Synergistic interactions of binary suspensions of magnetic anisotropic particles - Supplementary Information†

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1 The magnetic field of a uniformly magnetized ellipsoid

The magnetic potential outside the ellipsoidal particle ($\xi > 0$) generated by the magnetization along the $x-$axis of the particle results[1]

$$\phi = \frac{r_x r_y r_z}{2} x M_x \int_{\xi}^{\infty} \frac{1}{(\alpha + r_x^2)\sqrt{(\alpha + r_x^2)(\alpha + r_y^2)(\alpha + r_z^2)}} d\alpha. \tag{1}$$

Rearranging it, results

$$\phi = -\frac{r_x r_y r_z}{2} x (L_{r_x}(\xi) - L_{r_x}(\infty))\, M_x. \tag{2}$$

The magnetic field at position ξ outside the particle results equal to the negative gradient of the magnetic potential. The components of the magnetic field due to the magnetization along the $x-$direction results

$$H_x = \frac{r_x r_y r_z}{2} (L_{r_x}(\xi) - L_{r_x}(\infty)) M_x + \frac{r_x r_y r_z}{2} x \frac{\partial L_{r_x}(\xi)}{\partial x} M_x \tag{3}$$

$$H_y = \frac{r_x r_y r_z}{2} x \frac{\partial L_{r_x}(\xi)}{\partial y} M_x, \tag{4}$$

$$H_z = \frac{r_x r_y r_z}{2} x \frac{\partial L_{r_x}(\xi)}{\partial z} M_x. \tag{5}$$

Similarly, the magnetic potentials due to the magnetization along the y and z particle axes result in a similar functionality to Eq. (2). The magnetic field components due to the magnetization along the $y-$direction results

$$H_x = \frac{r_x r_y r_z}{2} y \frac{\partial L_{r_y}(\xi)}{\partial x} M_y, \tag{6}$$

$$H_y = \frac{r_x r_y r_z}{2} \left(L_{r_y}(\xi) - L_{r_y}(\infty)\right) M_y + \frac{r_x r_y r_z}{2} y \frac{\partial L_{r_y}(\xi)}{\partial y} M_y, \tag{7}$$

$$H_z = \frac{r_x r_y r_z}{2} y \frac{\partial L_{r_y}(\xi)}{\partial z} M_y. \tag{8}$$

The magnetic field components due to the magnetization along the $z-$direction results,

$$H_x = \frac{r_x r_y r_z}{2} z \frac{\partial L_{r_z}(\xi)}{\partial x} M_z, \tag{9}$$

$$H_y = \frac{r_x r_y r_z}{2} z \frac{\partial L_{r_z}(\xi)}{\partial y} M_z, \tag{10}$$

$$H_z = \frac{r_x r_y r_z}{2} M_z \left(L_{r_z}(\xi) - L_{r_z}(\infty)\right) + \frac{r_x r_y r_z}{2} z \frac{\partial L_{r_z}(\xi)}{\partial z} M_z. \tag{11}$$

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Therefore, the external field to the particle due to a magnetization $\mathbf{M} = (M_x, M_y, M_z)$ in an arbitrary direction in the particle frame results

$$\begin{aligned} H_x &= \frac{r_x r_y r_z}{2}\left([L_{r_x}(\xi) - L_{r_x}(\infty)] + x\frac{\partial L_{r_x}(\xi)}{\partial x}\right) M_x \\ &+ \frac{r_x r_y r_z}{2}\left(y\frac{\partial L_{r_y}(\xi)}{\partial x}\right) M_y \\ &+ \frac{r_x r_y r_z}{2}\left(z\frac{\partial L_{r_z}(\xi)}{\partial x}\right) M_z, \end{aligned} \tag{12}$$

$$\begin{aligned} H_y &= \frac{r_x r_y r_z}{2}\left(x\frac{\partial L_{r_x}(\xi)}{\partial y}\right) M_x \\ &+ \frac{r_x r_y r_z}{2}\left(\left[L_{r_y}(\xi) - L_{r_y}(\infty)\right] + y\frac{\partial L_{r_y}(\xi)}{\partial y}\right) M_y \\ &+ \frac{r_x r_y r_z}{2}\left(z\frac{\partial L_{r_z}(\xi)}{\partial y}\right) M_z, \end{aligned} \tag{13}$$

$$\begin{aligned} H_z &= \frac{r_x r_y r_z}{2}\left(x\frac{\partial L_{r_x}(\xi)}{\partial z}\right) M_x \\ &+ \frac{r_x r_y r_z}{2}\left(y\frac{\partial L_{r_y}(\xi)}{\partial z}\right) M_y \\ &+ \frac{r_x r_y r_z}{2}\left(\left[L_{r_z}(\xi) - L_{r_z}(\infty)\right] + z\frac{\partial L_{r_z}(\xi)}{\partial z}\right) M_z. \end{aligned} \tag{14}$$

