Formation and Fluctuation of Two-dimensional Dodecagonal Quasicrystal SUPPLEMENTARY INFORMATION

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1 Supporting video

Network rearrangement corresponding to Fig.11 in the main text from the time step 3000×10^4 to 3800×10^4 .

2 Results for isotropic particle system



Figure S1: Change in local structures of isotropic particles during annealing simulations at $\rho_a = 0.825$. (a) Ratio of the local structure in time; the line represents the data of a simulation, the shade area corresponds to the 95% confidence intervals of 10 independent simulations. The growth of the DDQC is divided into three stages. (b-d) The snapshots and their Fourier transformations. (e) Two kinds of dodecagonal motif as shown in the boxes in (d), they are interchangeable by rotating the particles on the first ring $\pm 30^{\circ}$. (f) The packing of the dodecagonal motifs of the structure in (d). (g) Orientations of the tiling edges connected by the particle positions of the structure in (d); the colours of the edges correspond to the six unit vectors shown above.



Figure S2: Interchanges between the local structures during the growth made of isotropic particles at $\rho_a = 0.825$ by decomposition of the "in" and "out" amount of selected pairs of local structures (a). (b-e) The corresponding data inside the box in (a). The graph show moving average of 15 data points.



Figure S3: Dependence of ratio of local structure of isotropic particles on temperature in (a) annealing simulations, (b) fixed temperature simulations at $\rho_{\rm a} = 0.825$. The dashed lines estimate the critical temperature T^* .



Figure S4: The reverse transition from DDQC to the Z structure for isotropic particles. (a) Structural change of particle composition during the reverse (continuous line) and annealing (dotted line). (b-c) Snapshots showing the expansion of Z particles region during the reverse process taken at T = 0.8625. (d) Displacements of particles from snapshot (b) to (c). Some rotations of the first neighbours of Z particles (marked yellow) can be observed. (e) Histogram of the rotation angle of the neighbours of Z particles in (d); dashed lines indicate peaks at $\pm 15^{\circ}$.