

**Supplementary Information: Evidence of Vacancy Ordered  
Structures in  $\text{PuO}_{2-x}$  and  $\text{AmO}_{2-x}$  from First-principles  
Calculations**

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(Dated: April 11, 2023)

TABLE S1. The structural details (Lattice type, Space group, and Lattice parameters) of the three predicted ground states of  $\text{PuO}_{2-x}$ .

Predicted GS	Lattice Parameters	Atomic Positions
$\text{PuO}_2$	$a=b=3.847, c=5.515 \text{ \AA}$	Pu (2a) 0 0 0
I4/mmm (139)	$\alpha=\beta=\gamma=90.0$	O (4a) 0 1/2 1/4
$\text{Pu}_8\text{O}_{15}$	$a=7.757, b=11.028,$	Pu (4c) 0.37851 0.37813 0.75677, Pu (4c) 0.73551 0.11419 0.00004
C2 (5)	$c=8.670 \text{ \AA},$	Pu (4c) 0.13654 0.38036 0.27326, Pu (2b) 0.00000 0.62824 0.50000
	$\beta=116.30$	Pu (2b) 0.00000 0.12836 0.50000
	$V_0=667.15$	O (2a) 0.00000 0.00534 0.00000, O (2a) 0.00000 0.75661 0.00000
		O (2a) 0.00000 0.46794 0.00000, (4c) 0.64797 0.25223 0.75571
		O (4c) 0.11521 0.49773 0.73638, (4c) 0.74113 0.25357 0.48134
		O (4c) 0.75384 0.00574 0.50826, (4c) 0.10730 0.25223 0.75476
		O (4c) 0.62139 0.49760 0.73718
$\text{Pu}_6\text{O}_{11}$	$a=12.240, b=5.531,$	Pu (4c) 0.66594 0.32681 0.58421, Pu (4c) -0.00172 0.31584 0.73545
C2 (5)	$c=7.853 \text{ \AA},$	Pu (4c) 0.32190 0.35606 -0.08926,
	$\beta=108.6$	O (4c) 0.64867 0.08574 0.83784, O (4c) 0.34775 0.07446 0.68654
		O (2b) 0.00000 0.57120 0.50000, O (4c) 0.82477 0.08173 0.64809
		O (2b) 0.00000 0.06526 0.50000, O (4c) 0.83727 0.08406 0.18325
		O (2a) 0.00000 0.13084 0.00000
$\text{Pu}_8\text{O}_{14}$	$a=b=7.804,$	Pu (4k) 0.76482 0.50000 0.72208, Pu (4j) 0.73833 0.00000 0.77773
P-4m2 (115)	$c=5.592 \text{ \AA},$	O (2g) 0.00000 0.50000 0.45827, O (4i) 0.78938 0.78938 0.50000
	$V_0=340.58$	O (1c) 0.50000 0.50000 0.50000, O (4h) 0.73802 0.73802 0.00000
		O (2g) 0.00000 0.50000 -0.01481, O (1a) 0.00000 0.00000 0.00000
$\text{Pu}_2\text{O}_3$	$a=b=3.935, c=5.723 \text{ \AA}$	Pu (2g) 0 1/2 0.27719
P-4m2 (115)	$\alpha=\beta=\gamma=90.0$	O (1d) 0 0 1/2, O (1b) 1/2 1/2 0
		O (1c) 1/2 1/2 1/2

