Electronic Supplementary Material (ESI) for Soft Matter.

1 Supplementary figure

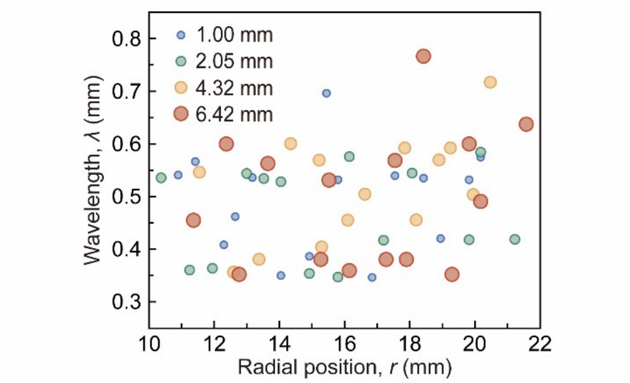

Fig. S1 Wavelength variation trend of pedal waves along radial coordinate r while collecting varying-sized particles.

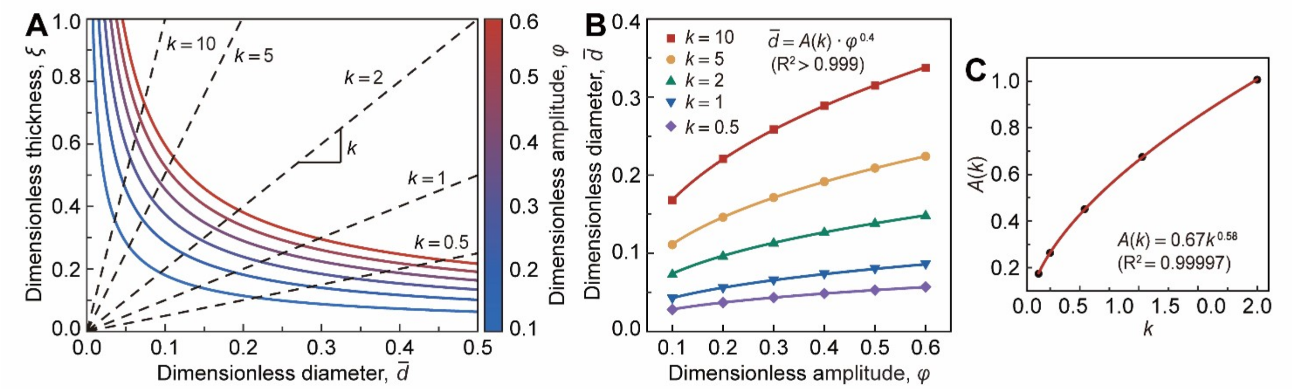

Fig. S2 (A) Pencil of ξ-$\bar{d}$ curves and orthogonal trajectories. (B)-(C) Relationship between $\bar{d}$ and φ in varying slopes k of orthogonal trajectories.

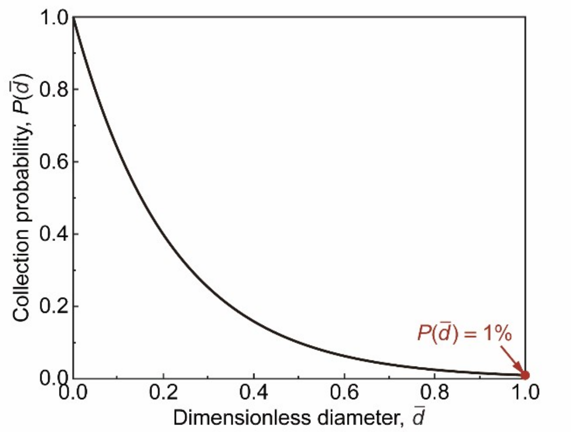

Fig. S3 Probability of particle collection satisfies the exponential distribution. The probability $P(\bar{d})$ is set to 1% at $\bar{d}$ = 1.0, which means an imaginarily large particle located in a diameter equivalent to the funnel radius cannot to collect by the snail.