Thus, the field outside the ellipsoidal particle with an arbitrarily oriented magnetization $\mathbf{M}$ can be expressed as

$$\mathbf{H} = \frac{r_x r_y r_z}{2}\mathscr{G} \cdot \mathbf{M}, \tag{15}$$

where $\mathscr{G}$ is a tensor, which is equivalent to the Green tensor in ellipsoidal coordinates with components

$$\mathscr{G}_{ij} = \delta_{ij}\left[L_{r_j}(\xi) - L_{r_j}(\infty)\right] + x_j\frac{\partial L_{r_j}(\xi)}{\partial x_i}, \tag{16}$$

where δ_{ij} represents the identity tensor and L_{r_j} is a scalar function. Additionally, $\frac{\partial L_{r_j}(\xi)}{\partial x_i} = \frac{\partial L_{r_j}(\xi)}{\partial \xi}\frac{\partial \xi}{\partial x_i}$, and

$$\frac{\partial \xi}{\partial x_i} = \frac{2x_i}{(r_i^2+\xi)} \Big/ \left(\frac{x^2}{(r_x^2+\xi)^2} + \frac{y^2}{(r_y^2+\xi)^2} + \frac{z^2}{(r_z^2+\xi)^2}\right), \tag{17}$$

$$\frac{\partial L_{r_j}(\xi)}{\partial \xi} = F_{r_j}(\xi). \tag{18}$$

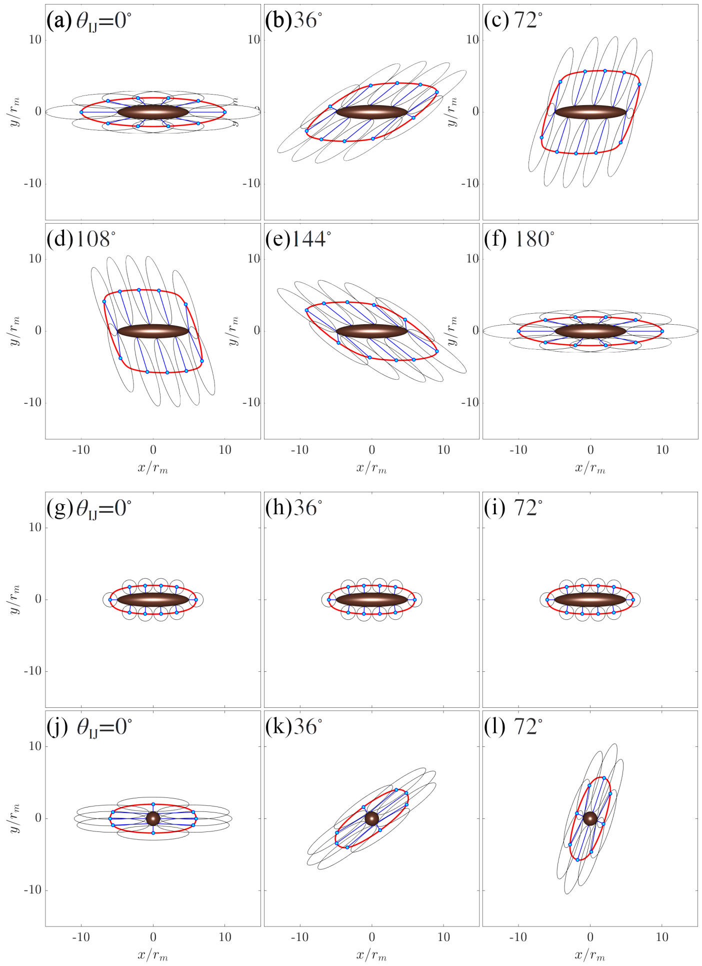

Fig. S 1 Excluded volume of (a)-(f) monodisperse ellipsoids and (g)-(l) binary ellipsoid-sphere systems with different orientations of particle J.