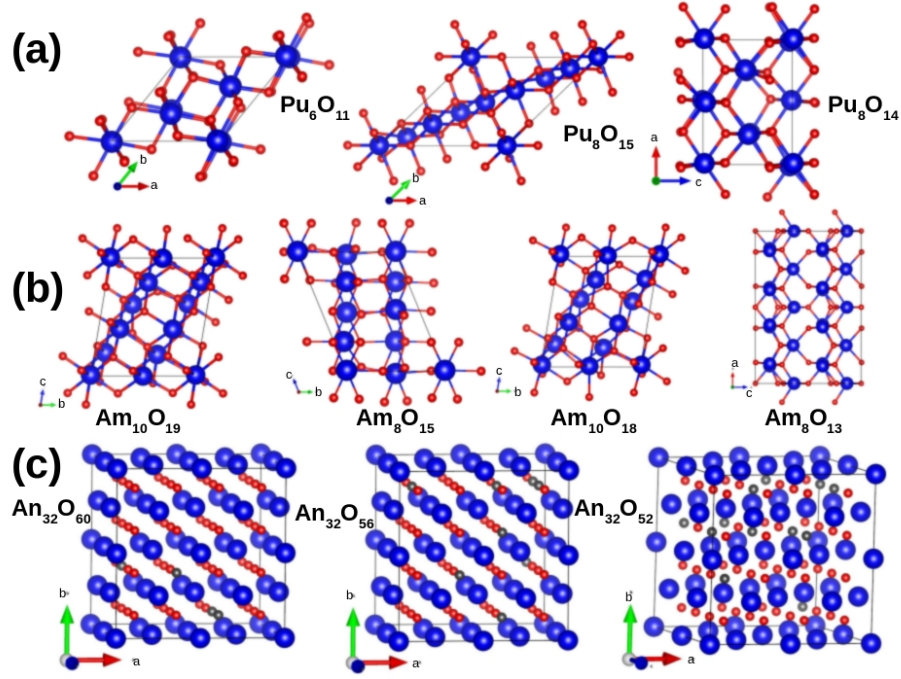


FIG. S1. The CE determined vacancy ordered structures of (a)  $\text{PuO}_{2-x}$  (b)  $\text{AmO}_{2-x}$  and (c) SQS generated structures are shown here. Oxygen and An (Pu/Am) atoms are shown in red and blue, respectively.

$$\text{Elastic constant matrices for } \text{AnO}_{2-x} \text{ in GPa} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix}$$

$$\text{PuO}_2 = \begin{bmatrix} 372.965 & 110.649 & 109.661 & -0.362 & 0.078 & 0.507 \\ 110.649 & 372.332 & 109.350 & 0.043 & 0.087 & 0.032 \\ 109.661 & 109.350 & 378.729 & 0.099 & 0.141 & 0.513 \\ -0.362 & 0.043 & 0.099 & 69.770 & 0.079 & 0.512 \\ 0.078 & 0.087 & 0.141 & 0.079 & 69.535 & 0.099 \\ 0.507 & 0.032 & 0.513 & 0.512 & 0.099 & 68.382 \end{bmatrix}$$