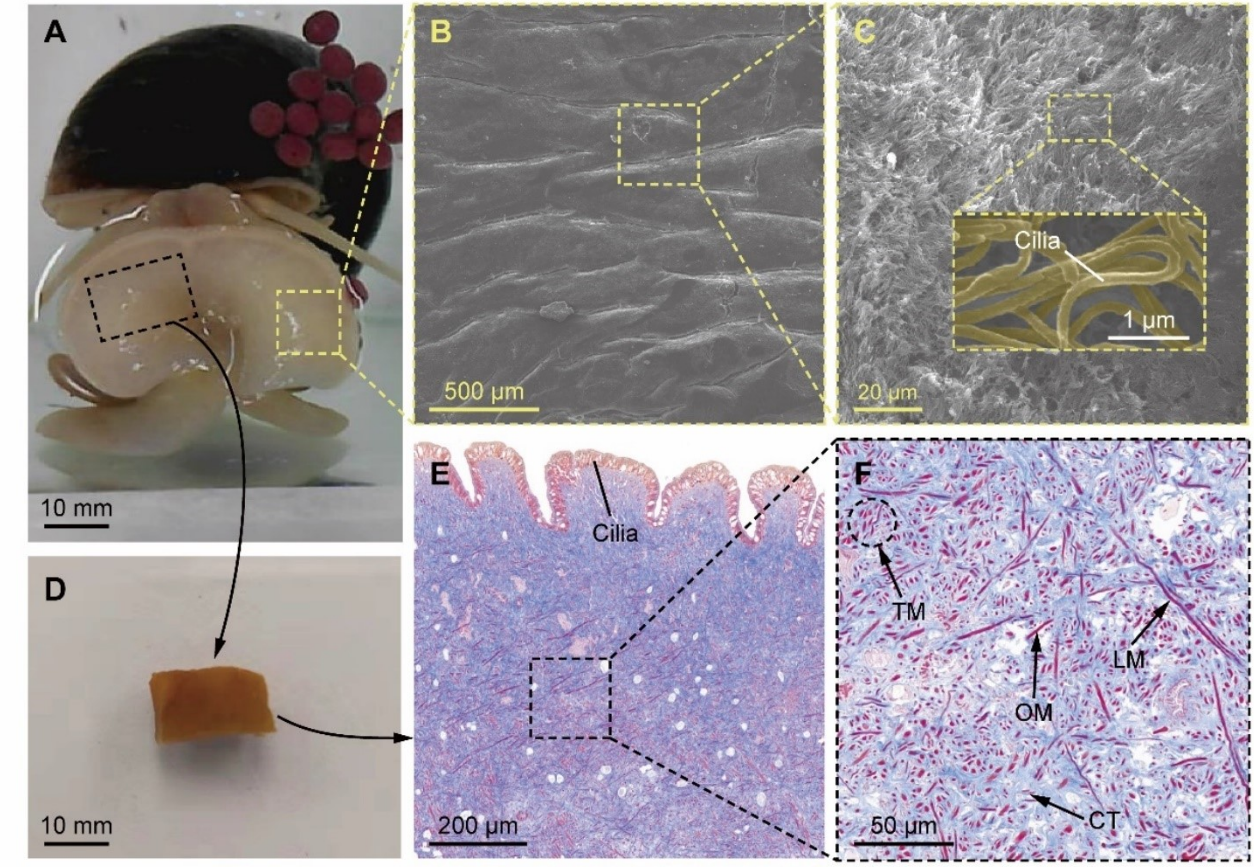

Fig. S4 (A) Natural feeding behavior of a water snail that uses its foot to collect particles floating on the water surface. (B)-(C) SEM images of the snail foot surface. The cilia are densely distributed on textured surface of snail foot. (D) Samples for preparing foot muscle slices. (E)-(F) Distribution of foot muscles of the snail. Thin longitudinal section of the muscle stained with masson dye solution, showing transverse muscles (TM), oblique muscles (OM) , longitudinal muscles and connective tissue (CT) surrounding the muscles.

