

Supplementary Material for "Pressure-induced Shape-shifting of Helical Bacteria"

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NOTATION

Symbol	Parameter
p	Pressure
p_x^*	Pressure at the critical point of x
z	Helix height
L_c	Contour length (tube's central axis)
L_f	Contour length (reinforced string)
r	Tube radius
R	Helix radius
L	Helical pitch (tube)
L_r	Helical pitch (reinforced string)
ζ	Helical angle (tube)
τ	Torsion of the tube's surface
α	Helical angle (reinforced string)
n	Number of turns (tube)
n_r	Number of turns reinforced line
t	Shell thickness
E	3D Young's modulus
Y	2D Young's modulus
ν	Poisson's ratio
μ	Second Lamé coefficient
I	Moment of inertia for thin shell
J	Moment of twist for thin shell

I. COMPUTATIONAL MODEL

A. Construction of the cell

The main tube defining the cell is generated by positioning vertices on the surface of a cylinder to obtain a perfectly-ordered equilateral triangular mesh organized in consecutive rings. Two hemispherical caps are added to the top and the bottom of the cylinder matching the radius of the main body. The generation of each cap starts by placing point particles randomly on a hemispherical shell. The number of particles per cap is calculated to have an average distance between particles equal to that of the main body. After randomly positioning the vertices on the spherical shell, their location is equilibrated by means of a Monte Carlo simulation considering a repulsive interaction between particles given by a Weeks-Chandler-Anderson potential. We obtain the final mesh from the Delaunay-based algorithm Advancing Front Surface Reconstruction routine on the Computational Geometry Algorithms Library (CGAL). We confirmed that the specific topology of the mesh does not affect our results by generating the cylindrical main body following the same protocol used for the caps. This results in vertices located randomly and presenting multiple topological defects, yet organized as nearly-equilateral triangles. We found the same helical properties for both meshes. Helical meshes to be used as input relax configurations were generated following the same procedure here described for the random cylinders but considering the parameterization of a helix.

The selection of vertices for the helical reinforcement relies on finding the best match between a theoretical helix parameterized as a function of its arc length and the cumulative distance between consecutively-connected vertices. That is, the i th vertex of the helix is obtained considering the helix:

$$\mathbf{r}_h = r_0 \cos(il_0c) \mathbf{i} + r_0 \sin(il_0c) \mathbf{j} + Lil_0c \mathbf{k}, \quad (1)$$

where r_0 and L are the radius and the pitch in the undeformed configuration and $c = a(r_0^2 + L^2)^{-1/2}$. The parameter $a = \sqrt{3}/2$ accounts for the mesh geometry and l_0 is the bond length of the mesh. The vertex i is selected among the coordinates of the nearest-neighbours of vertex $i - 1$, \mathbf{r}_i , as that with the minimum distance to the theoretical helix, $\arg \min_i \|\mathbf{r}_h - \mathbf{r}_i\|$. Concentric vertices to the main helix vertices can be selected for edge reinforcement, increasing the width w of the reinforced helical stripe (Fig. S10a). The results in the main section were obtained considering the minimal width possible given the degree of refinement of our mesh.

The dimensions of the spherocylinder and the mechanical properties of the cell can be found in Table S1.

B. Numerical minimisation

In our simulation toolbox, the total energy of the pressurized shell is described by:

$$E = E_s + E_b - pV, \quad (2)$$

with E_s the stretching energy, p is the pressure difference and V the volume enclosed by the shell. The stretching energy of the mesh is given by the harmonic potential

$$E_s = \frac{k_s}{2} \sum_{\langle i,j \rangle} (r_{ij} - r_{ij}^0)^2, \quad (3)$$

where the sum runs over all pairs of connected nodes i, j constituting an edge of the mesh and $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$. Despite the simplicity of this discrete model, it maps to an (equivalent) continuous elastic medium with 2D Young modulus

$$Y = \frac{2}{\sqrt{3}} k_s, \quad (4)$$

and with a constant Poisson ratio imposed by the triangular geometry of the mesh, given by $\nu = 1/3$ [1]. The 2D Young modulus Y is related to the 3D Young modulus E by $Y = Et$, with t the thickness of the shell. The stiffness of the edges connecting vertices identified as part of the helical region is reinforced by a factor $K = 10^3$. This large value was selected to ensure the insensibility of the reinforced region and should not be confused with a requirement for such specific increase in stiffness on a real system.

The bending stiffness is given by

$$E_b = k_b \sum_{\langle \alpha, \beta \rangle} (1 - \hat{\mathbf{n}}_\alpha \cdot \hat{\mathbf{n}}_\beta), \quad (5)$$

Parameter	Value
Cell configuration	
Cell radius (unpressurized), r_0	0.20 μm
Cell length, L_{c_0}	12.5 μm
Helix angle, α	40-88°
Number of vertices, N	10712
Mechanical parameters	
Cell wall Young modulus (3D), E	30 MPa
Cell wall thickness, t	2.0 nm
Reinforcing factor in helical region, K	10^3
Turgor pressure, p	0.1-1.5 atm

TABLE S1. Simulation parameters. The mechanical parameters were selected to match the described values of Gram negative bacteria [4–6]. The unpressurised radius results in r between 0.20-0.57 μm upon pressurisation (0.0-1.0 atm), in the range of experimentally-observed values for *Helicobacter pylori* [7].

where the sum is over the pair of triangles α, β sharing an edge, $\hat{\mathbf{n}}$ are their respective normal vectors and k_b is a bending stiffness. For a cylindrical object, the following relation between the latter and the continuous bending rigidity κ is [1, 2]:

$$\kappa = \frac{\sqrt{3}}{2} k_b. \quad (6)$$

where the κ is a function of the Young modulus and the thickness [3]:

$$\kappa = \frac{Eh^3}{12(1-\nu^2)}. \quad (7)$$

Hence, the discrete stretching and bending stiffnesses of the mesh model are defined by considering the Young modulus and thickness of Gram negative bacteria. Importantly, in Gram negative bacteria the thickness is small compared with radius of the cell. In such case, the stretching energy is much larger than the bending energy, $E_s \gg E_b$ and thus the bending contribution is mostly negligible in non-reinforced regions. We consider that the bending stiffness k_b of a pair of triangles sharing an edge connecting vertices identified as part of the helical region is reinforced by a factor K .

To calculate the volume V enclosed by a non-convex polyhedron we use the method described in [8]. The total volume enclosed by the surface is given by

$$V = \frac{1}{6} \sum_{t=N_t} \det(\mathbf{T}_t), \quad (8)$$

where the sum is over all the triangles of the surface N_t . The matrix \mathbf{T} is constructed from the coordinates of the vertices defining the triangle, $\mathbf{T} = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$ with $\mathbf{v}_i^T = [x_i, y_i, z_i]^T$ the coordinates of the vertex i .

We minimize the total energy (Eq. 2) using a custom non-linear conjugate gradient written in C++ using the secant-method for line search [9]. We consider analytical expressions for all the gradients. The total minimization routine for a final objective pressure is performed by iteratively increasing the pressure in a step-wise fashion until convergence. The code is freely available on the public repository in [10].

The mechanical parameters of the model can be found in Table S1.

C. Analysis of helical parameters

A three dimensional helix can be described by the radius of the tube r , as well as by the pitch L , the helical radius R and the total number of turns n of the helix. Additional quantities such as the contour length of the helix L_c , the height z or the end-to-end distance can be inferred from the former parameters. All the aforementioned parameters can be extracted from the medial axis of the helix. Therefore, we relied on extracting the medial axis as the first step in the characterization of the helix geometry.

C.1. Medial axis

We start by aligning the cell to the \mathbf{k} axis by means of the Rodrigues' rotation formula. To determine the coordinates of the nodes conforming the medial axis \mathbf{x}_i we used a tailored adaptation of the shrinking-ball algorithm [11]. This algorithm is general: it does not depend on any particular organization or topology of the mesh. However, we can exploit the specific topology of the mesh used in our simulations, where the vertices of the tube are organized as consecutive hoops. In this case, we can use the center of mass of each ring as a node of the medial axis. We obtained the same helical parameters by using the shrinking ball algorithm or the center of mass of the hoops for the determination of the medial axis (difference $< 5\%$).

C.2. Geometric parameters

From the nodes defining the medial axis of the tube, we determine the radius of the tube r as the distance between a vertex of the mesh to i) the line defined by the two nearest medial axis nodes to the given vertex (for the shrinking-ball medial axis) or ii) to the corresponding hoop center of mass (for hoop-based medial axis). Though the shrinking ball algorithm returns N medial axis nodes as well as the radius of the inscribed ball on the tube for each vertex, we noted that for the degree of refinement of our mesh this approach tends to underestimate the real radius of the tube.

The radius of the helix R is given by the radius of a node in the plane perpendicular to the axial direction of the helix \mathbf{x}_i^\perp . The local radius of the helix is then:

$$R = \|\mathbf{x}_i^\perp\| \quad (9)$$

with $\mathbf{x}_i^\perp = \mathbf{x}_i - (\mathbf{x}_i \cdot \hat{\mathbf{k}})\hat{\mathbf{k}}$. In a similar manner, we calculate the pitch L as

$$L_i = \frac{z_i}{\theta_i}, \quad (10)$$

where $z_i = \mathbf{t}_i \cdot \hat{\mathbf{k}}$ with $\mathbf{t}_i = \mathbf{x}_i - \mathbf{x}_{i-1}$. The angle θ represents the rotation on the plane normal to $\hat{\mathbf{k}}$. It is obtained as $\theta_i = \cos^{-1}(\hat{\mathbf{x}}_i^\perp \cdot \hat{\mathbf{x}}_{i-1}^\perp)$. The cumulative angle Θ is the total number of turns of the helical vessel, $\Theta = \sum \theta_i$ (and then $n = 2\pi\Theta$). The cumulative increase in extension $Z = \sum z_i$ as a function of the cumulative angle is fitted to a linear function to obtain an average pitch. Still, our approach allows to characterize the properties of the helices defined locally.

C.3. Torsion

In this work we refer as torsion to the twist of the surface of the tube, i.e. the rate of rotation of the material frame along the tangent to the medial axis, and it should not be confused with the torsion of a one dimensional helix [12]. We determine the torsion of the surface by taking advantage of the geometry of the mesh, composed of consecutive hoops organized to form a perfect hexatic structure. Hence, in the undeformed configuration, a vertex at hoop h , interacts with two vertexes of the hoop $h+1$. We call the coordinates of a given vertex and its two upper neighbours to its right and to its left \mathbf{r}_i , \mathbf{r}_i^r and \mathbf{r}_i^l , respectively. We make these points coplanar by subtracting the coordinates of the medial axis at h (for \mathbf{r}_i) or at $h+1$ (for \mathbf{r}_i^l and \mathbf{r}_i^r). In the undeformed configuration $\|\mathbf{r}_i^l - \mathbf{r}_i\| \equiv \|\mathbf{r}_i^r - \mathbf{r}_i\| = l_0/2$, with l_0 the bond length of the mesh. Thus, we can determine the torsion by minimizing the objective function for the distance between vertices:

$$f(\phi) = \sum_{i=1}^{N_h} (\|\mathbf{r}_i - \mathcal{R}\mathbf{r}_i^r\| - s)^2 + \sum_{i=1}^{N_h} (\|\mathbf{r}_i - \mathcal{R}\mathbf{r}_i^l\| - s)^2, \quad (11)$$

where the sum runs over all the vertices i of a hoop h , and the matrix $\mathcal{R} = \mathcal{R}(\phi, \hat{\mathbf{t}}_h)$ is a rotation matrix around the tangent vector to the medial axis $\hat{\mathbf{t}}_h$ at the hoop h . The value of the minimum is $s = \pi r_h / N_h$, with r_h the radius at hoop h . We used the golden-section search algorithm bracketed on the interval $[-\pi, \pi]$ to determine the argument of the minimum $f(\phi)$. This procedure returns the local torsion between two hoop h and $h+1$. We find the final torsion of the object from a linear fit to the cumulative sum of the torsion $\Phi = \sum \phi_i$ for all the consecutive hoops as a function of the cumulative length between hoops in the *undeformed* configuration, $\tau = \Phi / L_{c_0}$.

D. Torsion analysis

We examined if surface torsion τ is involved in the relocation of the reinforced string to the internal region of the helix (Fig. S9). We find that τ grows super-linearly with pressure and its absolute value, for a given pressure, is inversely proportional to the reinforcement angle α . This is consistent with the expected limit of no-torsion (only bending) for a reinforced tube with $\alpha = 90^\circ$. Pressurization for low α is torsion-dominated whilst large α is bending-dominated, yet the contribution of the complementary term is not negligible in terms of the total energy. Importantly, simulations show that τ has a smooth dependency on \bar{p} and that no change of τ during pressurization can be associated with a reduction of z , thus discarding torsion as a major determinant of the shortening transition.

We tested these observations experimentally: we clearly observe torsion of the surface upon pressurization for $\alpha = 50^\circ$ (Fig. S20), in agreement with our computational observations. The separation of a bending dominated and a torsion dominate regime are in agreement with the results obtained by others in fibers-reinforced soft actuators [13, 14].

II. ANALYTICAL MODEL

A. Dependency of cell radius and contour length on pressure

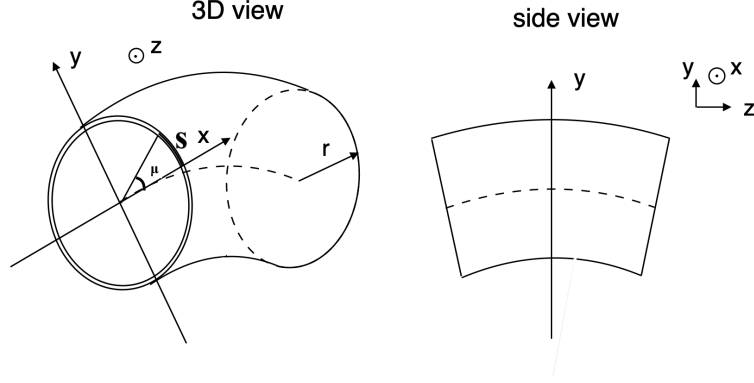


FIG. S1. Parametrization in curved tubes: 3D and side views.