Notes and references

1 J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Company, Inc., New York, 1941, p. 615.

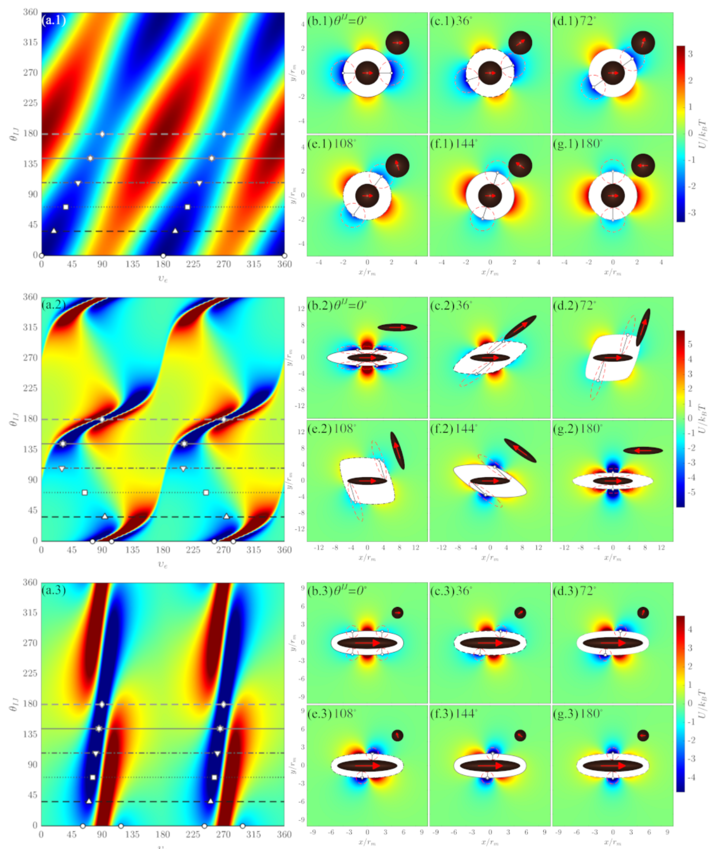

Fig. S 2 Dipolar interaction energy using point-dipole model between two magnetized ellipsoids ($\beta_s = 10$) with different orientations and aspect ratios, (a1)-(g1) $r_x/r_m = 1$, (a2) - (g2) 5, and (a3) - (g3) 5 and 1. In both particles, the magnetization is aligned along the $x-$axis of the particle. The white area represents the excluded volume between particles at the corresponding orientation.

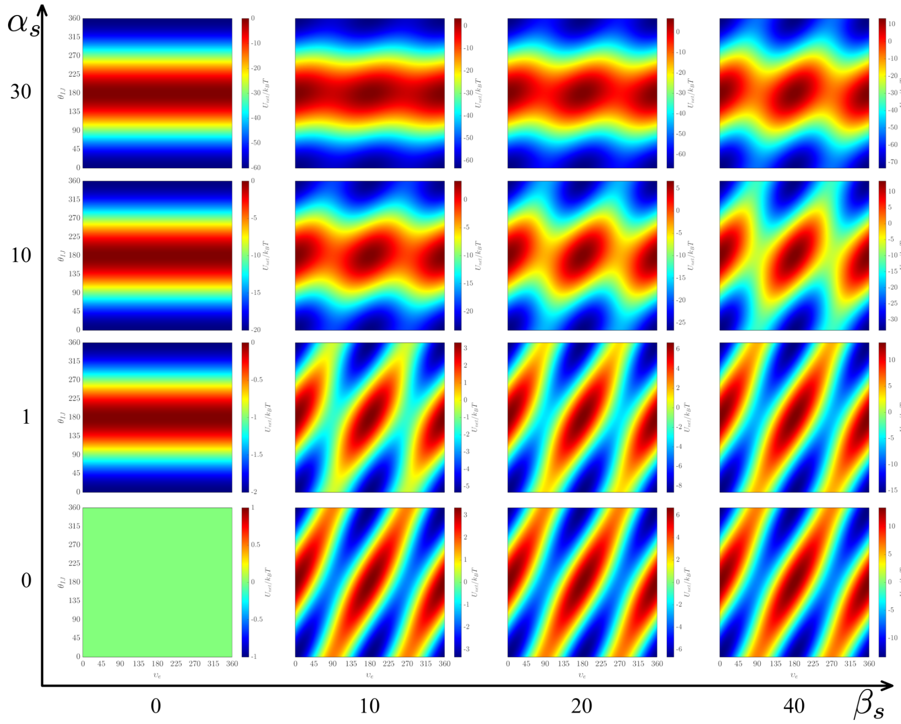

Fig. S 3 Dipolar interaction energy between two magnetized spheres $r_x/r_m = 1$ as a function of dipole-dipole interaction parameter β_s and dipole-field interaction parameter α_s. In both particles, the magnetization is aligned along the $x-$axis of the particle. The field is along the $x-$axis in the laboratory space.