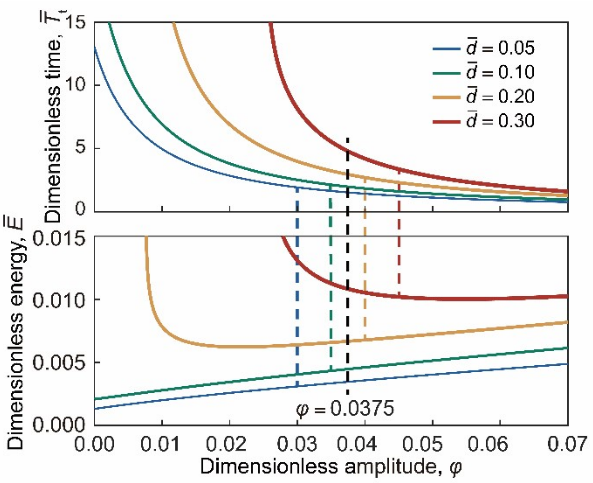

Fig. S5 Transport time and energy consumption when transporting different particles by varying amplitudes. The dotted line is used to compare the transport time and energy consumption at a specific amplitude, in which the colored lines indicate that varying amplitudes are used to transport different particles and the black line indicate that a specific amplitude is used to transport.

Supplemental information

S1. Particle velocity

Considering a two-dimensional situation, in the frame fixed to the foot surface wave, applying the lubrication approximation to the incompressible Navier-Stokes equation, we have

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 u_r}{\partial z^2}, \ \frac{\partial p}{\partial z} = 0, \qquad \text{Equation S1}$$

in which u_r is the component of velocity along the r-direction, and p is the pressure. First, we rationalize the flow of mucus covering the foot surface. The boundary conditions for Equation S1 can be expressed as $u_r = -V_w$ at $z = f$, as well as $\partial u_r/\partial z = 0$ at $z = h$, where f and h are the vertical position of the foot surface and the gas-liquid interface, respectively. Since p depends upon merely r rather than z, $\partial p/\partial r$ can be written as dp/dr. Integrating Equation S1 in the r-direction twice with respect to z, while using the boundary conditions, we arrive at the velocity of the mucus flow as

$$u_r = -V_w - \frac{1}{2\mu}\frac{dp}{dr}\left[(h-f)^2 - (h-z)^2\right]. \qquad \text{Equation S2}$$

Further, integrating the velocity with respect to z, we get the rate of volume flow at each cross-section as

$$q = \int_f^h u_r dz = -V_w(h-f) + \frac{(h-f)^3}{3\mu}\frac{dp}{dr}, \qquad \text{Equation S3}$$

which is a time-independent constant because the mucus thickness is time-invariant in the wave frame.

When the free surface remains approximately flat and undisturbed, the mucus is validated to be actuated by the pedal waves, rather than the hydrostatic pressure or surface tension[1]. In other words, the location of the free surface can be approximated as a critical constant h_0. We may derive a relationship of $h \neq h_0$, otherwise the pressure would be uniform and there would be no flow on the no-slip surface. There might have two ultimate states to the free surface location. For the first state where the capillary number Ca tends to infinity, the interface shape exactly conforms to the wavy shape of the foot. In the other state with $Ca \approx 0$, the mucus-air interface becomes flat[2]. To get an approximately flat free surface, the change in mucus thickness is required to satisfy $|h - h_0| \ll f_0$. According to the momentum equations in the r-direction, viscous stresses must be balanced by the pressure gradient. By balancing the terms and rearranging, an approximately flat free surface is given by (3.9) of[1]:

$$\frac{\rho g (h_0 - f_0)^2}{\sigma} \gg \frac{\lambda}{f_0} Ca,\ Ca \equiv \frac{\mu U}{\sigma} \ll \frac{(h_0 - f_0)^2 f_0}{\lambda^3}. \qquad \text{Equation S4}$$