Inspired by the observation from the simulation that the radius of the tube r and the contour length of the central axis L_c have little dependency on the reinforcement angle α (Figure S14), we use an empirical expression for $r(p)$ and $L_c(p)$ in the main text as functions of p (independent of α). Here we present a model for the expression of functions $r(p)$ and $L_c(p)$. Since the results hardly depend on the helical reinforcement angle α , we begin with an extreme case in which the reinforced line is completely straight, lying in the longitudinal direction of the cylinder. Thus after pressurization the vessel will curve into a banana shape as illustrated in Fig. S1. We assume that the length of this reinforced straight line remains fixed and the curved tube after pressurization has a uniform radius.

We begin with the general elastic energy expression for a continuous material:

$$\Phi = \frac{1}{2} \int N^{\alpha\beta} E_{\alpha\beta} d\xi_1 d\xi_2, \quad (12)$$

with

$$N^{\alpha\beta} = \frac{Et}{1-\nu^2} [(1-\nu)E^{\alpha\beta} + \nu E_\gamma^\gamma g^{\alpha\beta}]. \quad (13)$$

Here we consider only the stretching energy and neglect the bending energy. We use the parametrization of $\xi_1 = s$ and $\xi_2 = z$ as shown in Fig. S1. s is the distance measured along the curve of the cross-section in the undeformed state and z is the distance along the longitudinal direction of the tube. We further define $\mu = \frac{s}{r_0}$ as the corresponding dimensionless parameter. We define r_0 and L_0 as the cylinder radius and length in the undeformed state, r and L_c as the cylinder radius and length, and $\kappa = \frac{1}{R}$ as the curvature of the center line in the deformed state. Then the metric tensor g equals the identity matrix,

$$g = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Notice the constraint that the straight reinforced line on the tube has a fixed length. By using geometric relations, we have the expression for the strain tensor:

$$\begin{aligned} E^{\alpha\beta} &= E_{\alpha\beta} = \frac{1}{2}(G_{\alpha\beta} - g_{\alpha\beta}) \\ &= \frac{1}{2} \begin{pmatrix} \left(\frac{r}{r_0}\right)^2 - 1 & 0 \\ 0 & \left(\frac{R+r \cos \mu}{R-r}\right)^2 - 1 \end{pmatrix}. \end{aligned} \quad (14)$$

The off-diagonal elements are zero since there is no shear in this deformation. The diagonal term indicates how much the material is stretched in the circumferential and longitudinal directions. We define λ_α as the stretching rate in the

corresponding directions [12]. This number λ equals 1 when there is no stretching, bigger than 1 for actual stretching and smaller than 1 for compression. Thus we have

$$\begin{aligned}\lambda_s &= \frac{|d\mathbf{r}'_s|}{|dr_s|} = \sqrt{1 + 2E_{ss}} = \frac{r(p)}{r_0}, \\ \lambda_z &= \frac{|d\mathbf{r}'_z|}{|dr_z|} = \sqrt{1 + 2E_{zz}} = \frac{R + r \cos \mu}{R - r}.\end{aligned}\quad (15)$$

Notice that under the approximation of small strain $|E_{\alpha\alpha} - 1| \ll 1$, the rate of stretching becomes $\lambda_\alpha - 1 \approx E_{\alpha\alpha}$.

Consider the elastic energy of one single spring, $\Delta E = \frac{1}{2}k\Delta l^2$. Using the stretching rate λ and assuming that the spring lies along the stretching direction, we can rewrite the energy expression: $\Delta E = \frac{1}{2}kl^2(\lambda - 1)^2$, in which l is the length of the spring in the undeformed state. We compare this to the general elastic energy expression we have (Eq. 12). Thus instead of using $E_{\alpha\beta} = (\lambda^2 - 1)/2$, we define $\hat{E}_{\alpha\beta} = \lambda - 1$. This goes back to the linear elasticity model in the region of small strain. Then we have

$$\hat{E}^{\alpha\beta} = \hat{E}_{\alpha\beta} = \begin{pmatrix} \frac{r}{r_0} - 1 & 0 \\ 0 & \frac{R+r \cos \mu}{R-r} - 1 \end{pmatrix}.\quad (16)$$

We are now in the position to rewrite the general energy expression using $r(p)$ and $L_c(p)$ and minimize the total energy $U = \Phi - pV$ with regard to the two parameters. Defining dimensionless parameters $\bar{p} = \frac{pr_0}{Et}$, $\bar{U} = \frac{2U}{\pi r_0 L_0 Et}$, $\alpha = \frac{r}{r_0}$ and $\beta = \frac{L_c}{L_0}$ we have:

$$\begin{aligned}\bar{U} &= \frac{2}{1 - \nu^2}(\alpha - 1)^2 + \frac{3}{1 - \nu^2}(\beta - 1)^2 \\ &+ \frac{4\nu}{1 - \nu^2}(\alpha - 1)(\beta - 1) - 2\bar{p}\alpha^2\beta.\end{aligned}\quad (17)$$

Energy minimization gives us:

$$\begin{aligned}\frac{\partial \bar{U}}{\partial \alpha} &= \frac{4(\alpha - 1)}{1 - \nu^2} + \frac{4\nu(\beta - 1)}{1 - \nu^2} - 4\bar{p}\alpha\beta = 0, \\ \frac{\partial \bar{U}}{\partial \beta} &= \frac{6(\beta - 1)}{1 - \nu^2} + \frac{4\nu(\alpha - 1)}{1 - \nu^2} - 2\bar{p}\alpha^2 = 0.\end{aligned}\quad (18)$$

This would lead to a third order equation and can be solved analytically. Here we use commercial software (Mathematica) to plot the solutions without showing the lengthy expressions. For the values of ν and Et , the mapping between a continuous material and the triangular mesh in the limit of small deformation [1] predicts:

$$\begin{aligned}\nu &= \frac{1}{3} \\ Et &= \frac{2}{\sqrt{3}}k \approx 1.155k\end{aligned}\quad (19)$$

Then we have the results shown in Fig. S16 (a). This doesn't agree well with simulation data, indicating that we are not in the small deformation region. We find the value of ν and Et that describes the non-linearity of the mesh well:

$$\begin{aligned}\nu &= \frac{1}{5} \\ Et &= 1.44k_s\end{aligned}\quad (20)$$

The result is shown in Fig. S16 (b).

B. Calculation of the bending and twisting magnitude of a helix deformed from a cylinder

In this section we calculate the bending and twisting magnitude of a helix deformed from a cylinder using the Darboux vector. We introduce the material frame $(\vec{d}_1, \vec{d}_2, \vec{d}_3)$ in which \vec{d}_3 is the unit vector along the longitudinal direction and \vec{d}_1 and \vec{d}_2 are two unit vectors perpendicular to \vec{d}_3 and to each other. We assume that in the deformed

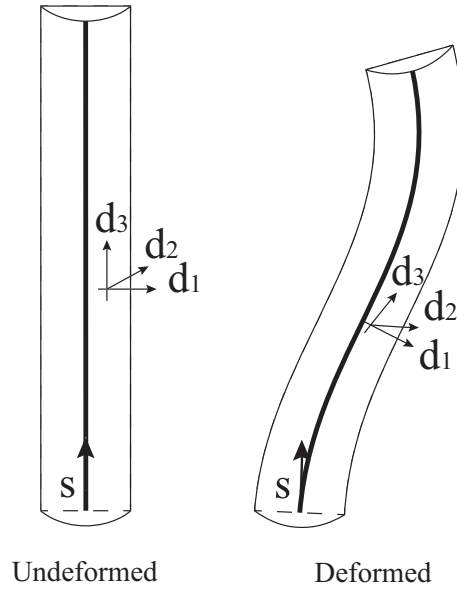


FIG. S2. The Darboux frame. Illustration of the material frame of a rod ($\vec{d}_1, \vec{d}_2, \vec{d}_3$).

state the three vectors are still perpendicular to each other. We use s to specify the position along the central axis. Then the condition of orthonormality requires [12]:

$$\vec{d}_i \cdot \vec{d}_i = 0 \text{ and } \frac{d(\vec{d}_i \cdot \vec{d}_j)}{ds} = 0. \quad (21)$$

Therefore, there exists three scalar functions such that the derivatives of the material frame can be described as:

$$\begin{aligned} \vec{d}_1'(s) &= \tau(s)\vec{d}_2(s) - \kappa_2(s)\vec{d}_3(s), \\ \vec{d}_2'(s) &= -\tau(s)\vec{d}_1(s) + \kappa_1(s)\vec{d}_3(s), \\ \vec{d}_3'(s) &= \kappa_2(s)\vec{d}_1(s) - \kappa_1(s)\vec{d}_2(s). \end{aligned} \quad (22)$$

Here prime ($'$) denotes the derivation with regards to s . We define the Darboux vector $\vec{\Omega}(s)$ as:

$$\vec{\Omega}(s) = \kappa_1(s)\vec{d}_1(s) + \kappa_2(s)\vec{d}_2(s) + \tau(s)\vec{d}_3(s), \quad (23)$$

so that Eq. 22 can be rewritten as

$$\vec{d}_i'(s) = \vec{\Omega}(s) \times \vec{d}_i(s). \quad (24)$$

In the following we want to calculate the Darboux vector $\vec{\Omega}$ of a general case in which a tube with a helical reinforced area specified by α deforms into a helix with a known shape. We utilize the assumption that the reinforced area lies at the inner most part of the helix.

We begin with the parameterization of the surface of a helix. The expression for a helix with parameterization (θ, ϕ) shown in Fig. S3 (c) is

$$\vec{r}(\theta, \phi) = \left((R + r \cos \phi) \cos \theta + \frac{Lr \sin \theta \sin \phi}{\sqrt{L^2 + R^2}}, (R + r \cos \phi) \sin \theta - \frac{Lr \cos \theta \sin \phi}{\sqrt{L^2 + R^2}}, L\theta + \frac{Rr \sin \phi}{\sqrt{L^2 + R^2}} \right). \quad (25)$$

We first express the material frame $(\vec{d}_1, \vec{d}_2, \vec{d}_3)$. Since $\vec{d}_3(s)$ is the unit vector along the central axis at position s , we express the function for the central axis of the helix:

$$\vec{x}(s) = (R \cos \theta, R \sin \theta, L\theta), \text{ with } s = \theta \sqrt{R^2 + L^2}. \quad (26)$$

Thus we have the expression for $\vec{d}_3(s)$:

$$\vec{d}_3 = \frac{d\vec{x}(s)}{ds} = \frac{R}{\sqrt{R^2 + L^2}} (-\sin \theta, \cos \theta, L/R) = (-\cos \zeta \sin \theta, \cos \zeta \cos \theta, \sin \zeta). \quad (27)$$

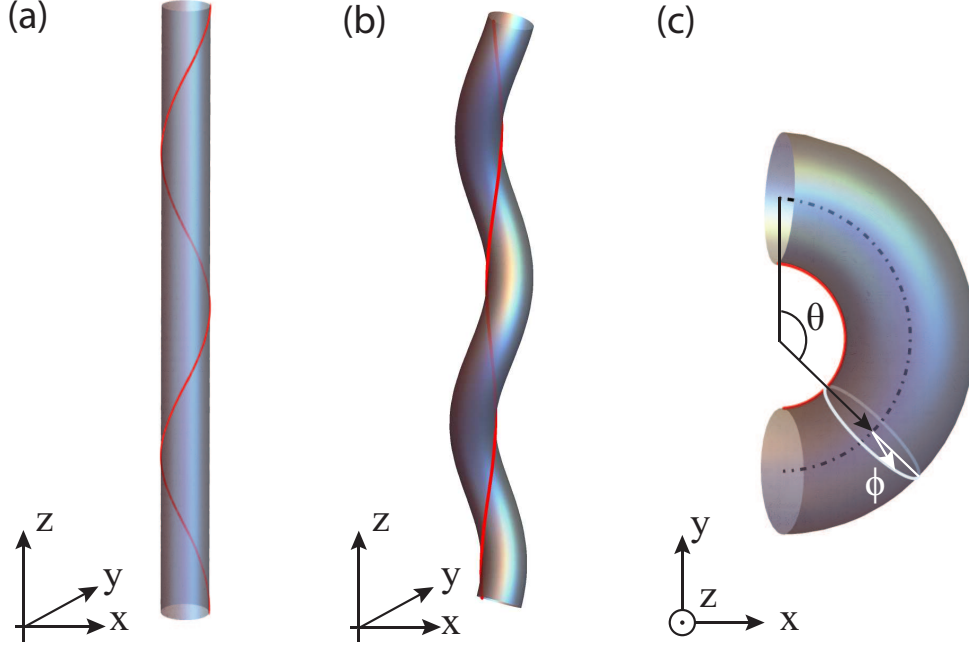


FIG. S3. Reinforced line considering different tube configurations. a) Illustration of the tube with the reinforced line in the undeformed state. b) Illustration of the tube in the shape of a helix with reinforced line (deformed state). c) Illustration showing the parameterization (θ, ϕ) .

For the last step, we use the definition of ζ ($\tan \zeta = L/R$).