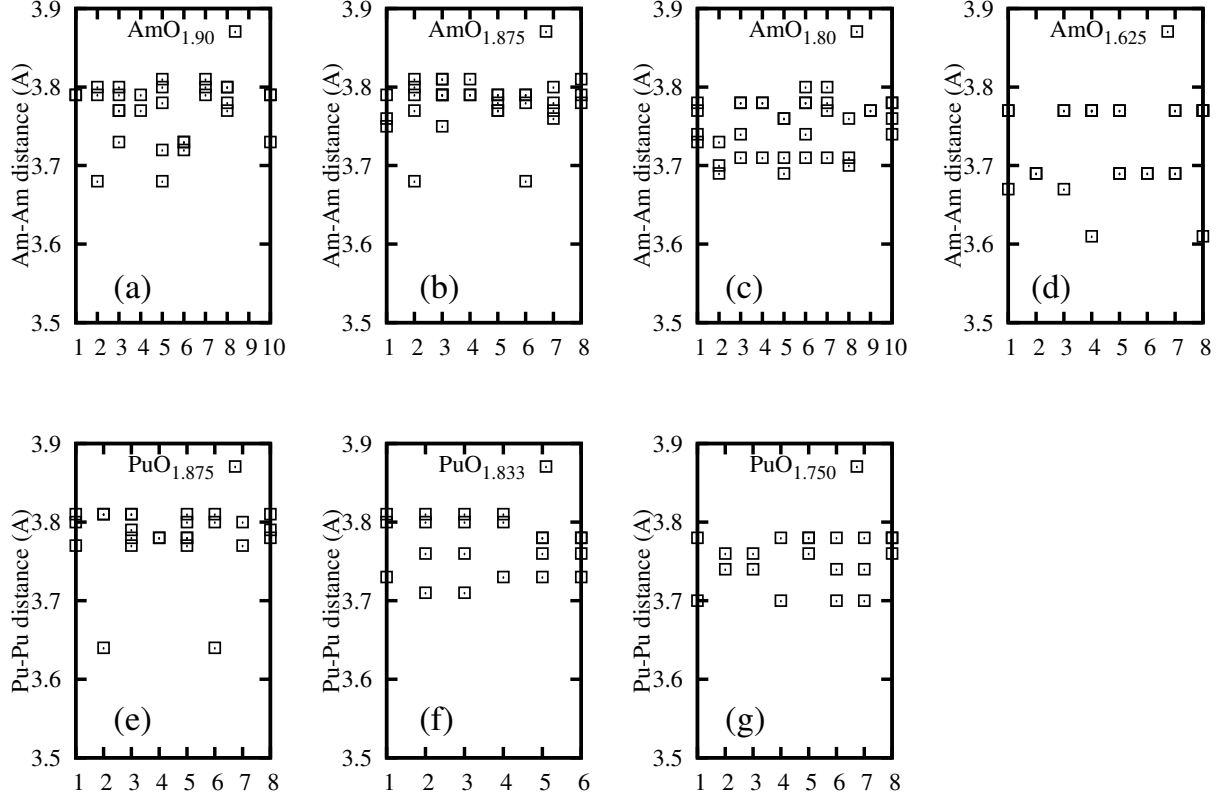


FIG. S2. Distribution of Ac-Ac distances in vacancy ordered structures (a)  $\text{Am}_{10}\text{O}_{19}$  ( $\text{AmO}_{1.90}$ ), (b)  $\text{Am}_8\text{O}_{15}$  ( $\text{AmO}_{1.875}$ ), (c)  $\text{Am}_{10}\text{O}_{18}$  ( $\text{AmO}_{1.8}$ ), (d)  $\text{Am}_8\text{O}_{13}$  ( $\text{AmO}_{1.625}$ ) and (e)  $\text{Pu}_8\text{O}_{15}$  ( $\text{PuO}_{1.875}$ ), (f)  $\text{Pu}_6\text{O}_{11}$  ( $\text{PuO}_{1.833}$ ), (g)  $\text{Pu}_8\text{O}_{14}$  ( $\text{PuO}_{1.75}$ ) in  $\text{AmO}_{2-x}$  and  $\text{PuO}_{2-x}$ , respectively. The x-axis shows the Number of Am/Pu atoms present in the vacancy ordered structures.

$$\text{Pu}_8\text{O}_{15} = \begin{bmatrix} 274.947 & 144.159 & 111.112 & -1.123 & 0.208 & 0.285 \\ 144.159 & 257.147 & 103.015 & 1.306 & -0.254 & -0.793 \\ 111.112 & 103.015 & 301.561 & 2.307 & 0.132 & -0.128 \\ -1.123 & 1.306 & 2.307 & 56.689 & 0.377 & 0.013 \\ 0.208 & -0.254 & 0.132 & 0.377 & 64.482 & 0.386 \\ 0.285 & -0.793 & -0.128 & 0.013 & 0.386 & 88.005 \end{bmatrix}$$

$$\text{Pu}_6\text{O}_{11} = \begin{bmatrix} 248.293 & 132.486 & 103.564 & -0.876 & 3.406 & 4.288 \\ 132.486 & 248.492 & 103.575 & -3.360 & 0.886 & 4.348 \\ 103.564 & 103.575 & 274.994 & -1.944 & 2.011 & -2.467 \\ -0.876 & -3.360 & -1.944 & 58.009 & 3.044 & -0.081 \\ 3.406 & 0.886 & 2.011 & 3.044 & 57.934 & 0.073 \\ 4.288 & 4.348 & -2.467 & -0.081 & 0.073 & 91.540 \end{bmatrix}$$