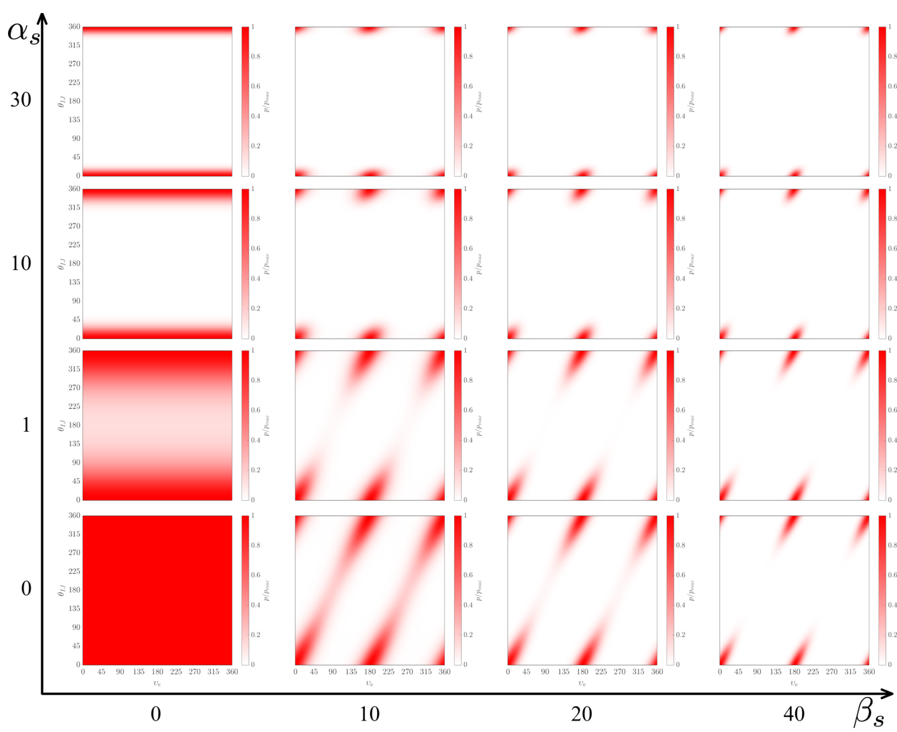

Fig. S 4 Normalized probability between two magnetized spheres $r_x/r_m = 1$ as a function of dipole-dipole interaction parameter β_s and dipole-field interaction parameter α_s. In both particles, the magnetization is aligned along the $x-$axis of the particle. The field is along the $x-$axis in the laboratory space.

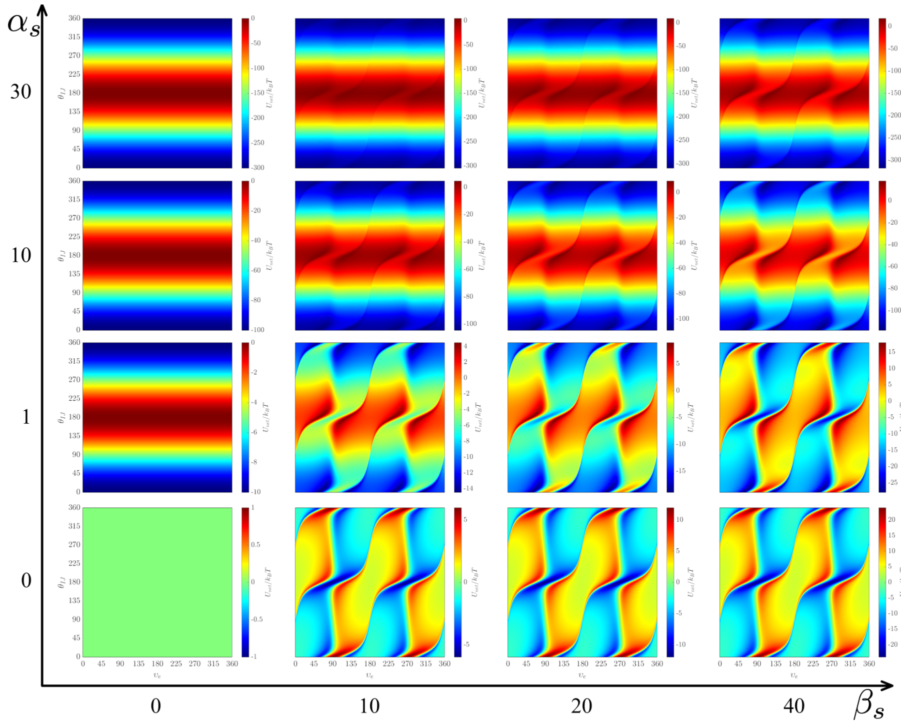

Fig. S 5 Dipolar interaction energy between two magnetized ellipsoids $r_x/r_m = 5$ as a function of dipole-dipole interaction parameter β_s and dipole-field interaction parameter α_s. In both particles, the magnetization is aligned along the $x-$axis of the particle. The field is along the $x-$axis in the laboratory space.