To come up with the expression for $\vec{d}_2(s)$, we specify a straight line as a reference line on the surface of undeformed cylindrical tube and calculate the function for this line in the deformed state. The vector $\vec{d}_2(s)$ can then be expressed as the vector starting at certain position s on the central axis pointing towards the reference line on the plain locally perpendicular to the central axis. In the undeformed state, the system is a cylindrical tube. We can parameterize the undeformed state by $\vec{r}(r = r_0, R = 0, L = L_r)$, in which \vec{r} is specified in Eq. 25:

$$\vec{r}_{cylinder}(\theta_1, \phi_1) = \vec{r}(r = r_0, R = 0, L = L_r) = (r_0 \cos(\theta_1 - \phi_1), r_0 \sin(\theta_1 - \phi_1), L_r \theta_1). \quad (28)$$

Here we use the parameterization (θ_1, ϕ_1) to specify the undeformed case. Thus, one possible reference line expression, considering that the reference line is straight in the undeformed state, is:

$$\vec{r}_{r,1} = (r_0, 0, L_r \theta_1) = \vec{r}_{cylinder}(\theta_1, \phi_1 = \theta_1). \quad (29)$$

To come up with the function of the reference line in the deformed state, we need to use the expression of the reinforced line in both the undeformed and deformed state. In the undeformed state, the expression of the reinforced line is:

$$\vec{r}_{f,1} = (r_0 \cos(\theta_1), r_0 \sin(\theta_1), L_r \theta_1) = \vec{r}_{cylinder}(\theta_1, \phi_1 = 0). \quad (30)$$

In the deformed state, the system becomes a helix. According to the parameterization (θ, ϕ) illustrated in Fig. S3 (c), $\phi = 0$ always corresponds to the outer-most part of the helix. Since we assume that the reinforced line always lies in the inner-most part of the helix, we can use $\phi_2 = \pi$ to describe the position of the reinforced line:

$$\vec{r}_{f,2} = \vec{r}(\theta_2, \phi_2 = \pi). \quad (31)$$

Here we use the parameterization (θ_2, ϕ_2) to specify the deformed (helix) case. In the modeling, we only consider three kinds of deformation: bending, twisting and uniform swelling. For the process of bending and twisting, given two points on the same cross-section, we know the difference in ϕ between them is preserved during deformation. This can be seen by noticing that any cross-section of the system remains undeformed during bending or twisting (or combined). These two kinds of deformation change the relative positioning between the cross-sections, but don't deform the cross-sections themselves. For the uniform swelling, the tube radius and contour length increase in a

uniform way. Thus the difference in angle ϕ for two points on the same cross-section is still preserved. We know the function of the reference line is:

$$\vec{r}_{r,2} = \vec{r}(\theta_2, \phi_2 = \pi + \theta_1) . \quad (32)$$

Note that θ_1 and θ_2 are linearly related, where θ_1 ranges from 0 to n_r and θ_2 ranges from 0 to n . Here, n_r is the number of turns of the reinforced line and n is the number of turns of the tube. Thus, we have

$$\theta_1 = e\theta_2 , \text{ with } e = \frac{n_r}{n} . \quad (33)$$

With the function of the reinforced line at hand, we can express the vector $\vec{d}_2(s)$ as the vector starting at a certain position s on the central axis pointing towards the reference line on the plain locally perpendicular to the central axis.

$$\begin{aligned} \vec{d}_2(s) &= \frac{1}{r} [\vec{r}_{r,2} - \vec{x}(s)] = \frac{1}{r} \left[\vec{r} \left(\theta_2, \phi_2 = \pi + \frac{n_r}{n} \theta_2 \right) - (R \cos \theta_2, R \sin \theta_2, L\theta_2) \right] , \\ &= (\cos(\pi + e\theta_2) \cos \theta_2 + \sin \theta_2 \sin(\pi + e\theta_2) \sin \zeta, \cos(\pi + e\theta_2) \sin \theta_2 - \cos \theta_2 \sin(\pi + e\theta_2) \sin \zeta, \sin(\pi + e\theta_2) \cos \zeta) . \end{aligned} \quad (34)$$

Again for the last step, we use the definition of ζ ($\tan \zeta = L/R$). To get \vec{d}_1 , we use $\vec{d}_1 = \vec{d}_2(s) \times \vec{d}_3(s)$. We can then calculate τ , κ_1 and κ_2 by

$$\begin{aligned} \tau &= \vec{d}_1(s) \cdot \vec{d}_2(s) = -\vec{d}_2(s) \cdot \vec{d}_1(s) , \\ \kappa_1 &= \vec{d}_2(s) \cdot \vec{d}_3(s) = -\vec{d}_3(s) \cdot \vec{d}_2(s) , \\ \kappa_2 &= -\vec{d}_1(s) \cdot \vec{d}_3(s) = \vec{d}_3(s) \cdot \vec{d}_1(s) . \end{aligned} \quad (35)$$

Finally we get the results:

$$\begin{aligned} \tau &= (e - \sin \zeta) \frac{1}{\sqrt{L^2 + R^2}} , \\ \kappa_1 &= -\cos \zeta \cos(e\phi) \frac{1}{\sqrt{L^2 + R^2}} , \\ \kappa_2 &= \cos \zeta \sin(e\phi) \frac{1}{\sqrt{L^2 + R^2}} . \end{aligned} \quad (36)$$

Note that during the calculation we used the relation $\frac{d\theta}{ds} = \frac{1}{\sqrt{L^2 + R^2}}$. So we have the twisting and bending magnitude:

$$\begin{aligned} \tau &= \left(\frac{n_r}{n} - \sin \zeta \right) \frac{1}{\sqrt{L^2 + R^2}} , \\ \kappa &= \sqrt{\kappa_1^2 + \kappa_2^2} = \cos \zeta \frac{1}{\sqrt{L^2 + R^2}} . \end{aligned} \quad (37)$$

C. Modeling helix deformation

Here we calculate the deformation of a cylindrical tube with a helical reinforced line after pressurization. A finite helix can be defined by four parameters: (n, L, R, r) . First we have the assumption that the length of the reinforced area is fixed: $L_f = L_f(p=0)$. This implies:

$$2\pi n_r \sqrt{L_r^2 + r_0^2} = 2\pi n \sqrt{L^2 + (R-r)^2} . \quad (38)$$

For $r(p)$ and $L_c(p)$, we can use the expression derived in Supplemental Material Sec. II(A). Under these three constraints we wish to minimize the energy:

$$\Phi = \frac{EI}{2} \left(\kappa^2 + \frac{3}{4} \tau^2 \right) , \quad (39)$$

in which (Eq. 37)

$$\begin{aligned}\tau &= \left(\frac{n_r}{n} - \sin \zeta \right) \frac{1}{\sqrt{L^2 + R^2}}, \\ \kappa &= \sqrt{\kappa_1^2 + \kappa_2^2} = \cos \zeta \frac{1}{\sqrt{L^2 + R^2}}.\end{aligned}\quad (40)$$

For simplification, we define the following parameters:

$$\begin{aligned}\sqrt{\eta} &= \frac{L_f}{L_c(p)}, \\ \gamma &= \frac{r(p)}{(1-\eta)L}, \\ g &= \frac{(1-\eta)\gamma}{1+\beta^2} = \frac{\beta \pm \sqrt{\eta(1+\beta^2)} - 1}{1+\beta^2}, \\ \hat{n}_0 &= \frac{2\pi n_r r(p)}{L_c(p)}, \\ \beta &= \cot(\zeta) = \frac{R}{L}.\end{aligned}\quad (41)$$

Using these we have the expression for N and $\beta = \cot(\zeta) = \frac{R}{L}$:

$$\beta(L) = \gamma \pm \sqrt{\gamma^2 \eta - 1} = \frac{1 \pm \sqrt{\eta - g^2}}{g}, \quad (42)$$

$$n(L) = \frac{L_c(p)}{2\pi r} g \sqrt{1 + \beta^2}. \quad (43)$$

We rewrite the energy expression:

$$\Phi \frac{2}{EI} \frac{L_c^2}{(2\pi)^2} = n^2 - \frac{n^2}{4(1+\beta^2)} + \frac{3n_r^2}{4} - \frac{3}{2} \frac{n}{\sqrt{1+\beta^2}} n_r. \quad (44)$$

Since we can express both β and n as a function of L , we require $\frac{d\Phi}{dL} = 0$ and get the following:

$$\begin{aligned}4g\sqrt{1+\beta^2} \left(\frac{dg}{dL} \sqrt{1+\beta^2} + g \frac{d\sqrt{1+\beta^2}}{dL} \right) - g \frac{dg}{dL} - 3\hat{n}_0 \frac{dg}{dL} &= 0 \\ \Leftrightarrow 4 \frac{dg}{dL} \frac{\eta + 1 \pm 2\sqrt{\eta - g^2}}{g} - 4 \frac{dg}{dL} \frac{1 + \eta \pm 2\sqrt{\eta - g^2} \pm \frac{g}{\sqrt{\eta - g^2}}}{g} - g \frac{dg}{dL} - 3\hat{n}_0 \frac{dg}{dL} &= 0 \\ \Leftrightarrow \pm \frac{4g}{\sqrt{\eta - g^2}} + g + 3\hat{n}_0 &= 0 \\ \Leftrightarrow 16g^2 = (g + 3\hat{n}_0)^2 (\eta - g^2) \\ \Leftrightarrow g^4 + 6\hat{n}_0 g^3 + (16 - \eta + 9\hat{n}_0^2) g^2 - 6\hat{n}_0 \eta g - 9\hat{n}_0^2 \eta &= 0.\end{aligned}\quad (45)$$

Thus we can solve for g at different pressures and number of turns n_0 . This would also give us the values of β and n . We can derive the expression $L = \frac{L_c(p)}{2\pi n \sqrt{1+\beta^2}}$ to calculate the values of L . Recall $R = L\beta$. Thus we solve the system of (n, L, R, r) . Throughout the parameter region considered, the equation for g only has one positive real root so there is no ambiguity. We select the minus sign in the $\beta(g)$ expression (Eq. 42). A comparison between the results of simulation and analytical formulation can be found in Fig. S9 and Fig. 2 in the main text.

D. Critical pressure for the maximum helix height

Due to the constraint that the reinforced path is not stretchable, we have:

$$L_f = \sqrt{z^2 + (2\pi n)^2 \cdot (R - r)^2}. \quad (46)$$

Thus the maximum of z is achieved when $R = r$. On the other hand, we have the expression for R :

$$R = \beta L = \beta \cdot \frac{L_c}{2\pi n \sqrt{1 + \beta^2}} = \frac{r}{g(\beta + 1/\beta)}, \quad (47)$$

in which

$$\beta = \frac{1 - \sqrt{\eta - g^2}}{g}. \quad (48)$$

Thus $R = r$ gives us

$$\eta^2 = \eta - g. \quad (49)$$

On the other hand, the equation for g (Eq. 45) gives us

$$16g^2 = (g + 3\hat{n}_0^2)^2(\eta - g). \quad (50)$$

Thus we can find g and η for the critical pressure of maximum height. Recall the definition $\eta = \frac{L_f^2}{L_c^2}$, which implies η is close to 1. We can assume $\eta = 1 - \varepsilon$ with $|\varepsilon|$ a small parameter, and solve for \hat{n}_0 up to the first order of ε . Thus Eq. 49 becomes $g = \varepsilon$. Using this relation in Eq. 50 and neglecting the higher order terms in ε , we have:

$$\hat{n}_0^2 = \varepsilon = 1 - \eta = 1 - \frac{L_f^2}{L_c^2}. \quad (51)$$

Recalling the definition $\hat{n}_0 = \frac{2\pi n_r r(p)}{L_c(p)}$ (Eq. 41), we have:

$$\begin{aligned} (2\pi n_r r)^2 &= L_c^2 - L_f^2 \\ \Leftrightarrow L_{c0}^2 \cot^2(\alpha) \frac{r^2}{r_0^2} &= L_c^2 - L_{c0}^2(1 + \cot^2(\alpha)). \end{aligned} \quad (52)$$

So we have the final result as shown in the main text:

$$\cot^2(\alpha) = \frac{(L_c/L_{c0})^2 - 1}{(r/r_0)^2 + 1}. \quad (53)$$

Here L_{c0} is the contour length in the undeformed state, which equals the helix height in the undeformed state. The right hand side depends on the material properties, which turns out to be nonlinear in our case of a triangular mesh. Eq. 53 also reveals that when α decreases, the increase of the left hand side ($\cot^2(\alpha)$) would in general make it harder for the right hand side (which equals zero in the zero pressure case) to match the left hand side. Thus the state of maximum height may not be achievable when α is too small for a limited range of pressures, as observed in the simulation. With the empirical $r(p)$ and $L_c(p)$ relation provided in Supplemental Material Sec. II(A), we can calculate the range of α that the maximum height would be achieved during a limited range of pressure. For the current pressure range we consider ($p \leq 1.5$ atm), the maximum height would be achieved only if $\alpha > 61^\circ$. We compare the critical pressure for the maximum height we got from Eq. 53 and from numerically calculating the critical pressure for the height predicted in this analytical framework (Fig. S4). For the large α region with $\alpha > 70^\circ$, the two agree well with each other, thus validating the assumptions we used. As the pressure increases, the contour length L_c increases while the length of the reinforced line L_f is fixed. Thus $|\varepsilon|$ increases as the pressure increases, making our assumption less accurate. This leads to the divergence of the two curves as pressure increases. The comparison between the theoretical prediction (Eq. 53) and the simulation result can be found in Fig. 4 in the main text.

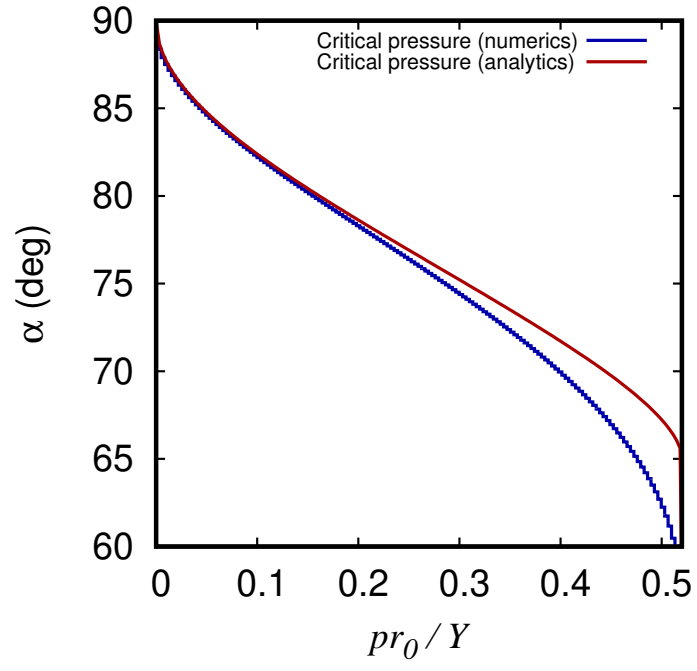


FIG. S4. Critical pressure as a function of the helical angle. Comparison between analytical results (Eq. 53) and numerical results (numerical calculation under the framework described in Supplemental Material Sec. II(C)) of the critical pressure for different values of α .

III. EXPERIMENTAL PROCEDURES

To create the rubber tubes, we print two mold parts using a 3D printer (Ultimaker 3 extended) and a PLA substance. The mold is a cylindrical cavity (30 cm length, 2 cm diameter) to which a solid tube with smaller diameter is inserted. The solid tube is centered by the cap and the pin at the tip (Fig. S5), in order to keep a fixed distance (1 mm) between the mold and the tube across the structure. The solid tube has a helical engraving with a specific pitch angle (ranging 25-85°), that supports the coiling of a red embroidery thread (1 mm diameter). The thread is attached at its ends to the solid tube using scotch tape.