We consider mucus has the same density and surface tension as water, while being 100 times in viscosity[1]. By introducing the numbers of the mucus thickness $H = 1$ mm, the amplitude $f_0 = 0.1$ mm, and the wavelength $\lambda = 0.5$ mm, we can extend Equation S4 into $Ca \equiv 10^{-3} \ll 0.4$ that satisfies the flat free surface approximation. So, in the following derivations, h can be replaced by h_0.

The rate of volume flow at each cross section in the laboratory frame is $Q = q + v_w(h_0 - f)$, then the volume flow averaged by total duration at each cross section can be calculated by integrating over the period: $Q^{mean} = \int_0^T Q dt \Big/ T = q + V_w h_0$. Substituting Q^{mean} into Equation S3, and introducing the dimensionless amplitude $\varphi = f_0/h_0$ and dimensionless volume flow $\theta = Q^{mean}/(V_w h_0)$, we get

$$\frac{h_0^2}{\mu V_w \lambda} dp = \frac{3\varphi\left(\theta - \frac{f}{f_0}\right)}{\lambda\left(\varphi \frac{f}{f_0} - 1\right)^3} dr. \qquad \text{Equation S5}$$

Actually, dimensionless amplitude φ is a small quantity. We rewrite right side of

Equation S5 by Taylor expansion, keeping it to the order of φ^3:

$$\frac{h_0^2}{\mu V_w \lambda} dp = \frac{1}{\lambda}\left[3\varphi\left(\frac{f}{f_0} - \theta\right) + 9\varphi^2 \frac{f}{f_0}\left(\frac{f}{f_0} - \theta\right) + 18\varphi^3 \left(\frac{f}{f_0}\right)^2 \left(\frac{f}{f_0} - \theta\right)\right] dr, \qquad \text{Equation S6}$$

Integrating both sides of the equation simultaneously, we get

$$\frac{h_0^2}{\mu V_w \lambda} \Delta p_\lambda = -3\theta\varphi + \frac{9}{2}\varphi^2 - 9\theta\varphi^3, \qquad \text{Equation S7}$$

where Δp_λ is the pressure rise within one wavelength. Considering the case of $\Delta p_\lambda = 0$, Equation S7 can be transformed into

$$\theta = \frac{3\varphi}{2(3\varphi^2 + 1)}. \qquad \text{Equation S8}$$

Next, we rationalize the flow of mucus around the particle. We consider that an oblate cylindrical floating particle moves radially along the foot without direct contact with the foot. Assuming that the particle is moving with velocity V_s, the boundary conditions for Equation S1 can be expressed as $u_r = -V_w$ at $z = f$, as well as $u_r = V_s - V_w$ at $z = h_s$, in which h_s is the position of the lower surface of the particle. Since we consider the shape of the particle as an oblate cylinder, the lower surface of the particle is flat and h_s keeps constant in the static equilibrium. Consistent with the derivation of the flow of mucus covering the foot surface, we arrive at the velocity of the local flow in the area between the lower surface of the particle and the surface of the foot as

$$u_r = -V_w + \frac{z - f}{h_s - f} V_s + \frac{1}{2\mu} \frac{dp}{dr} (z - h_s)(z - f). \qquad \text{Equation S9}$$

Further, the rate of volume flow at each cross-section can be expressed as

$$q_s = -V_w(h_s - f) + V_s h_s - \frac{(h_s - f)^3}{12\mu} \frac{dp}{dr}, \qquad \text{Equation S10}$$

which can be averaged by total duration in the laboratory frame is $Q_s^{mean} = q_s + V_w h_s$. Substituting Q_s^{mean} into Equation S10, then introducing the dimensionless amplitude

