_\pm\, a}{\sqrt{\cosh \tau - \mu}} \right|_{\tau_\pm} = \frac{1}{2} \sinh \tau_\pm \sum_{n=0}^{\infty} s_n(\tau_\pm) P_n(\mu) + (\cosh \tau - \mu) \sum_{n=0}^{\infty} s_n'(\tau_\pm) P_n(\mu) \tag{46}$$

which, after the projection on the Legendre basis $\langle P_n(\mu) | P_m(\mu) \rangle = 2/(2n+1)\delta_{nm}$ and the aid of integrals (16, 18) in Appendix A results in the set of equations:

$$\frac{1}{2} \sinh \tau_+ s_n(\tau_+) + \cosh \tau_+ s_n'(\tau_+) - \left(\frac{n}{2n-1} s_{n-1}'(\tau_+) + \frac{n+1}{2n+3} s_{n+1}'(\tau_+) \right) = 0, \tag{47}$$

$$\frac{1}{2}\sinh\tau_- s_n(\tau_-) + \cosh\tau_- s'_n(\tau_-) - \left(\frac{n}{2n-1}s'_{n-1}(\tau_-) + \frac{n+1}{2n+3}s'_{n+1}(\tau_-)\right) = \sqrt{2}a\alpha h_-^{n+1/2} \tag{48}$$

where we introduce specify the chemical properties of the particles according to the experimental configuration with $\alpha_+ = 0$ for the passive (47), and $\alpha_- = \alpha \neq 0$ for the active (48) particle. The evaluation of the slip velocity on each particle only involves the value of $s_n(\tau_\pm)$. In order to obtain a solution in the short separation regime we introduce the dimension-less parameter δ/Δ, with $\Delta = (R_+R_-)/(R_+ + R_-)$, a distance smaller than either R_+, or R_-. In this Appendix we present a solution for a_n, and b_n as a series expansion in δ/Δ

$$a_n = a_n^{(0)} + a_n^{(1/2)}\left(\frac{\delta}{\Delta}\right)^{1/2} + a_n^{(1)}\left(\frac{\delta}{\Delta}\right) + \mathcal{O}\left[\left(\frac{\delta}{\Delta}\right)^{3/2}\right] \tag{49}$$

$$b_n = b_n^{(0)} + b_n^{(1/2)}\left(\frac{\delta}{\Delta}\right)^{1/2} + b_n^{(1)}\left(\frac{\delta}{\Delta}\right) + \mathcal{O}\left[\left(\frac{\delta}{\Delta}\right)^{3/2}\right] \tag{50}$$

Now, by introducing a_n, and b_n into "(47) + (48)", and "(47) − (48)", and expanding to small δ/Δ with taking into account that the location of the surfaces, $h_\pm$, is also developed to first orders in the small parameter:

$$h_+ = 1 - \frac{\sqrt{2}R_-}{R_+ + R_-}\left(\frac{\delta}{\Delta}\right)^{1/2} + \mathcal{O}(\delta/\Delta); \quad h_- = 1 - \frac{\sqrt{2}R_+}{R_+ + R_-}\left(\frac{\delta}{\Delta}\right)^{1/2} + \mathcal{O}(\delta/\Delta) \tag{51}$$

The first element in δ from equation "(47) + (48)" corresponds to the finite difference equation:

$$n\, b_{n-1}^{(0)} - (2n+1)b_n^{(0)} + (n+1)b_{n+1}^{(0)} = 0 \tag{52}$$

This recurrence relation appears multiple times in the following derivation. For this reason we condensate it into EQ2$[b_n^{(0)}] = 0$. This recurrence equation presents two independent solutions. The fist one is a trivial solution with $b_n^{(0)} = C_1$. The solution to the equation is given in terms of the Harmonic numbers $H_n = \sum_{k=1}^n 1/k$:

$$n\left(\sum_{k=1}^{n}\frac{1}{k} - \frac{1}{n}\right) - (2n+1)\sum_{k=1}^{n}\frac{1}{k} + (n+1)\left(\sum_{k=1}^{n}\frac{1}{k} + \frac{1}{n+1}\right) = 0 \tag{53}$$

The general solution is obtained as a linear combination $b_n^{(0)} = C + DH_n$. The initial condition is determined by the $n = 0$ term that leads to $b_0^{(0)} = b_1^{(0)}$. Applying the condition to the equation, $C = C + D$, implies $D = 0$. Thus, $b_n^{(0)} = C$.