A liquid composition of tin-cured silicone rubber (Mold-Max NV-14, Ceramet, prepared according to instructions) is poured onto the two mold halves. The solid tube is placed on top of one of them, aligning pins and pressure clamps are used to seal the other half on top of it. The composition is left to cure overnight in a vertical position. After releasing the cured rubber tube from the mold, further sealing of the hole at the tip is performed with the same silicone rubber. Marks on the rest configuration of the rubber tube are made using a Sharpie pen.

The tube is then connected to a pressure source with a fine pressure regulator (IR2010 0.1-4bar, Baccara.co.il). Videos and images of the different configurations are taken.

Helical properties were obtained from snapshots at different pressures using the ImageJ software.

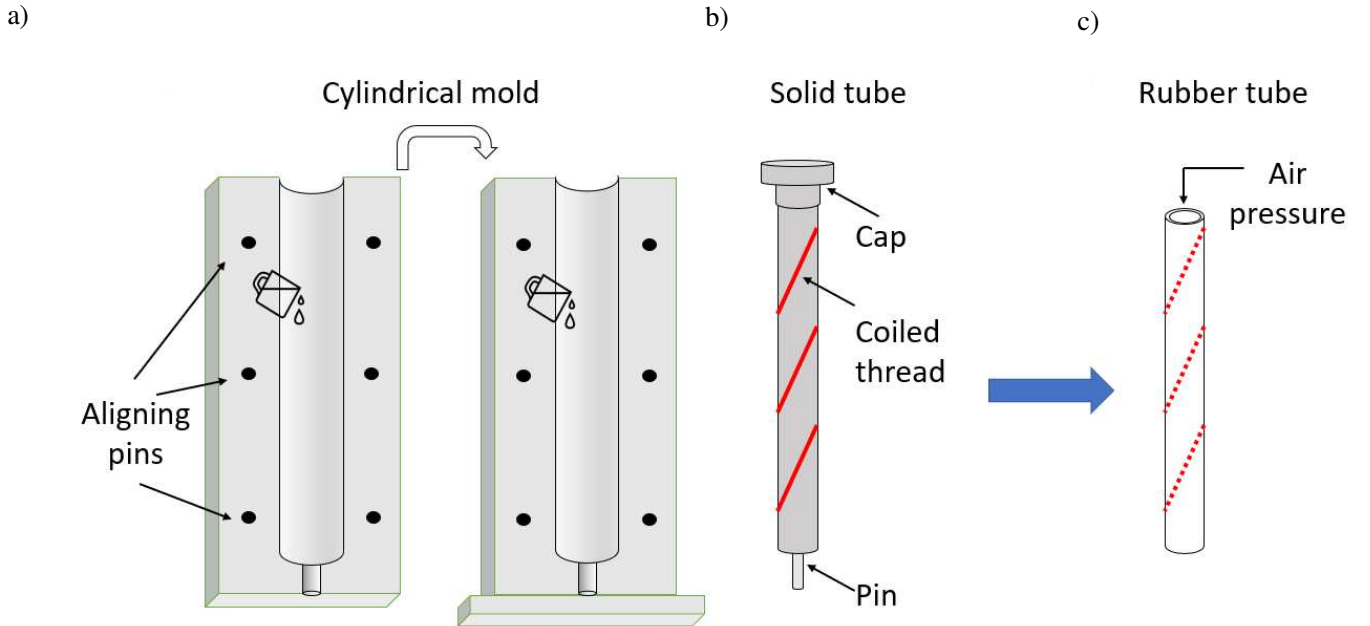


FIG. S5. Making of inflatable rubber tubes. a) The two halves of the 3D printed mold create a cylindrical cavity, to which the liquid rubber is poured. b) a 3D printed solid tube, with an unextensible thread coiled around it, is placed between the two mold halves. c) After over-night curing, the rubber tube with an embedded helical thread is released from the mold. Sealing the hole at the bottom with silicon, and connecting the top to a pressure source, the tube can now be inflated in a controlled manner.

IV. VARIATION OF OSMOLARITY IN THE STOMACH

It has been shown the osmolarity in the stomach of dogs increases by one order of magnitude after chemical-induction of gastric juice secretion, reaching the steady state ≈ 20 minutes post stimulation [15]. Later, [16] showed that the osmolarity 30 and 90 minutes post injections is similar, suggesting a rapid change after injection.

In addition, [16] showed that the osmolarity is highly-dependent on the ingested food. Most of the results shown by the authors concern the analysis of gastric juices after a steak meal, finding an osmolality on the stomach content of 232 mosm./kg. The authors, with the objective of testing the role of an hypertonic meal, conducted the same measurements after a milk and doughnut meal, finding an osmolarity almost two times larger (440 mosm./kg). A similar result was reported later by [17], where a wide range of osmolalities is observed in the stomach depending on the ionic strength of a drank solution, from 20 mosm./kg for water to 360 mosm./kg for an hypertonic beverage.

V. SUPPLEMENTARY FIGURES

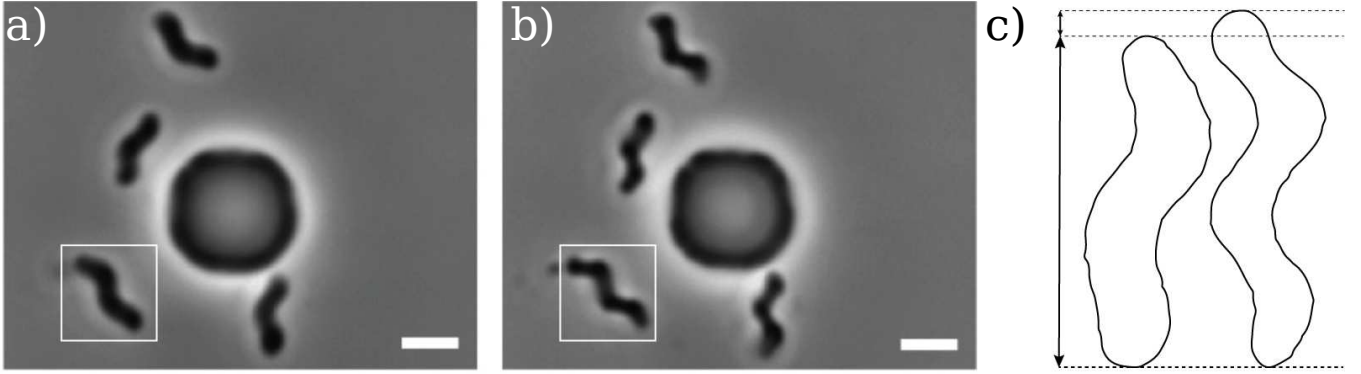
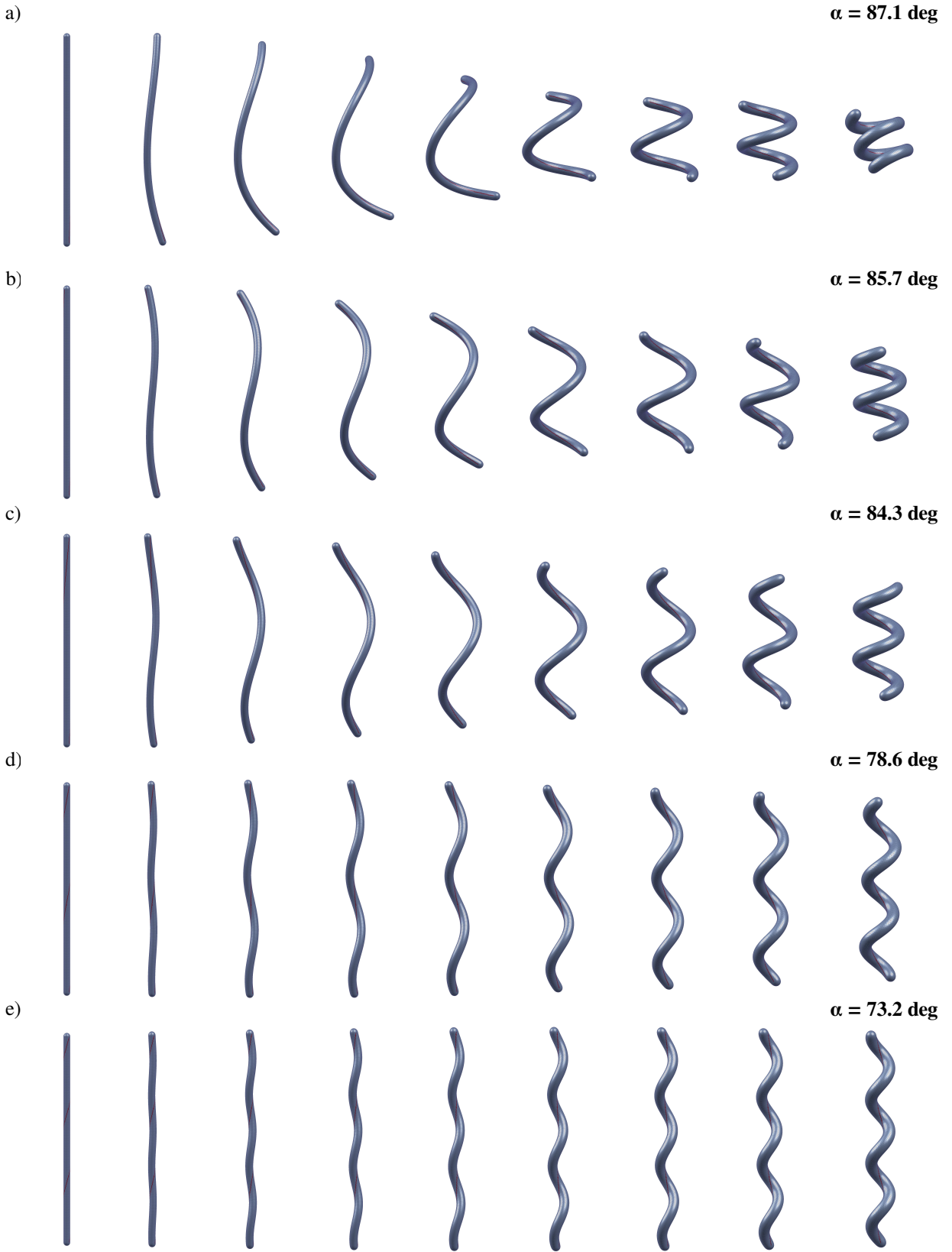


FIG. S6. Wild-type *H. pylori* cells a) before and b) after hyper-osmotic shocks with 200 mM NaCl. Figure adapted from [18]. c) The contours of one cell before and after hyper-osmotic shocks. The selected cell is labelled by the white square in panel a) and b). Note the decrease in radius and the lengthening (increase in height) after the osmotic shock.



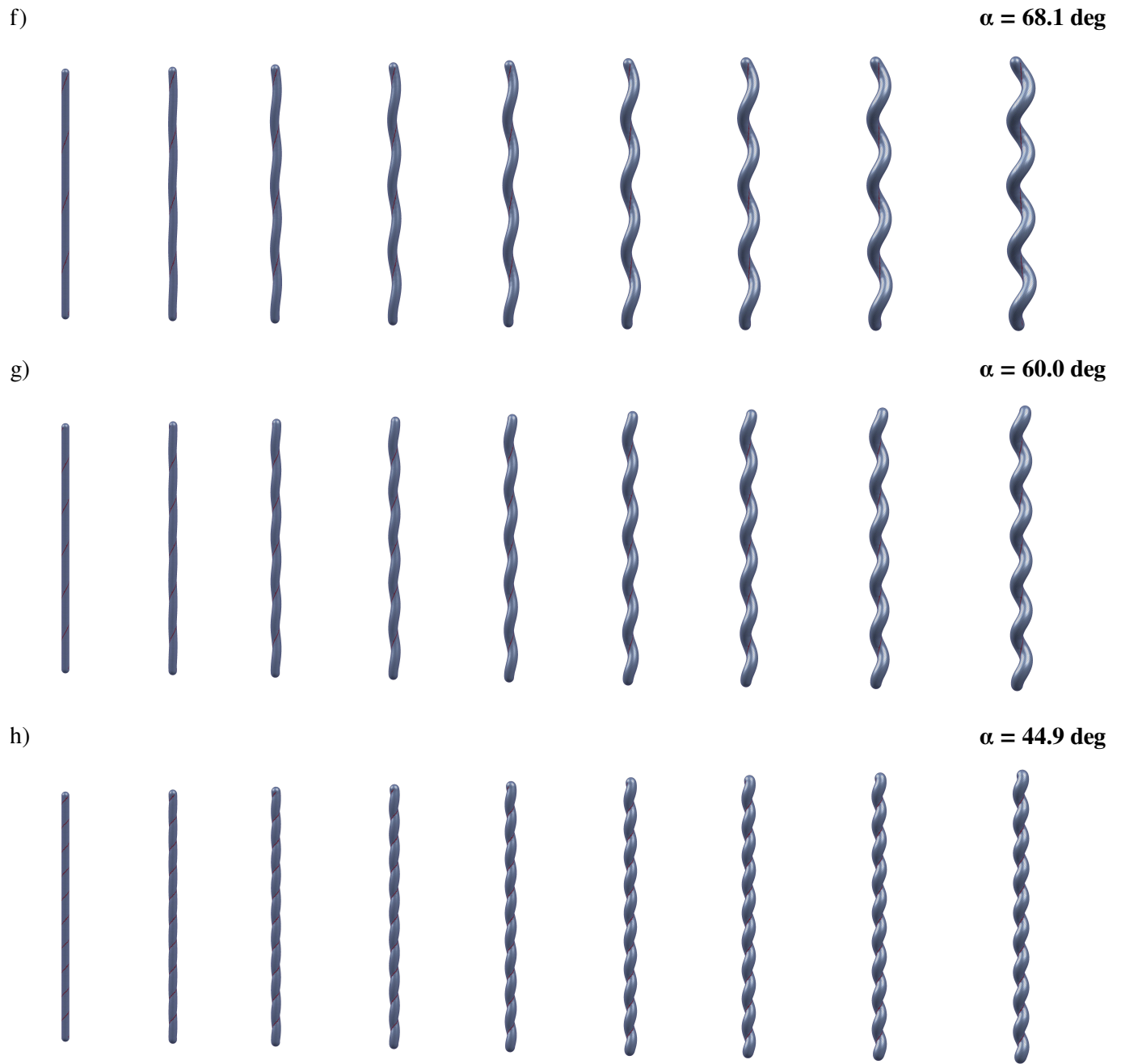


FIG. S7. Example minimal energy configurations for pressurized spherocylinders. Data corresponding to different reinforcing angles α for increasing rescaled pressures $\bar{p} = pr_0/Y$. The initial configuration as well as the mechanical properties of the cell are the same for every simulation. The reinforced regions is shown in red. Non-dimensional pressures, from left to right, $\bar{p} = 0, 0.10, 0.20, 0.27, 0.34, 0.41, 0.44, 0.47, 0.51$.