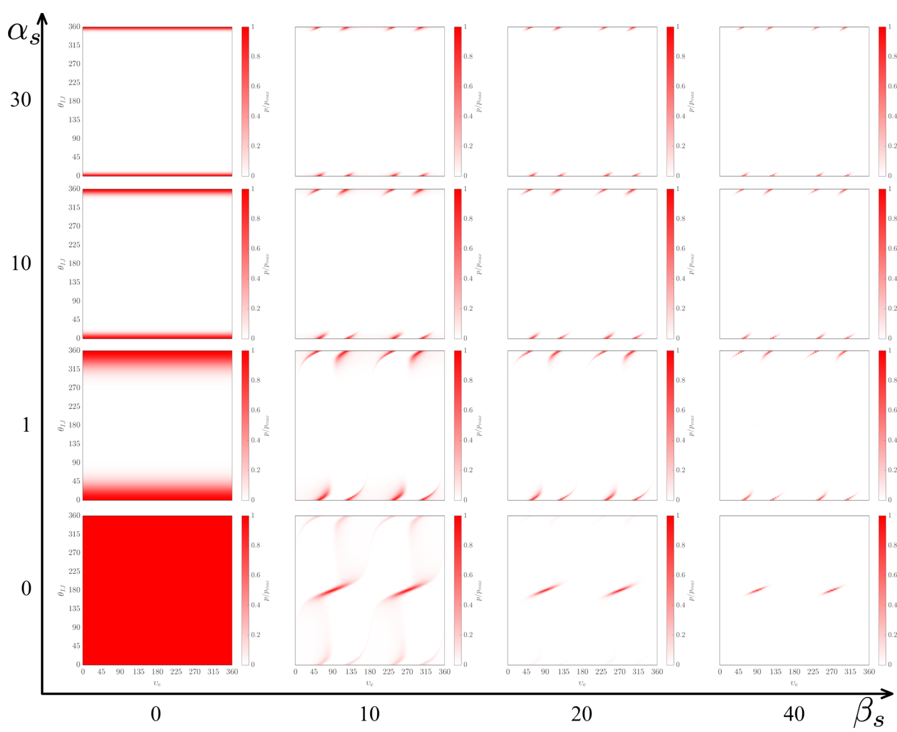

Fig. S 6 Normalized probability between two magnetized ellipsoids $r_x/r_m = 5$ as a function of dipole-dipole interaction parameter β_s and dipole-field interaction parameter α_s. In both particles, the magnetization is aligned along the $x-$axis of the particle. The field is along the $x-$axis in the laboratory space.

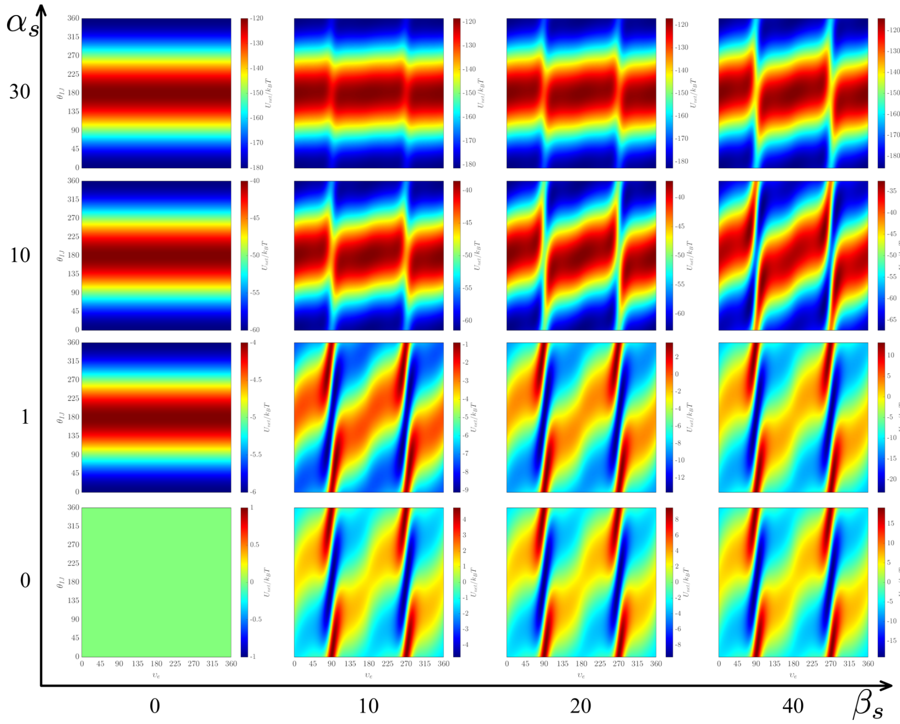

Fig. S 7 Dipolar interaction energy between a magnetized ellipsoid $r_x/r_m = 5$ and a magnetized sphere $r_x/r_m = 1$ as a function of dipole-dipole interaction parameter β_s and dipole-field interaction parameter α_s. In both particles, the magnetization is aligned along the $x-$axis of the particle. The field is along the $x-$axis in the laboratory space.

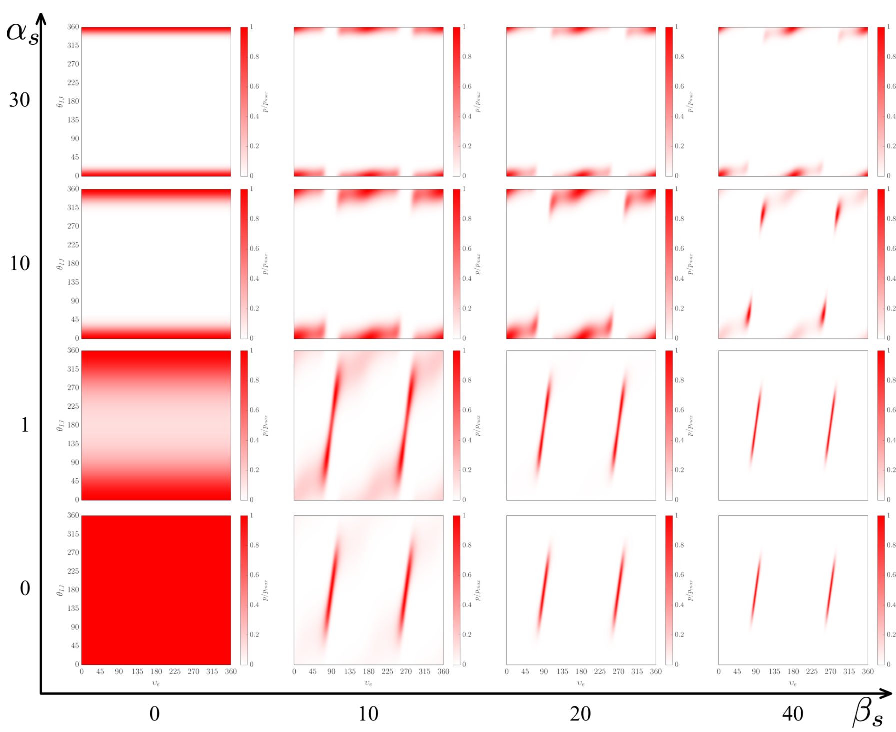

Fig. S 8 Normalized probability between a magnetized ellipsoid $r_x/r_m = 5$ and a magnetized sphere $r_x/r_m = 1$ as a function of dipole-dipole interaction parameter β_s and dipole-field interaction parameter α_s. In both particles, the magnetization is aligned along the $x-$axis of the particle. The field is along the $x-$axis in the laboratory space.