$\varphi_s = f_0/h_s$ and dimensionless volume flow $\theta_s = Q_s^{mean}/(V_w h_s)$ and dimensionless velocity $\chi = V_s/V_w$ into Equation S10, we get

$$\frac{h_s^2}{\mu V_w \lambda} dp = \frac{12\left(\theta_s - \varphi_s \frac{f}{f_0}\right) + 6\left(\varphi_s \frac{f}{f_0} - 1\right)\chi}{\lambda\left(\varphi_s \frac{f}{f_0} - 1\right)^3} dr, \qquad \text{Equation S11}$$

We rewrite right side of Equation S11 by Taylor expansion, keeping it to the order of φ_s^3:

$$\frac{h_s^2}{\mu V_w \lambda} dp = \frac{1}{\lambda}\left[6\chi + 12\varphi_s\left(\frac{f}{f_0}\chi + \frac{f}{f_0} - \theta_s\right.\right. dr, \qquad \text{Equation S12}$$

Integrating both sides of the equation, we get

$$\frac{h_s^2}{\mu V_w \lambda} \Delta p_\lambda = 6\chi - 12\theta_s \varphi_s + (9\chi + 18)\varphi_s^2 - 36\theta_s \varphi_s^3, \qquad \text{Equation S13}$$

Considering the case of $\Delta p_\lambda = 0$, Equation S13 can be transformed into

$$\theta_s = \frac{6\varphi_s^2 + \left(3\varphi_s^2 + 2\right)\chi}{4\varphi_s\left(3\varphi_s^2 + 1\right)}. \qquad \text{Equation S14}$$

In order to match the particle local flow with the global flow, we apply the results above to the axisymmetric configuration of the biological funnel. Extending from two-dimensional case to three-dimensional one, the width-integrated flux of whole foot flow $\Phi = 2\pi r Q^{mean}$ may be equal to the width-integrated flux of particle local flow $\Phi_s = dQ_s^{mean}$. Due to the flow loss of mucus passing through particles in tangential directions, the matching of the two flows should be written as $\Phi_s = (1 - K)\Phi$ where K is a leakiness coefficient K to measure the volume flow loss of mucus between the pedal wave and the lower surface of the particle in other non-radial directions.

Expressing the matching of the flow as $2\pi r\,(1-K)\,\theta = d\theta_s$ in dimensionless numbers and substituting Equation S8 and Equation S14 into it, we rearrange it to get

$$\chi = \frac{12\varphi^2\pi(3\varphi^2+\xi^2)}{(3\varphi^2+1)(3\varphi^2+2\xi^2)\xi d}\,(1-K)\,r - \frac{6\varphi^2}{3\varphi^2+2\xi^2}. \qquad \text{Equation S15}$$

where $\xi = \varphi/\varphi_s = h_s/h_0$ is a dimensionless number characterizing the relative magnitudes of h_s and h_0. We introduce two dimensionless numbers $\bar{r} = r/r_0$ and $\bar{d} = d/r_0$,which characterize the position of the particle on the surface of the foot and the relative size of the particle and the funnel, respectively, to make all the variables in Equation S15 dimensionless. In a dimensionless frame, the relationship between the velocity and the radial position on the foot surface of the particle can be expressed as

$$\chi = \frac{12\varphi^2\pi(3\varphi^2+\xi^2)}{(3\varphi^2+1)(3\varphi^2+2\xi^2)\xi \bar{d}}\,(1-K)\,\bar{r} - \frac{6\varphi^2}{3\varphi^2+2\xi^2}. \qquad \text{Equation S16}$$

S2. The supporting force for mucus

To calculate the supporting force provided by the mucus to the particles, we rewrite Equation S12 as follows

$$\frac{h_0^2}{\mu V_w \lambda}dp = \frac{1}{\lambda\eta^2}\left[6\chi + 12\frac{\varphi}{\xi}\left(\frac{f}{f_0}\chi + \frac{f}{f_0} - - (\right.\right. dr. \qquad \text{Equation S17}$$