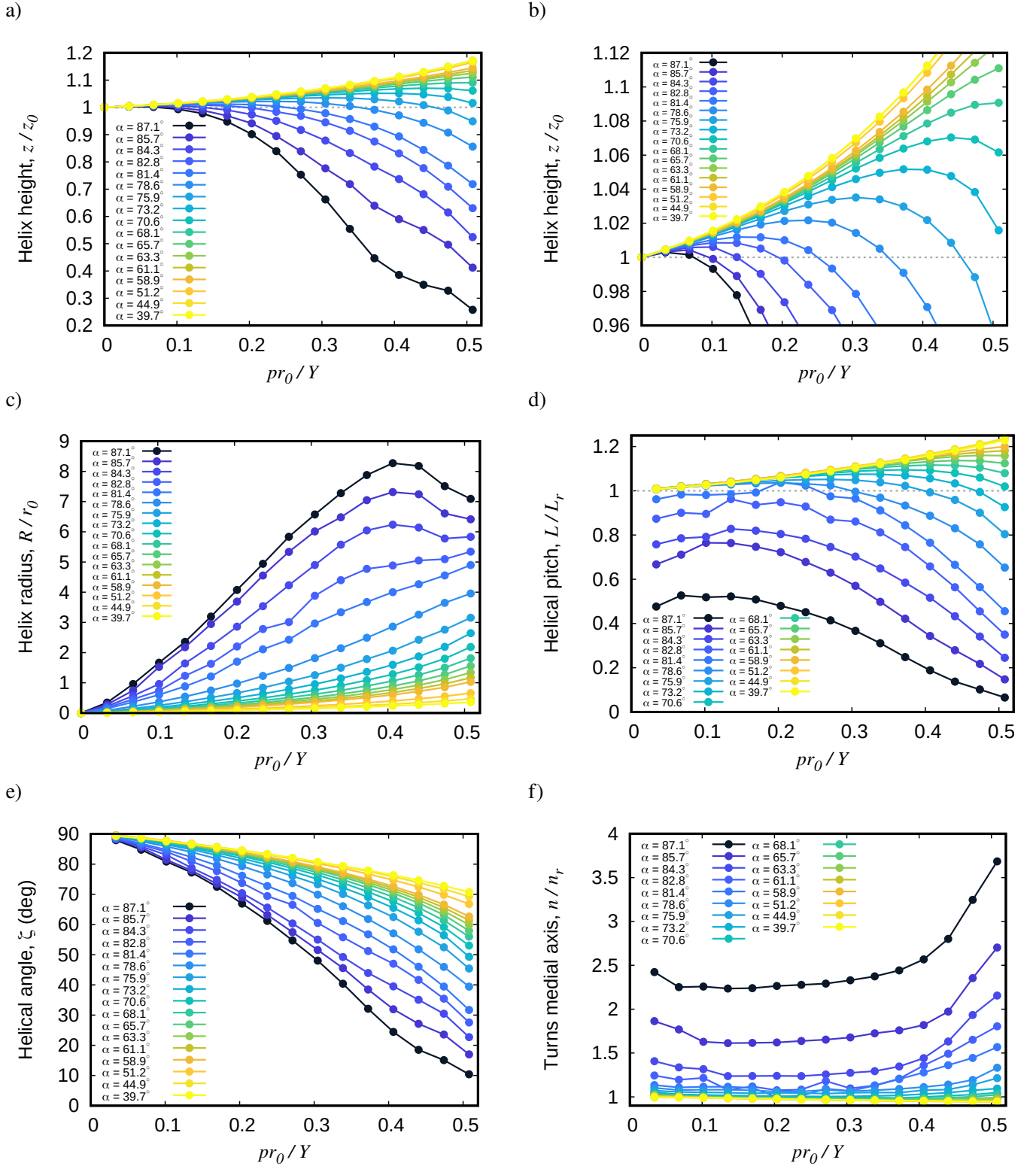


FIG. S8. Helical properties as a function of pressure and reinforcement angle. a) Helix height and b) zoomed version of showing the initial increase of height. c) Helical radius R . d) Helical pitch L normalized by the undeformed configuration (L_r). e) Helical angle, $\zeta = \text{atan}(L/R)$. f) Turns of the medial axis n normalized by the initial number of turns of the reinforced string in the undeformed configuration (n_r).

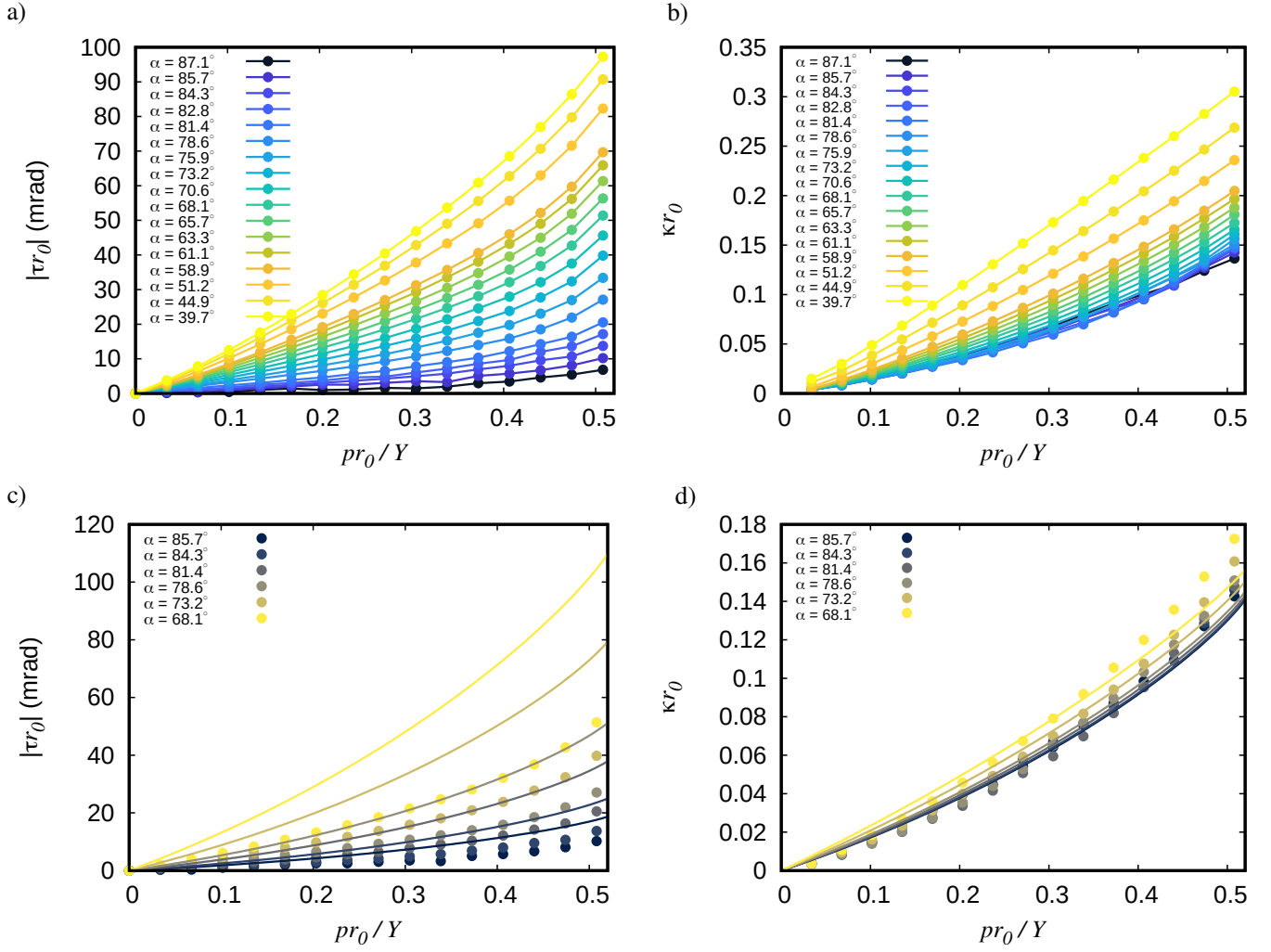


FIG. S9. Torsion and curvature dependency on pressure and reinforced angle. a) Absolute value of the torsion τ . The sign of the torsion is negative (left-handed, clockwise) and its opposite to that of the reinforced helix (right-handed, counter-clockwise). b) Curvature κ as a function of the rescaled pressure. c) Detail of the torsion τ for the steepest helical angles α . d) Detail of the curvature κ for the steepest angles α . Solid lines in a) and b) are a guide to the eye. Solid lines in c) and d) are theoretical predictions following Eq. 37.

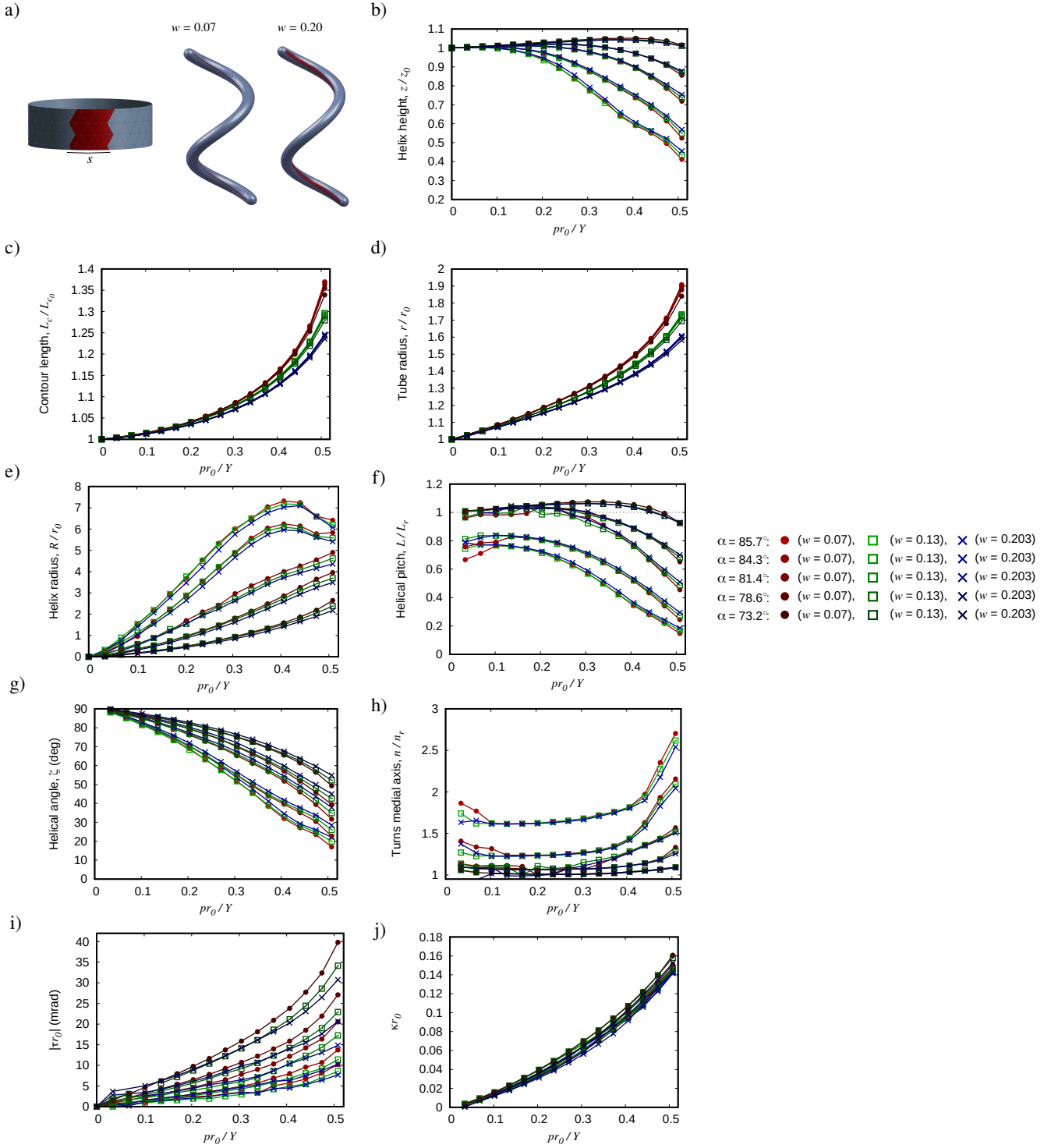


FIG. S10. Dependency of the width of reinforced string on helical parameters. a) *Left*. We define the reinforcement width $w := s/(2\pi r_0)$ as the fraction of material reinforced along the circumferential direction with respect to the perimeter of the cross-section. *Right*. Example of cells with reinforced angle $\alpha = 85.2^\circ$ at $\bar{p} = 0.41$ for $w = 0.07$ and $w = 0.20$. b) Helix height, z . c) Contour length $L_c(p)$. d) Tube radius, $r(p)$. e) Helix radius, R . f) Helical pitch, L . g) Helical angle, ζ . h) Number of turns of the medial axis, n . i) Surface torsion, τ . j) Average curvature of the medial axis, κ . Dimensions and mechanical parameters are shown in Table S1.