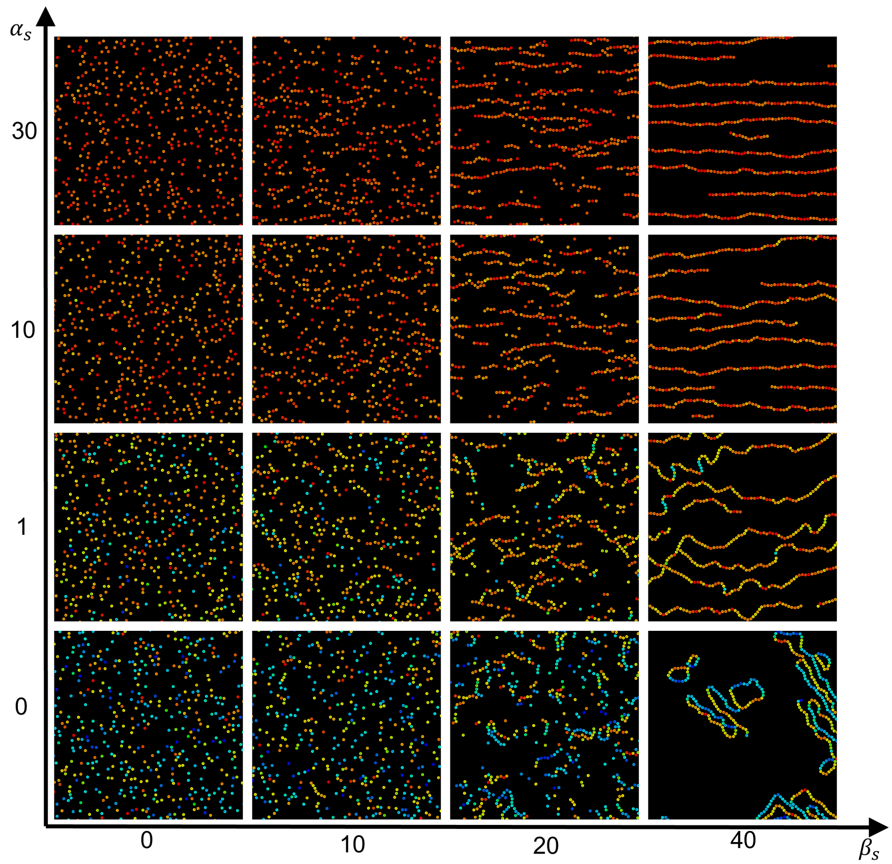

Fig. S 9 Snapshots of MC simulations for suspensions composed of spheres $(r_x/r_m = 1)$ in a two-dimensional confinement as a function of α_s and β_s. The colorbar is as shown in Fig. 3.

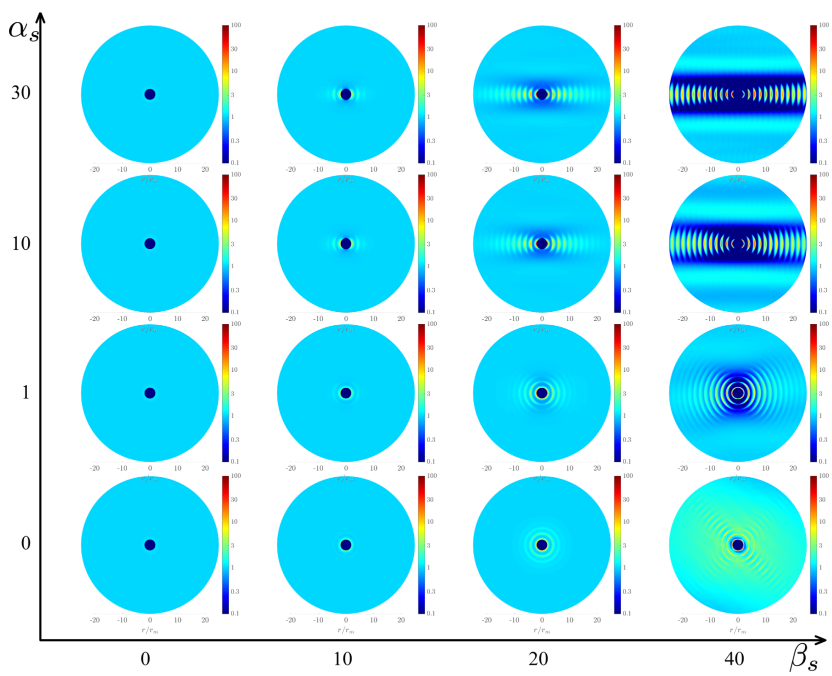

Fig. S 10 Pair distribution function for suspensions composed of spheres $(r_x/r_m = 1)$ in a two-dimensional confinement as a function of α_s and β_s..