The first term in δ/Δ in "(47) − (48)" is proportional to $(\delta/\Delta)^{1/2}$, and sets the relation between $a_n^{(0)}$:

$$(2n-1)n\, a_{n-1}^{(0)} - [3 + 4n(n+1)]\, a_n^{(0)} + (2n+3)(n+1)a_{n+1}^{(0)} = \frac{4\sqrt{2}R_+R_-\alpha}{R_+ + R_-} \tag{54}$$

which can be compacted as EQ1$[a_n^{(0)}] = K^{(0)} = 4\sqrt{2}R_+R_-\alpha/(R_+ + R_-)$. With EQ1$[a_n^{(0)}]$ the lhs in (54). The first terms are given by the equations:

$$-3a_0^{(0)} + 3a_1^{(0)} = K^{(0)}; \quad a_0^{(0)} - 11a_1^{(0)} + 10a_2^{(0)} = K^{(0)}; 6a_1^{(0)} - 27a_2^{(0)} + 21a_3^{(0)} = K^{(0)}; \ldots \tag{55}$$

We now proceed to solve the equation for $K^{(0)} \neq 0$, since $K^{(0)} = 0$ leads to the trivial solution. This difference equation is solved in the limit of large n. To do so, we follow Bender and Orszag[14] and transform the recurrence equation into a second order differential to obtain it's leading behavior. It is convenient to write the difference equation following the form $a_{n+2}^{(0)} + p(n)a_{n+1}^{(0)} + q(n)a_n^{(0)}$. First, we separate the $n = 0$ term ($a_0^{(0)} = -K^{(0)}/3 + a_1^{(0)}$). Then, we shift the indices so that we no longer have $n - 1$ as an index and write the difference equation for $w_n = a_n^{(0)}/K^{(0)}$ to ease notation.

$$(2n+5)(n+2)w_{n+2} - [3 + 4(n+1)(n+2)]w_{n+1} + (2n+1)(n+1)w_n = 1 \tag{56}$$

Dividing by $(2n+5)(n+2)$ and expanding to large n we obtain:

$$w_{n+2} - \left(2 - \frac{3}{n} + \frac{9}{n^2} + \ldots\right)w_{n+1} + \left(1 - \frac{3}{n} + \frac{9}{n^2} + \ldots\right)w_n = \frac{1}{2n^2} - \frac{9}{4n^3} + \ldots \tag{57}$$

To solve the recurrence equation we keep in mind the following substitution rules.

$$n \leftrightarrow x; \quad w_n \leftrightarrow y(x); \quad D[w_n] = w_{n+1} - w_n \leftrightarrow y'(x); \quad w_{n+2} + w_n - 2w_{n+1} \leftrightarrow y''(x) \tag{58}$$

where the finite difference $D[\omega_n]$ plays the role of the first order derivative in the correspondent differential equation. The equation is transformed into:

$$xy''(x) + 3y'(x) = 1/(2x). \tag{59}$$

The solution to the homogeneous part of the differential equation, $x\, y_h''(x) + 3y_h'(x) = 0$, is solved by the substitution $y_h'(x) = z(x)$, and thus

$$y_h(x) = \frac{-C}{2x^2} + D \tag{60}$$

The general solution is obtained by adding a particular solution. We assume an algebraic form for the particular solution, $z_p = Ax^c$, and establish the condition for A, and c, $Acx^c + 3Ax^c = 1/2x^{-1}$. The relation is by $c = -1$, and $A = -1/4$. Then $y_p = 1/4 \ln x$ is a particular solution to the differential equation. We write the general solution as

$$y(x) = \frac{1}{4}\ln(x) + D - \frac{C}{x^2} \tag{61}$$

The translation of $\ln x$ to a function of n is commonly done by the substitution to the digamma function, $\psi(n) = \Gamma'(n)/\Gamma(n)$, since $D[\psi(n)] = 1/n$. The digamma function has the asymptotic behaviour $\psi(n) \sim \ln n - 2/n$, and in the large n limit we substitute $\ln x \leftrightarrow \psi(n)$ with no loss of accuracy. Altogether, we have:

$$w_n \approx D + \frac{1}{4}\psi(n) - \frac{C}{n^2} \approx D + \frac{1}{4}\ln(n) - \frac{1}{8n} + \dots \tag{62}$$