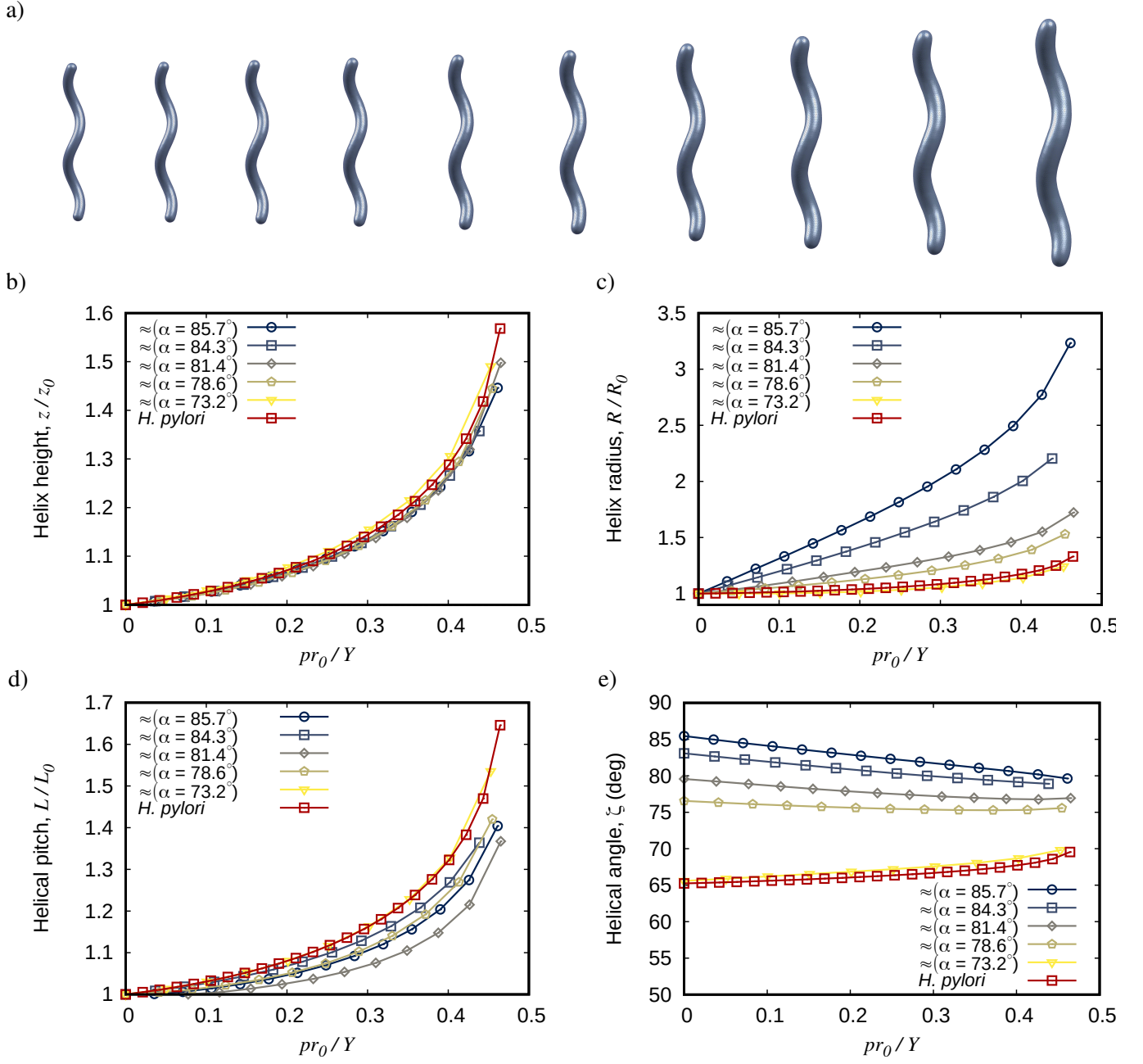


FIG. S11. a) Example minimal energy configurations for a pressurized helix. In the examples shown, the initial configuration was generated considering the mean helical parameter observed in turgid living *H. pylori* [7]. Non-dimensional pressures, from left to right, $\bar{p} = 0, 0.08, 0.15, 0.21, 0.27, 0.34, 0.40, 0.44, 0.46, 0.49$. b) Helix height, z . c) Helical radius, R . d) Helical pitch, L . e) Helical angle, ζ . The labels $\approx (\alpha x)$ indicate an initial helical configuration with the parameters obtained at the maximum extension from the simulations of the reinforced tube with reinforcement angle x .

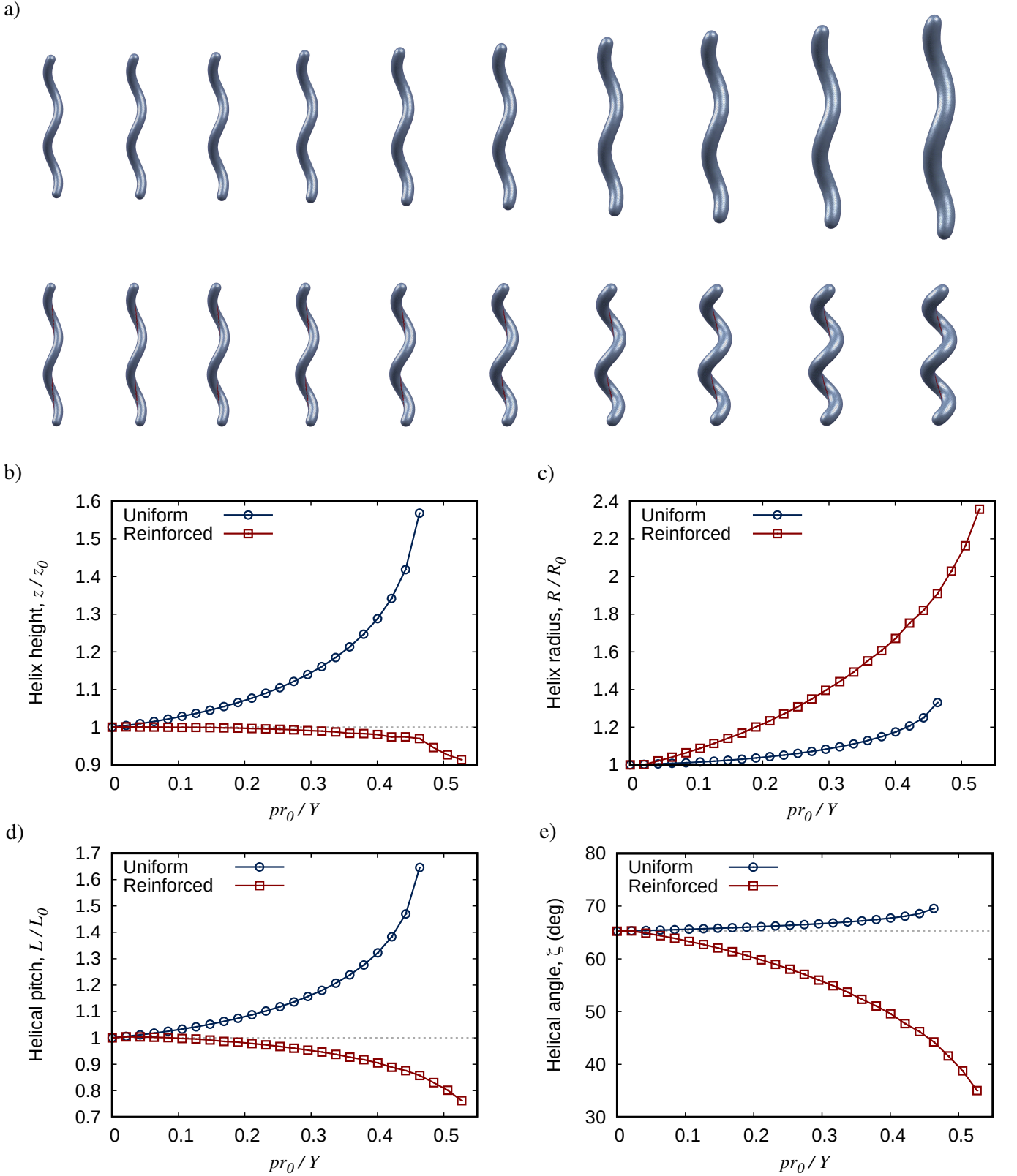


FIG. S12. a) Example minimal energy configurations in pressurized helices. *Top*: Uniform mechanics. *Bottom*: Reinforced on the concave region (red line). Non-dimensional pressures, from left to right, $\bar{p} = 0, 0.08, 0.15, 0.21, 0.27, 0.34, 0.40, 0.44, 0.46, 0.49$. b) Helix height, z . c) Helical radius, R . d) Helical pitch, L . e) Helical angle, ζ . For the reinforced case we obtain, at $p = 2.0$ atm ($\bar{p} = 0.42$), the following helical parameters: $r = 187$ nm, $R = 263$ nm, $L = 1.8$ $\mu\text{m}/\text{turn}$ and 2 helical turns. The initial configuration was generated considering the parameter observed in turgid living *H. pylori* [7].

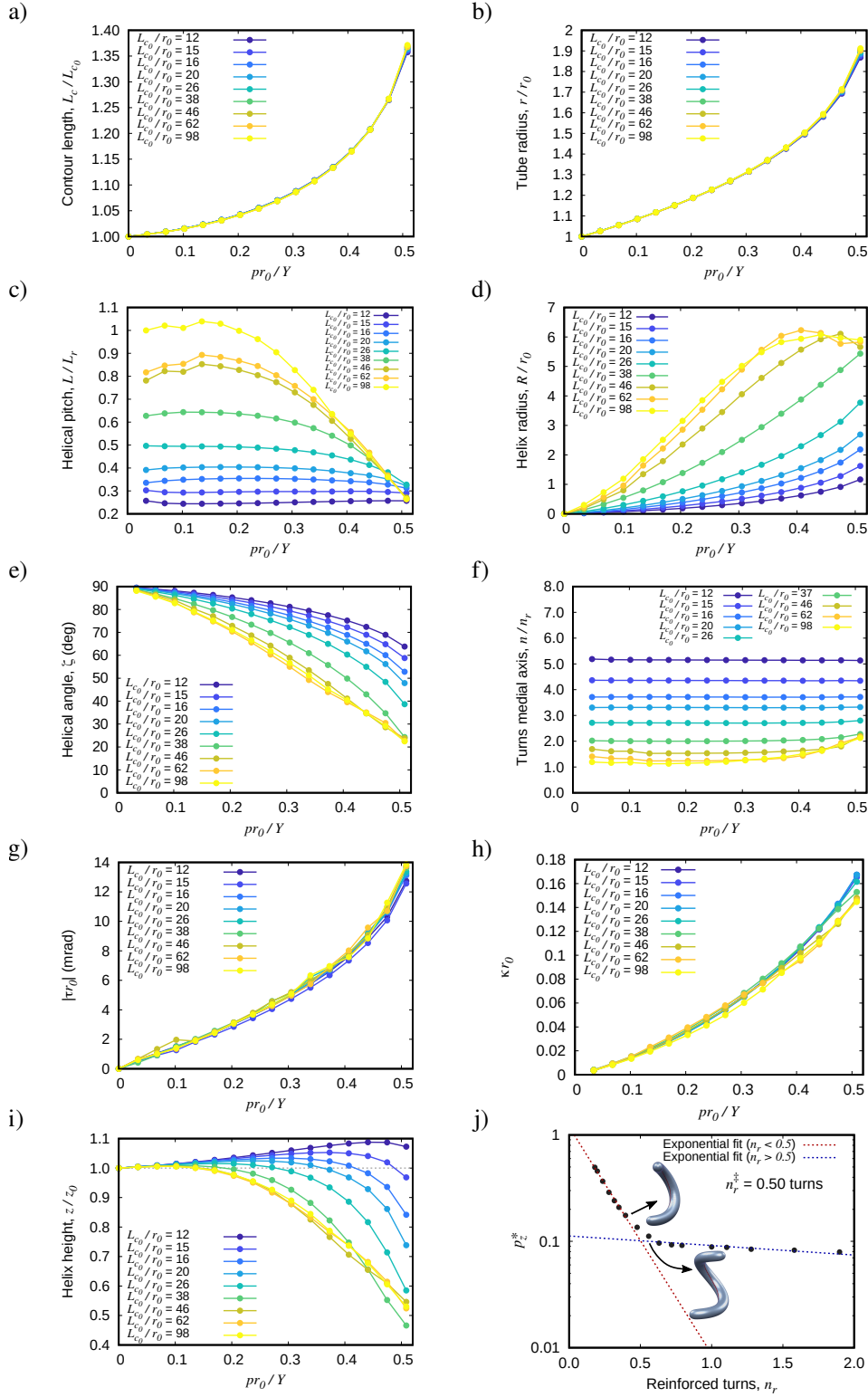


FIG. S13. Initial length and reinforced number of turns during pressure-induced formation of helices. a) Contour length $L_c(p)$. b) Tube radius, $r(p)$. c) Helical pitch, L . d) Helix radius, R . e) Helical angle, ζ . f) Number of turns of the medial axis, n . g) Surface torsion, τ . h) Average curvature of the medial axis, κ . i) Height of the main body, z . j) Critical pressure resulting in maximum height as a function of the initial number of reinforced turns n_r (the number of turns reinforced with helical angle α in a tube with initial length L_{c_0} is given by $n_r = \frac{L_{c_0}}{2\pi r_0 \tan \alpha}$). Dashed lines are exponential fits for $n_r < 0.5$ (red) or $n_r > 0.5$. The intersection between both fits occurs at $n_r^{\ddagger} \approx 0.5$. *Insets.* Minimal energy configurations for $n_r = 0.40$ and $n_r = 0.56$ showing the soft transition between C-shape to helix occurring at $n_r \approx 0.5$ ($\bar{p} = 0.47$). Data for different aspect ratios L_{c_0}/r_0 , with constant r_0 , for a reinforcement helical angle $\alpha = 84.3^\circ$. The threshold $n_r = 0.5$ occurs at $L_{c_0}/r_0 = 32$. Simulation parameters are shown in Table S1.