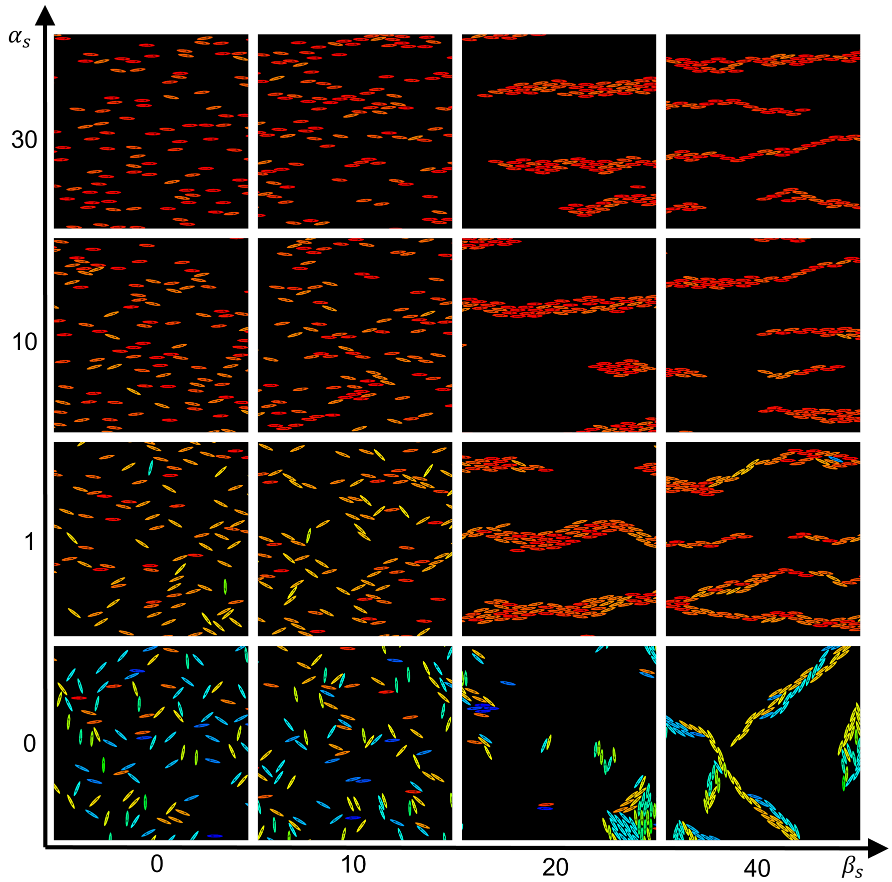

Fig. S 11 Snapshots of MC simulations for suspensions composed of ellipsoids $(r_x/r_m = 5)$ in a two-dimensional confinement as a function of α_s and β_s. The colorbar is as shown in Fig. 3.

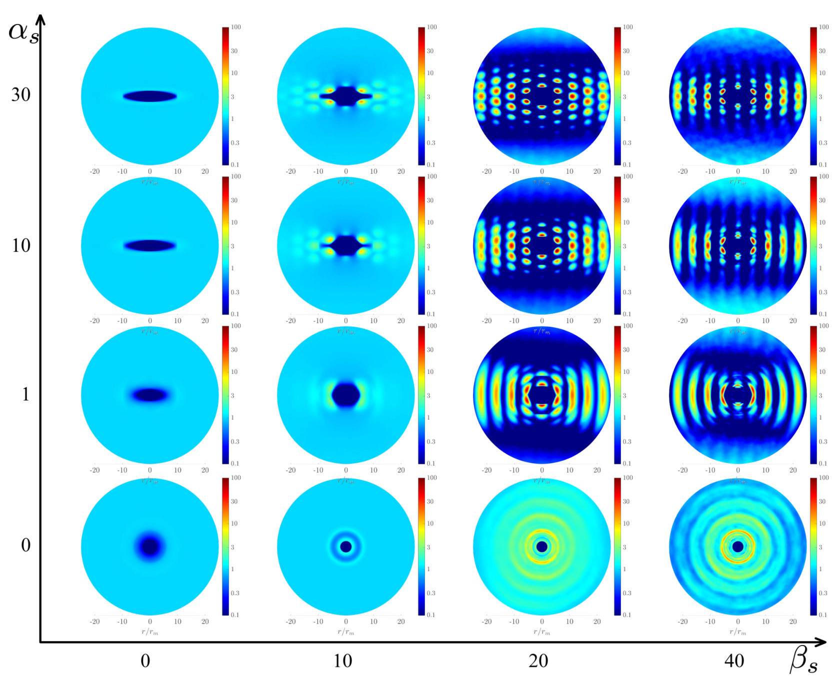

Fig. S 12 Pair distribution function for suspensions composed of ellipsoids $(r_x/r_m = 5)$ in a two-dimensional confinement as a function of α_s and β_s..