We then return to $a_n^{(0)}$

$$\boxed{a_n^{(0)} = \frac{K^{(0)}}{4}\psi(n) = \frac{\sqrt{2}R_+R_-\alpha}{R_+ + R_-}\psi(n), \qquad n \geq 1} \tag{63}$$

Now we separately compute $a_0^{(0)}$ using the previously excluded first term of the series, $a_0^{(0)} = -K^{(0)}/3 + a_1^{(0)}$.

$$a_0^{(0)} = -\frac{\sqrt{2}R_+R_-\alpha}{R_+ + R_-}\left(\psi(1) + \frac{4}{3}\right) = -\frac{\sqrt{2}R_+R_-\alpha}{R_+ + R_-}\left(\frac{4}{3} + \gamma\right) \tag{64}$$

The first correction in $(\delta/\Delta)^{1/2}$ in equation "(47) + (48)" defines the first non-zero b_n terms as a function of $a_n^{(0)}$, and the production term α:

$$\frac{R_+ - R_-}{2\sqrt{2}(R_+ + R_-)}\text{EQ1}\left[a_n^{(0)}\right] - \frac{2R_+R_-\alpha}{R_+ + R_-} = \text{EQ2}[b_n^{(1/2)}] \tag{65}$$

In order to solve $b_n^{(1/2)}$ we substitute (54) and simplify the lhs as

$$\frac{R_+ - R_-}{R_+ + R_-}\frac{2R_+R_-\alpha}{R_+ + R_-} - 2\frac{R_+R_-\alpha}{R_+ + R_-} = 2\frac{R_+R_-\alpha}{(R_+ + R_-)^2}(R_+ - R_- + R_+ + R_-) = \frac{4R_+^2R_-\alpha}{(R_+ + R_-)^2} \tag{66}$$

And reduces (65) to $\text{EQ2}[b_n^{(1/2)}] = G^{(1/2)}$, with $G^{(1/2)} = 4R_+^2R_-\alpha/(R_+ + R_-)^2$. The solution follows the procedure previously described with $b_0^{(1/2)} = b_1^{(1/2)} - G^{(1/2)}$. The recurrence relation is solved for $\omega_n = b_n^{(1/2)}/G^{(1/2)}$ with the $n > 0$ terms given by:

$$(n+1)\omega_n - (2n+3)\omega_{n+1} + (n+2)\omega_{n+2} = 1, \tag{67}$$

dividing by $n+2$ and reordering terms we arrive at

$$\omega_{n+2} - \frac{2n+3}{n+2}\omega_{n+1} + \frac{n+1}{n+2}\omega_n = w_{n+2} - 2\omega_{n+1} + \omega_n + \left(2 - \frac{2n+3}{n+2}\right)\omega_{n+1} - \left(1 - \frac{n+1}{n+2}\right)\omega_n, \tag{68}$$

a finite difference equation of the form

$$D^2[\omega_n] + \frac{1}{n+2} D[\omega_n] = \frac{1}{n+2}. \tag{69}$$

We already know its homogeneous solution $w_n = C + DH_n$. Then the solution to the recurrence equation is obtained by adding a particular solution to the equation, $\omega_n^p = n$ with $D[n] = 1,\ D^2[n] = 0$. After substitution the particular solution satisfies: $0 + 1/(n+1) = 1/(n+2)$. The solution to this equation is then given by

$$\omega_n = C + DH_n + n \tag{70}$$

With the first term given by the relation $\omega_1 = C + D + 1 = \omega_0 + 1 = C + 1$ so $D = 0$ with arbitrary C:

$$\boxed{b_n^{(1/2)} = \frac{4R_+^2 R_- \alpha}{(R_+ + R_-)^2} n} \tag{71}$$

With the first non-negative terms in

$$a_n^{(0)} = \sqrt{2}\frac{R_+ R_- \alpha}{R_+ + R_-}\,\psi(n+1),\ n > 0, \qquad a_n^{(1/2)} = -\frac{4}{3}\frac{R_+^2 R_- \alpha}{(R_+ + R_-)^2}\,n \tag{72}$$

$$a_0^{(0)} = -\left(\frac{1}{3} + \gamma\right)\frac{R_+ R_- \alpha}{R_+ + R_-}, \quad b_n^{(0)} = 0, \quad b_n^{(1/2)} = -\frac{4R_+ R_-^2 \alpha}{(R_+ + R_-)^2}\,n \tag{73}$$