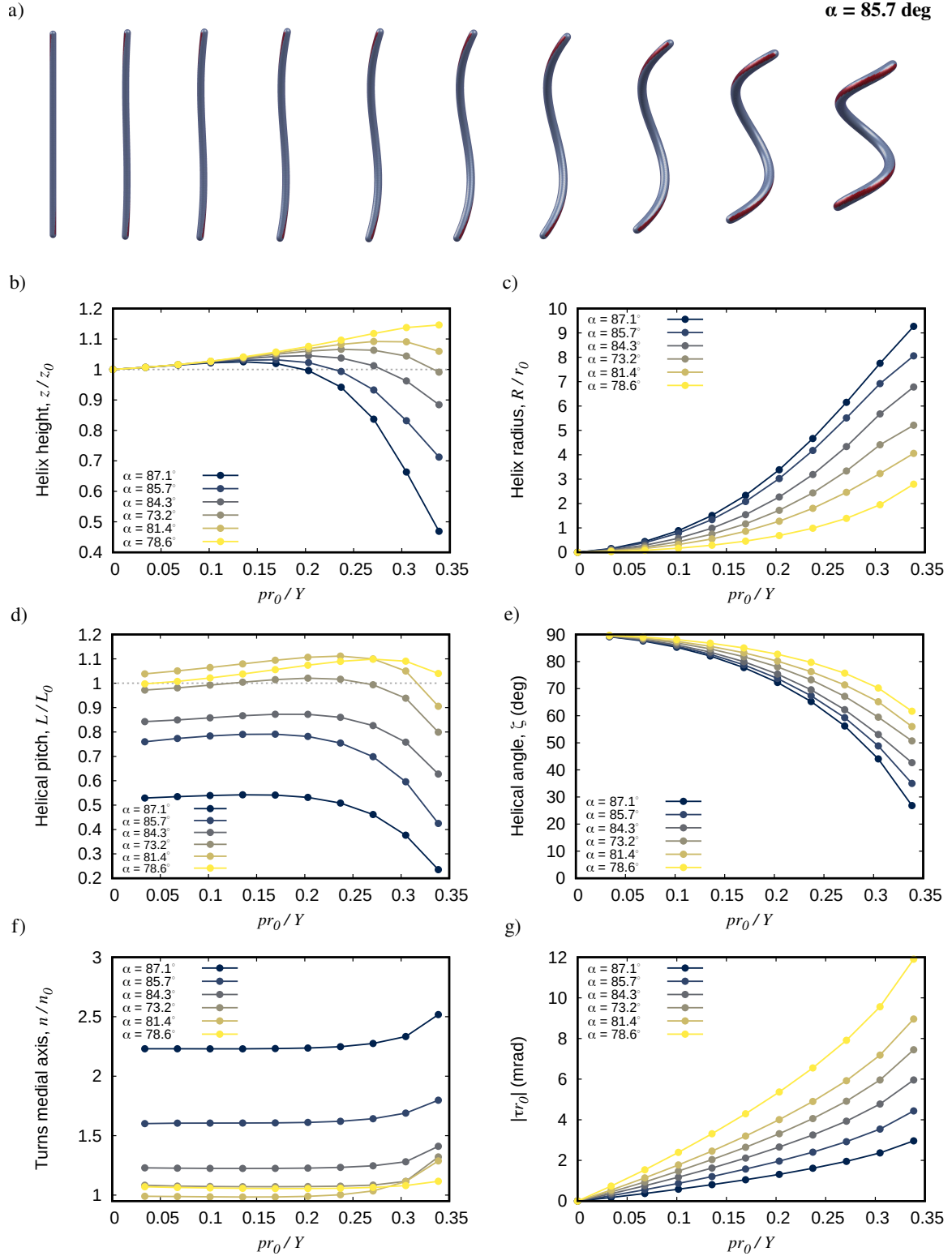


FIG. S14. Pressurization of tubes with helical weakening. a) Snapshot of minimal energy configurations for weakening angle $\alpha = 85.7^\circ$. After pressurization, the weakened region (red) is located on the outside (convex) area. Non-dimensional pressures, from left to right, $\bar{p} = 0, 0.07, 0.10, 0.14, 0.17, 0.20, 0.24, 0.27, 0.30, 0.34$. b) Height of the helix z , c) Helical radius, R , d) Helical pitch, L , e) Helical angle, ζ , f) Number of turns of the medial axis, n , and g) Surface torsion, τ , as a function of pressure and weakening angle. The weakening is adjusted by reducing the 2D Young modulus Y of the helical domain ($K = 0.33$). Lines connecting dots are a guide to the eye.

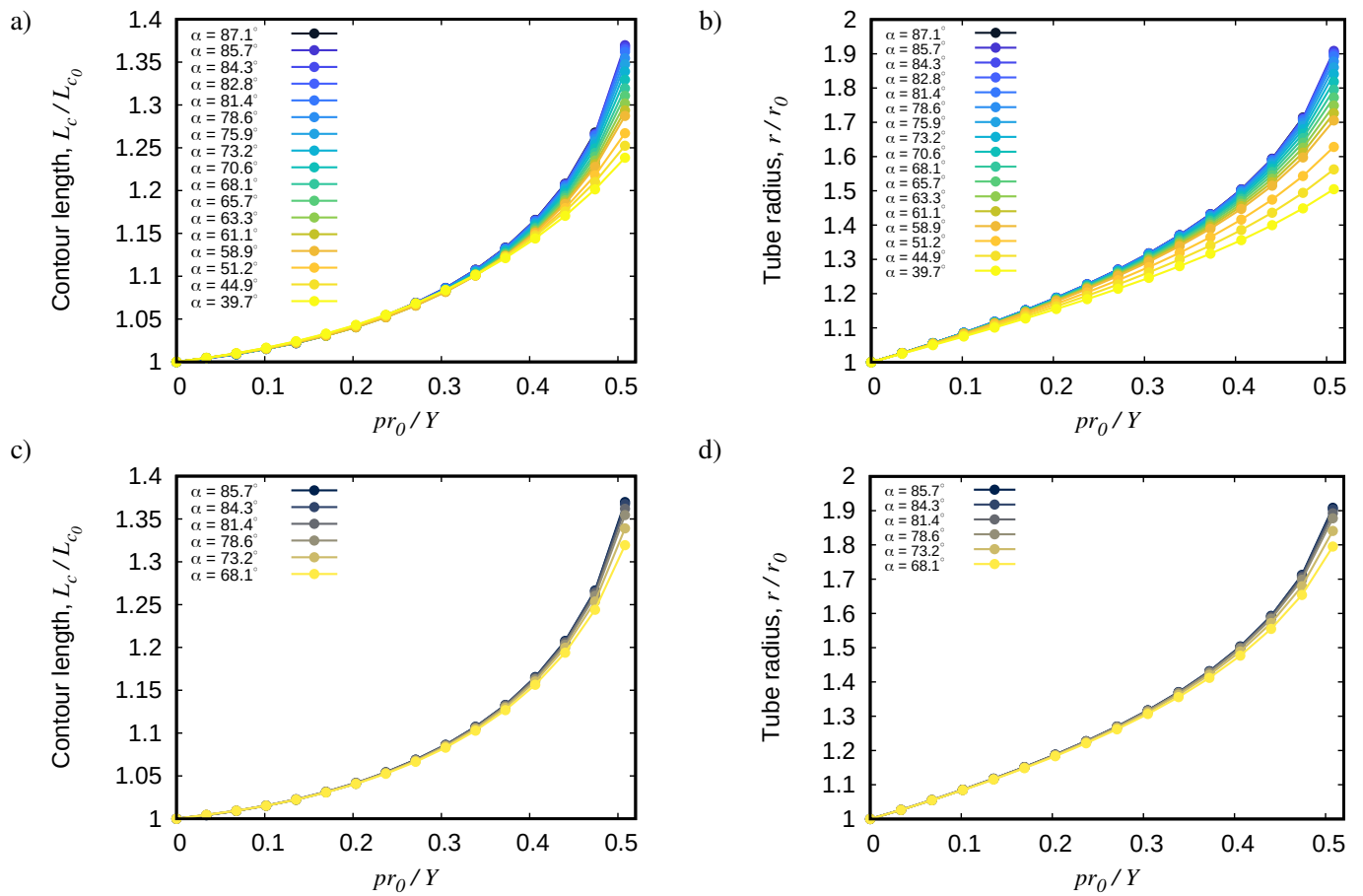


FIG. S15. Contour length and radius under pressure. Response for different reinforcement angles. Little differences are observed on $L_c(p)$ and $r(p)$ for $\alpha > 60^\circ$.

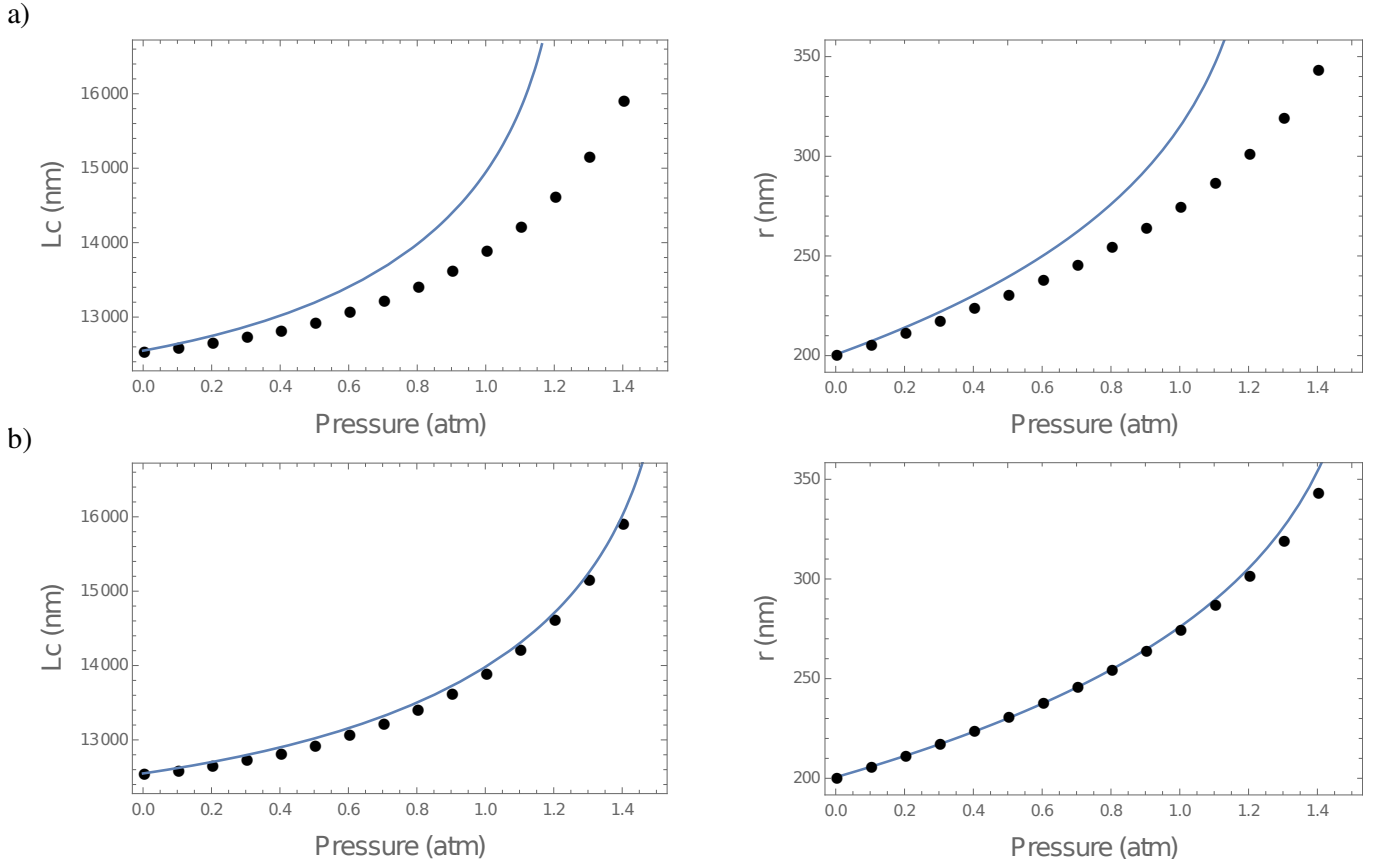


FIG. S16. Modeling of $r(p)$ and $L_c(p)$. Comparison between simulation data (dots) and modeling results (lines) of the overall swelling of the system (contour length L_c and radius r). (a) Modeling with 2D Young modulus $Y = Et = 1.16k_s$ and Poisson's ratio $\nu = 1/3$. A mapping between a continuous material and the triangular mesh predict this relationship to hold in the limit of small deformation. (b) Modeling considering a 2D Young modulus $Y = 1.44k_s$ and Poisson's ratio $\nu = 1/5$. Notice that we are using p instead of pr_0/Y , L_c instead of L_c/L_{c0} and r instead of r/r_0 .

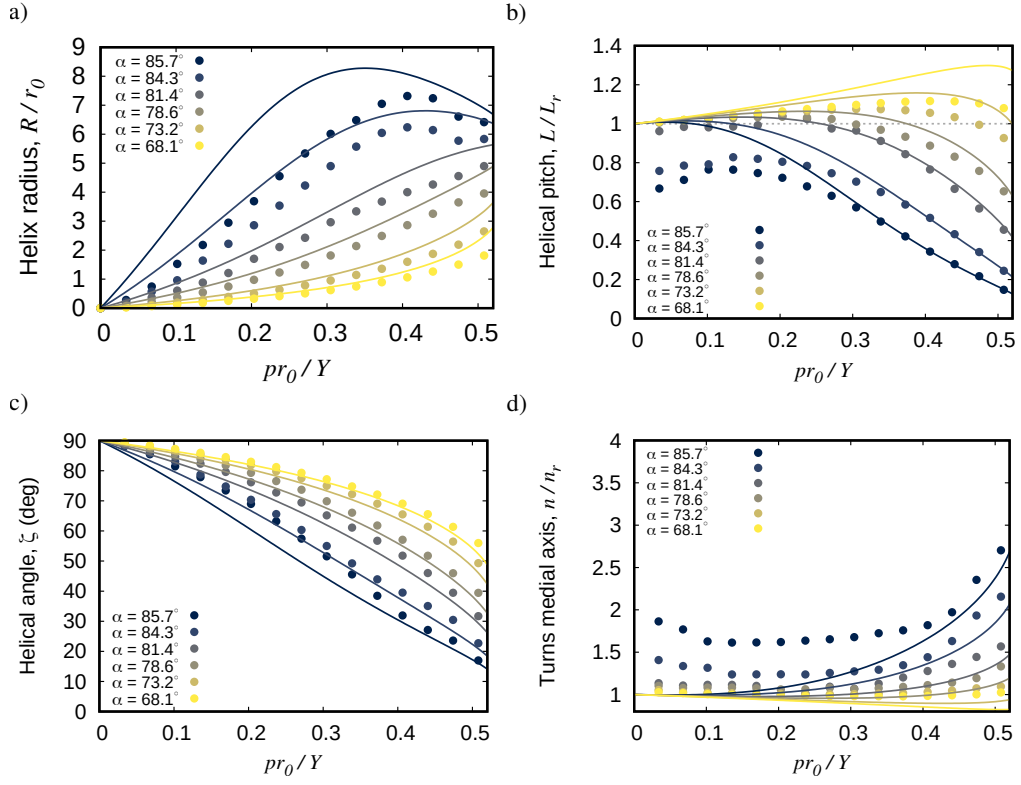


FIG. S17. Dependence of helical parameters on pressure $\bar{p} = pr_0/Y$. a) Helical radius (R). b) Helical pitch (L) normalized by the helical pitch of the pitch of the reinforced string in the undeformed configuration L_r . c) Helical angle $\zeta := \text{atan}(L/R)$. d) Number of turns of the medial axis n . The result is normalized by the number of turns of the reinforced string in the undeformed configuration n_r . Symbols are the results from numerical simulation; solid lines are model predictions.