Substituting Equation S16 into Equation S17 and integrating both sides simultaneously, we can get the pressure p. To eliminate the effect of time, we calculate the time-averaged pressure distribution of the water film under the particle according to $p^{mean} = \int_0^T pdt \Big/ T$ and arrive at dimensionless time-averaged pressure as

$$\frac{h_0^2}{\mu V_w \lambda}p^{mean} = \frac{18\pi K\varphi^2(3\varphi^2+\xi^2)}{\eta^5(3\varphi^2+1)d\lambda}\left(r_0^2 - r^2\right), \qquad \text{Equation S18}$$

in which we confirmed that the constant term due to the integration by setting the boundary condition $p^{mean}|_{r=r_0}=0$ and r_0 is the position of the snail foot edge. For simplification, in the following text, we rewrite p^{mean} as p. Using $\bar{r}$ and $\bar{d}$ to make all the variables in Equation S18 dimensionless, the dimensionless pressure acting on the lower surface of the particle along the radial direction of the foot can be expressed as

$$\frac{h_0^2}{\mu V_w r_0}p = \frac{18\pi K\varphi^2(3\varphi^2+\xi^2)}{\xi^5(3\varphi^2+1)\bar{d}}(1-\bar{r}^2). \qquad \text{Equation S19}$$

S3. Numerical solution of eqn (5)

The numerical results show that all ξ-$\bar{d}$ curves exhibit inverse proportional principles with varying φ. Examining the slope variation of each curve along the coordinate $\bar{d}$, we notice that the orthogonal trajectories of ξ-$\bar{d}$ curve family are the lines through the origin (Fig. S2A). We intend to record the intersections of the curves with orthogonal trajectories in varying slopes k and express the relationship between $\bar{d}$ and φ at these points with an approximate equation.

First, we assume that $\bar{d}$ is proportional to the power of φ, which can be expressed as

$$\bar{d} = A\varphi^B. \qquad \text{Equation S20}$$

Fitting $\bar{d}$ with Equation S20 in varying k and comparing the fitting results (Tab. S1), we notice that the parameter A is sensitive to k, while B is not. Therefore, we keep the parameter B constant at B = 0.4 and rewrite Equation S20 as

$$\bar{d} = A\varphi^{0.4}. \qquad \text{Equation S21}$$

According to results of Equation S21 fitting (Fig. S2B), we consider A to be a function of k. To understand the varying in slopes of the orthogonal trajectories, we fit $A(k)$ in a form of a power function (Fig. S2C), written as

$$A(k) = mk^n. \qquad \text{Equation S22}$$

Substitute the fitting result into Equation S21, the relationship between $\bar{d}$ and φ can be expressed as

$$\bar{d} = 0.67k^{0.58} \cdot \varphi^{0.4}. \qquad \text{Equation S23}$$

Tab. S1 Results of Equation S20 fitting

k	0.5	1	2	5	10
A	0.174	0.263	0.451	0.675	1.007
$\Delta A/\Delta k$	/	17.8%	18.5%	11.1%	8.8%
B	0.397	0.396	0.394	0.391	0.388
$\Delta B/\Delta k$	/	0.2%	0.2%	0.1%	0.1%
R^2	0.99999	0.99999	0.99998	0.99996	0.99995