(74)

B.1 Computing the velocity

We are interested in a development of the chemical field on the surface of the spheres, thus evaluating $s_n(\tau)$ at $\tau_\pm$ (or $h_\pm$).

$$s_n(\tau_+) = \cosh\tau_+\, a_n + \sinh\tau_+\, b_n = a_n^{(0)} + \left[a_n^{(1/2)} + \sqrt{2}\frac{(2n+1)R_-}{R_+ + R_-} b_n^{(0)}\right]\left(\frac{\delta}{\Delta}\right)^{1/2} + \cdots \tag{75}$$

$$s_n(\tau_-) = \cosh\tau_-\, a_n + \sinh\tau_-\, b_n = a_n^{(0)} + \left[a_n^{(1/2)} - \sqrt{2}\frac{(2n+1)R_+}{R_+ + R_-} b_n^{(0)}\right]\left(\frac{\delta}{\Delta}\right)^{1/2} + \cdots \tag{76}$$

Since $b_n^{(0)} = 0$ the only remaining relevant terms are $a_n^{(0)}$, and $a_n^{(1/2)}$.

$$V_- = -\frac{\sqrt{2}}{2}\frac{M_-}{R_-}\frac{1-h_-^2}{h_-^{3/2}}\sum_{n=0}^{\infty}\left[n-(n+1)h_-^2\right]h_-^n\left[a_n^{(0)} + a_n^{(1/2)}\left(\frac{\delta}{\Delta}\right)^{1/2}\right] \tag{77}$$

$$V_+ = -\frac{\sqrt{2}}{2}\frac{M_+}{R_+}\frac{1-h_+^2}{h_+^{3/2}}\sum_{n=0}^{\infty}\left[n-(n+1)h_+^2\right]h_+^n\left[a_n^{(0)} + a_n^{(1/2)}\left(\frac{\delta}{\Delta}\right)^{1/2}\right] \tag{78}$$

With the behavior given by

$$a_n^{(0)} = \frac{\sqrt{2}R_+R_-\alpha}{R_+ + R_-}\psi(n+1), \quad n \geq 1; \qquad a_0^{(0)} \approx -(\gamma + 1/3)\frac{\sqrt{2}R_+R_-\alpha}{R_+ + R_-} \tag{79}$$

$$a_n^{(1/2)} = -\frac{4}{3}\frac{R_+^2 R_-\alpha}{(R_+ + R_-)^2}n \tag{80}$$

The summation involved in the process is the following one

$$S_{\ln} = \sum_{n=1}^{\infty}\left[n-(n+1)h_\pm^2\right]h_\pm^n\psi(n+1) = \frac{h_\pm}{1-h_\pm}\left[(1-\gamma)(1+h_\pm) + \gamma h_\pm^2 - \ln(1-h_\pm)\right] \tag{81}$$

If we proceed order by order:

$$\begin{aligned} s_n(\tau_+) =& a_n^{(0)} + \left[a_n^{(1/2)} + \sqrt{2}\frac{(2n+1)R_-}{R_+ + R_-} b_n^{(0)}\right]\left(\frac{\delta}{\Delta}\right)^{1/2} + \\ &+ \left(\frac{\delta}{\Delta}\right)\left[\frac{{R_-}^2 n^2 a_n^{(0)}}{(R_+ + R_-)^2} + a_n^{(1)} + \frac{\sqrt{2}R_-}{R_+ + R_-} n b_n^{(1/2)}\right] + \ldots \end{aligned} \tag{82}$$

$$\begin{aligned} s_n(\tau_-) =& a_n^{(0)} + \left[a_n^{(1/2)} - \sqrt{2}\frac{(2n+1)R_+}{R_+ + R_-} b_n^{(0)}\right]\left(\frac{\delta}{\Delta}\right)^{1/2} + \\ &+ \left(\frac{\delta}{\Delta}\right)\left[\frac{R_+^2 n^2 a_n^{(0)}}{(R_+ + R_-)^2} + a_n^{(1)} - \frac{\sqrt{2}R_+}{R_+ + R_-} n b_n^{(1/2)}\right] \end{aligned} \tag{83}$$