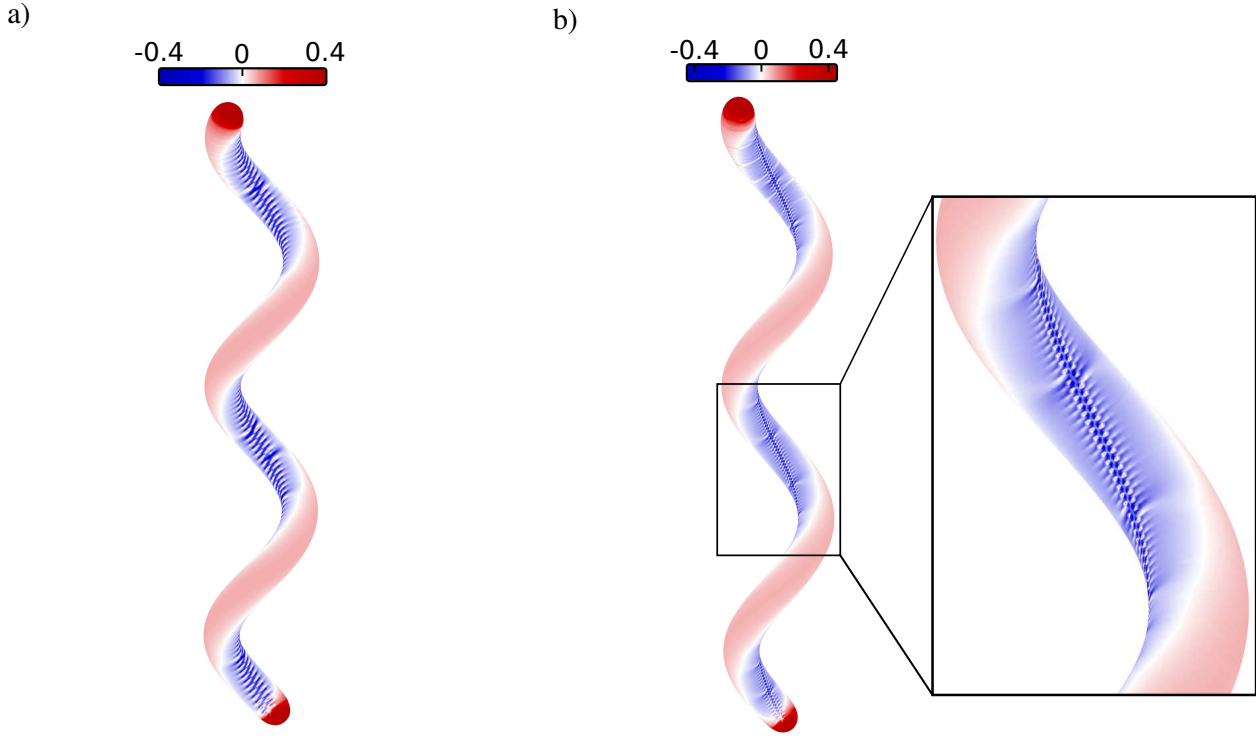


FIG. S18. Wrinkling during pressurization. a) Gaussian curvature in a pressurized helix to $\bar{p} = 0.47$ ($\alpha = 78.6^\circ$). b) Refined mesh with $N = 35652$ vertices pressurized to $\bar{p} = 0.47$ ($\alpha = 78.6^\circ$). Detail of the concave region highlighting the periodic wrinkling pattern. Gaussian curvature κ_G was determined as described in [19]. The indicated values correspond to rescaled curvatures given by $\bar{\kappa}_G := \kappa_G r_0^2$. The mechanical properties and the dimensions of the helix are indicated in Table S1.

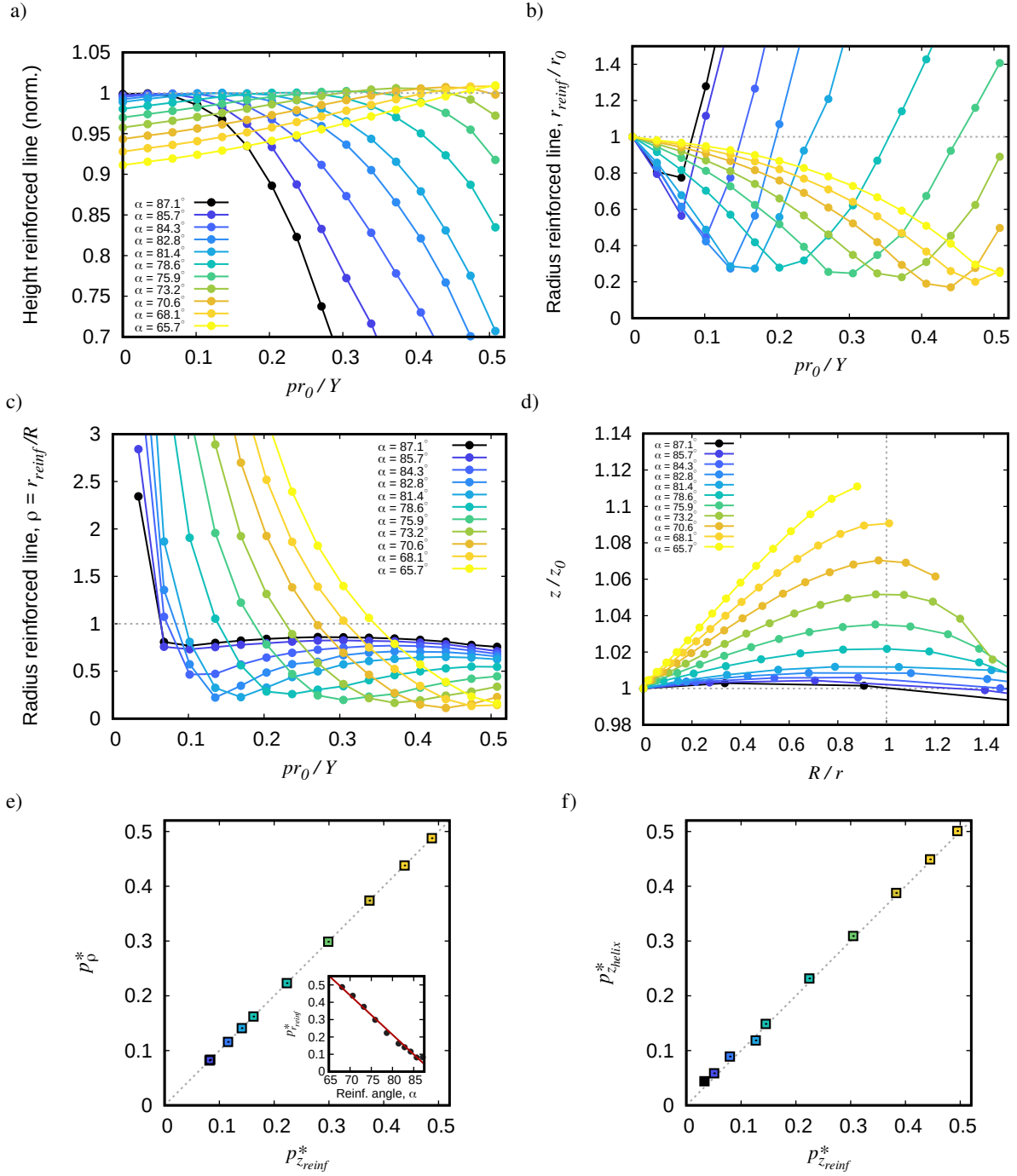


FIG. S19. Configuration of the reinforced line during pressurization. a) Maximal height of the reinforced string normalized by the theoretical contour length of the string $L_f = L_{c0} \sqrt{1 + \cot^2 \alpha}$. Detail showing the initial linear increase of the height with pressure as a result of straightening with the long axis before shortening. b) Average radius of the reinforced line r_{reinif} , measured with respect to the long axis, normalized by the initial tube radius r_0 . c) Same as in panel b) but normalized by the radius of the helix, where we define $\rho = r_{reinif}/R$. Values of $\rho < 1$ indicate a localization of the reinforced string on the internal region of the helix. d) Height as a function of the ratio between the radius of the tube r and the radius of the helix R . The maximum height is reached for $R \approx r$. e) Critical pressure p_ρ^* of occurrence of the minimal radius of the reinforced string as a function of the pressure of maximum string height, $p_{z_{reinif}}^*$. *Inset.* Dependency of $p_{z_{reinif}}^*$ on the reinforcement angle. Red line is a linear fit. f) Pressure of maximal height of the helix main body p_z^* as a function of $p_{z_{reinif}}^*$. The dashed lines with slope one in e) and f) are a guide to the eye.

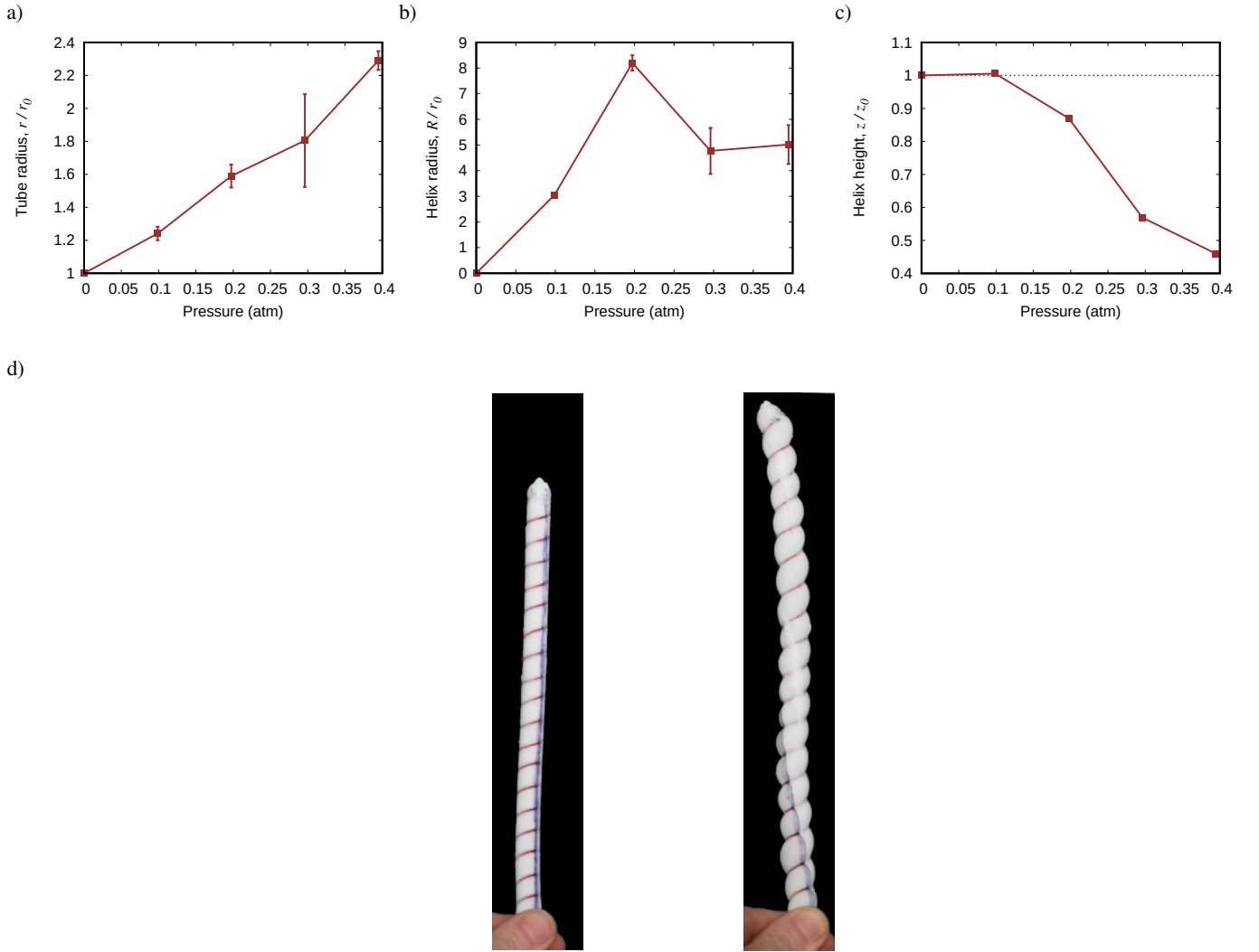


FIG. S20. Helical properties of reinforced balloons. a) Tube radius r , b) helix radius R , and c) helical height z as a function of the pressure in balloons with steep reinforcement angle ($\alpha = 73^\circ$). Error bars are the standard deviation of measurements on different regions of the pressurized balloon shown in the main text. d) Low reinforcing angles ($\alpha = 15^\circ$) show clearly noticeable torsion but no height reduction. *Left.* Initial configuration at zero pressure. *Right.* Configuration at $p \approx 2$ atm. The reinforced string is visible in red. The line in blue is a guide to the eye, drawn as a straight line in the undeformed configuration.

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