S4. Transport time and mass intake rate

Since $\chi = -d\bar{r}/d\bar{t}$ where duration time t is dimensionless to $\bar{t} = t \cdot V_w/r_0$ and the negative sign indicates that the particle is moving from the edge of the foot to the center of the foot, we rewrite Equation S16 as

$$-\frac{d\bar{r}}{d\bar{t}} = C_1\bar{r} + C_2, \qquad \text{Equation S24}$$

where parameters C_1 and C_2 can be expressed as

$$C_1 = \frac{12\phi^2\pi(3\phi^2 + \xi^2)}{(3\phi^2 + 1)(3\phi^2 + 2\xi^2)\xi\bar{d}}(1 - K),$$

$$C_2 = -\frac{6\phi^2}{3\phi^2 + 2\xi^2}. \qquad \text{Equation S25}$$

We consider that the moment when the particles are transported inward from the edge of the pedal wave is $\bar{t} = 0$, namely the initial condition of Equation S24 is $\bar{r}|_{\bar{t}=0} = 1$ Solving the equation with this initial condition, we arrive at the explicit relationship between $\bar{r}$ and $\bar{t}$ as

$$\bar{r} = -\frac{C_2}{C_1} + e^{-C_1\bar{t}}\left(1 + \frac{C_2}{C_1}\right). \qquad \text{Equation S26}$$

Assuming that the particle is transported to the position $\bar{r} = \bar{r}_t$ and just leave the flat portion of the funnel at $\bar{t} = \bar{T}_t$, we can rearrange Equation S26 as

$$\bar{T}_t = -\frac{1}{C_1}\ln\left(\frac{\bar{r}_t + \frac{C_2}{C_1}}{1 + \frac{C_2}{C_1}}\right), \qquad \text{Equation S27}$$

where $\bar{T}_t$ is the dimensionless form of the particle transport time T_t.

S5. Energy cost of changing waveforms

To quantify energy cost during the snail altering the waveform, we model the snail foot composed of a linear elastic material. For steady waves, in the case of small deformation, the dimensionless power that the snail output to maintain the forced vibration of the foot muscles can be expressed as

$$\varepsilon \sim \frac{32\pi^2 H^2}{16^2 + D^2}, \qquad \text{Equation S28}$$

in which H and D are dimensionless parameters identifying the characteristic strength of forced vibration and the stiffness of the foot muscles, respectively[3]. According to previous models that specify the wall displacement in a sinusoidal form[4-6], we have the connections between ϕ and D as

$$\varphi\frac{f}{f_0} \sim D\frac{16\cos\left(\frac{2\pi}{\lambda}r\right) + D\sin\left(\frac{2\pi}{\lambda}r\right)}{16^2 + D^2}. \qquad \text{Equation S29}$$

Integrating both sides of the equation simultaneously, we get $\varphi \sim D$. Therefore, the dimensionless power can be expressed as

$$\varepsilon \sim \varphi^2. \qquad \text{Equation S30}$$

Further, we can calculate the energy consumption of the snail during a transport by $\bar{E} = \varepsilon \cdot \bar{T}_t$ where $\bar{E}$ is the dimensionless energy consumption. We compared the changes in dimensionless transport time and dimensionless energy consumption when transporting the particles in different sizes by different amplitudes (Fig. S5). As the amplitude increases, the transport time for a single particle will decrease, but the energy consumption will increase, which requires the snail to balance time cost and energy cost. Taking $\overline{d}$ = 0.05 as an example, compared with φ = 0.03, using the amplitude in φ = 0.0375 can only help the snail save 15.4% of the energy but the snail has to spend 29.3% more time to transport the particle.

References

1. S. Joo, S. Jung, S. Lee, R. H. Cowie and D. Takagi, *Journal of the Royal Society Interface*, 2020, **17**, 20200139.
2. S. Lee, J. W. Bush, A. Hosoi and E. Lauga, *Physics of Fluids*, 2008, **20**, 082106.
3. D. Takagi and N. Balmforth, *Journal of fluid mechanics*, 2011, **672**, 219-244.
4. Y. Fung and C. Yih, 1968.
5. A. H. Shapiro, M. Y. Jaffrin and S. L. Weinberg, *Journal of fluid mechanics*, 1969, **37**, 799-825.
6. T.-F. Zien and S. Ostrach, *Journal of Biomechanics*, 1970, **3**, 63-75.