At very short separation distances we approximate $s_n(\tau_\pm) = a_n^{(0)}$. Then, the velocity is given by $V_\pm$

$$V_\pm = -\frac{\sqrt{2}}{2}\frac{M_\pm}{R_\pm}\sqrt{2}\frac{R_+R_-\alpha}{R_+ + R_-}\frac{1-h_\pm^2}{h_\pm^{3/2}}S_{\ln}^{(0)} \tag{84}$$

For the active particle, with $V_a = V_-$, we have a velocity contribution

$$V_a = -\frac{M_a R_p \alpha}{R_a + R_p} \left[-4 + 2\gamma + \ln \frac{2R_p\delta}{R_a(R_a + R_p)} \right] + \mathcal{O}\left[(\delta/\Delta)^{1/2} \right] \tag{85}$$

For the passive particle, $V_p = V_+$, we have a velocity contribution

$$V_p = -\frac{M_p R_a \alpha}{R_a + R_p} \left[-4 + 2\gamma + \ln \frac{2R_a\delta}{R_p(R_a + R_p)} \right] + \mathcal{O}\left[(\delta/\Delta)^{1/2} \right] \tag{86}$$

where we already substitute the subindex $+$(-) for the subscripts p(a) for the passive(active).

C SUPPORTING MOVIES

- **Supplementary Movie 1:**
Assembly of one hematite particle (long axis $= 1.8\mu\mathrm{m}$, short axis $= 1.3\mu\mathrm{m}$) and a passive silica sphere (diameter $1\mu\mathrm{m}$) via light illumination in a mixture of water and hydrogen peroxide (9% by Vol.). The blue light $450\mathrm{nm} < \lambda < 490\mathrm{nm}$ with power $P = 5\mathrm{mW}$ is applied after 3.5 seconds. This videoclip corresponds to Figure 1 of the main text.

- **Supplementary Movie 2:**
Observation of the velocity reversal with the size of the passive particle. The videoclip is composed of two part: The left video corresponds to a passive particle with diameter $1\mu\mathrm{m}$ and one active ellipsoid (long axis $= 1.8\mu\mathrm{m}$, short axis $= 1.3\mu\mathrm{m}$). The hybrid system propels with the hematite particle in the front of it after light illumination. The right video corresponds to a larger passive particle with a diameter of $4\mu\mathrm{m}$ and the composite pair propels with the passive particle in front of it. This videoclip corresponds to Figure 2 of the main text (illumination parameters: $P = 5\mathrm{mW}$, $450\mathrm{nm} < \lambda < 490\mathrm{nm}$).

- **Supplementary Movie 3:**
Transport of composite particles made of one passive particle (diameter $4\mu\mathrm{m}$) and different numbers (N_a) of hematite particles (long axis $= 1.8\mu\mathrm{m}$, short axis $= 1.3\mu\mathrm{m}$): Top left ($\mathrm{N}_a = 2$), Top right ($\mathrm{N}_a = 3$), Bottom left ($\mathrm{N}_a = 4$) and Bottom right ($\mathrm{N}_a = 5$). In all cases the hybrid systems propel with the large passive particle in front of it. This videoclip corresponds to Figure 5(a) and (b) of the main text (illumination parameters: $P = 5\mathrm{mW}$, $450\mathrm{nm} < \lambda < 490\mathrm{nm}$).

- **Supplementary Movie 4:**
Transport of composite particles made of one active hematite particle (long axis $= 1.8\mu\mathrm{m}$, short axis $= 1.3\mu\mathrm{m}$) and different numbers (N_p) of passive particles all with a diameter of $1\mu\mathrm{m}$: Top left ($\mathrm{N}_p = 2$), Top right ($\mathrm{N}_p = 4$), Bottom left ($\mathrm{N}_p = 5$) and Bottom right ($\mathrm{N}_p = 6$). In the first three cases the hybrid systems propel with the active hematite particle in front of it. In the last video (Bottom left) the composite particle stops the propulsion since the passive particles completely encircle the hematite one. This videoclip corresponds to Figure 5(c) and (d) of the main text (illumination parameters: $P = 5\mathrm{mW}$, $450\mathrm{nm} < \lambda < 490\mathrm{nm}$).

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