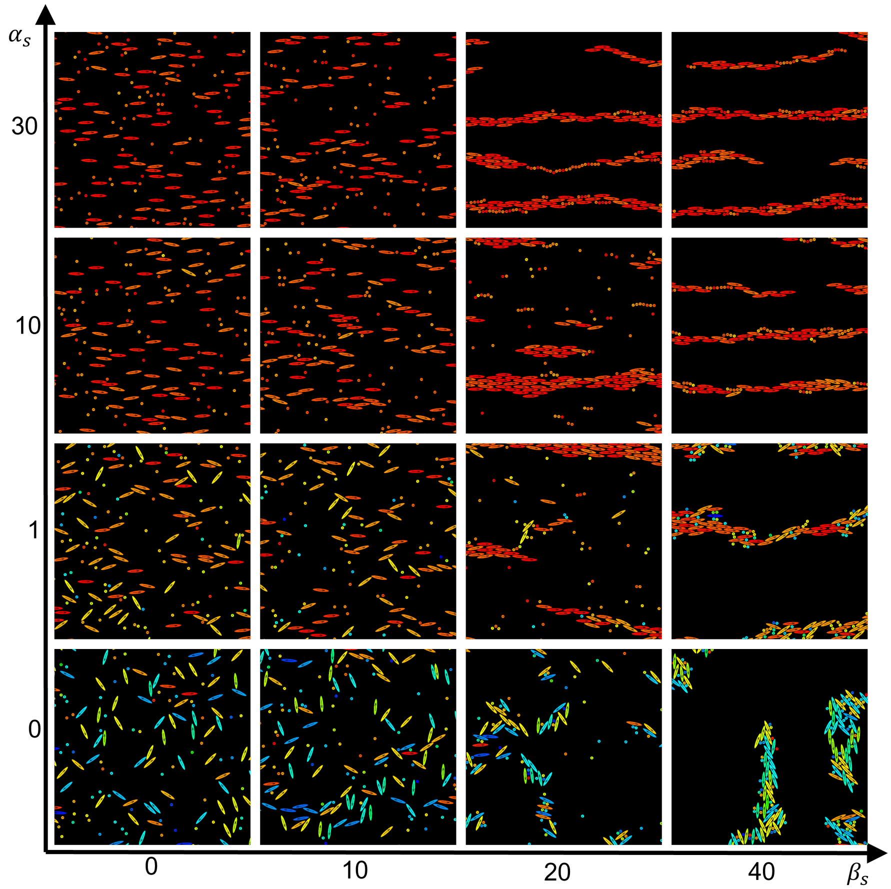

Fig. S 13 Snapshots of MC simulations for suspensions composed of a mixture of spheres $(r_x/r_m = 1)$ and ellipsoids $(r_x/r_m = 5)$ in a two-dimensional confinement as a function of α_s and β_s. The colorbar is as shown in Fig. 3.

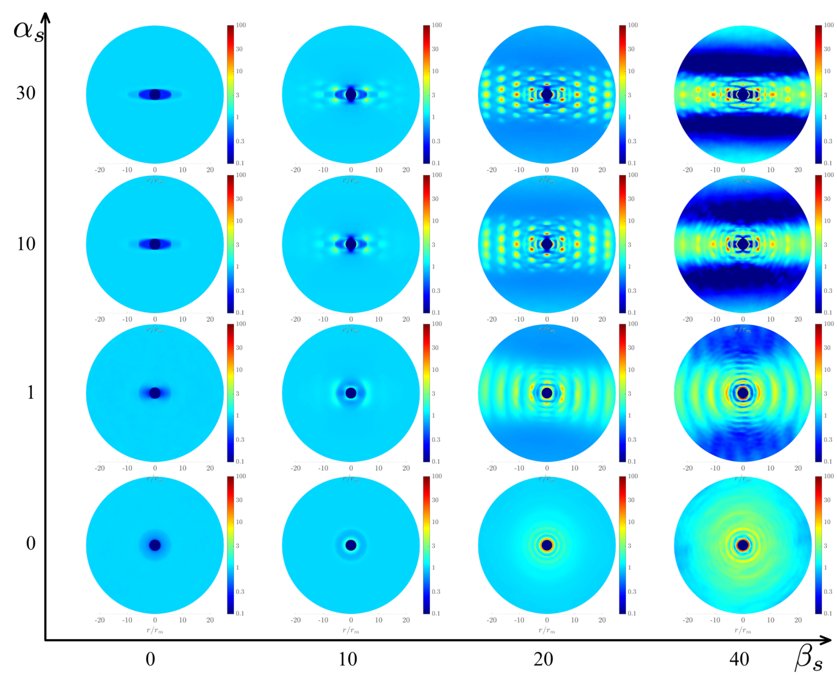

Fig. S 14 Pair distribution function for suspensions composed of a mixture of spheres $(r_x/r_m = 1)$ and ellipsoids $(r_x/r_m = 5)$ in a two-dimensional confinement as a function of α_s and β_s.