$$\text{Pu}_8\text{O}_{14} (\text{P-4m2}) = \begin{bmatrix} 224.290 & 111.405 & 101.879 & 0.000 & 0.000 & -0.097 \\ 111.405 & 224.290 & 101.879 & 0.000 & 0.000 & -0.096 \\ 101.879 & 101.879 & 258.176 & 0.000 & 0.000 & -0.085 \\ 0.000 & 0.000 & 0.000 & 55.723 & 0.150 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.150 & 55.739 & 0.000 \\ -0.097 & -0.096 & -0.085 & 0.000 & 0.000 & 66.343 \end{bmatrix}$$

$$\text{Pu}_2\text{O}_3 = \begin{bmatrix} 170.270 & 98.521 & 98.216 & -0.211 & 0.022 & 0.118 \\ 98.521 & 168.513 & 98.048 & 0.239 & 0.132 & -0.141 \\ 98.216 & 98.048 & 169.275 & -0.054 & -0.080 & 0.056 \\ -0.211 & 0.239 & -0.054 & 47.831 & -0.636 & 0.664 \\ 0.022 & 0.132 & -0.080 & -0.636 & 48.121 & -0.516 \\ 0.118 & -0.141 & 0.056 & 0.664 & -0.516 & 48.037 \end{bmatrix}$$

$$\text{Am}_{10}\text{O}_{19} = \begin{bmatrix} 258.826 & 149.574 & 107.321 & -0.128 & 2.325 & -0.025 \\ 149.574 & 257.532 & 106.521 & -0.441 & -1.753 & -2.003 \\ 107.321 & 106.521 & 297.723 & -0.598 & -0.692 & 2.792 \\ -0.128 & -0.441 & -0.598 & 52.287 & 1.108 & -0.807 \\ 2.325 & -1.753 & -0.692 & 1.108 & 58.733 & 2.055 \\ -0.025 & -2.003 & 2.792 & -0.807 & 2.055 & 97.090 \end{bmatrix}$$

$$\text{Am}_8\text{O}_{15} = \begin{bmatrix} 264.370 & 144.133 & 105.103 & 2.519 & -3.344 & -0.190 \\ 144.133 & 241.498 & 105.925 & -1.156 & -1.932 & -2.931 \\ 105.103 & 105.925 & 292.172 & -0.790 & 0.767 & 2.440 \\ 2.519 & -1.156 & -0.790 & 59.482 & 0.122 & -0.005 \\ -3.344 & -1.932 & 0.767 & 0.122 & 60.606 & 0.709 \\ -0.190 & -2.931 & 2.440 & -0.005 & 0.709 & 87.048 \end{bmatrix}$$

$$\text{Am}_{10}\text{O}_{18} = \begin{bmatrix} 229.415 & 116.782 & 91.602 & -3.932 & -8.159 & -0.806 \\ 116.782 & 197.330 & 78.551 & -0.580 & -2.781 & -1.968 \\ 91.602 & 78.551 & 239.688 & -2.374 & -4.362 & 3.744 \\ -3.932 & -0.580 & -2.374 & 46.542 & -5.088 & -0.455 \\ -8.159 & -2.781 & -4.362 & -5.088 & 56.171 & 0.985 \\ -0.806 & -1.968 & 3.744 & -0.455 & 0.985 & 78.348 \end{bmatrix}$$

$$\text{Am}_8\text{O}_{13} = \begin{bmatrix} 184.236 & 94.951 & 74.587 & 0.031 & -0.139 & -0.063 \\ 94.951 & 186.346 & 74.904 & -0.070 & -0.097 & -0.089 \\ 74.587 & 74.904 & 198.344 & -0.031 & -0.073 & -0.098 \\ 0.031 & -0.070 & -0.031 & 38.055 & 0.041 & -0.024 \\ -0.139 & -0.097 & -0.073 & 0.041 & 38.641 & -0.112 \\ -0.063 & -0.089 & -0.098 & -0.024 & -0.112 & 63.964 \end{bmatrix}$$

Elastic constant matrices for  $\text{Am}_2\text{O}_3$  (Sp. Gr. P-3m1) in GPa =

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 \\ C_{12} & C_{11} & C_{13} & -C_{14} & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ C_{14} & -C_{14} & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & C_{14} \\ 0 & 0 & 0 & 0 & C_{14} & C_{66} \end{bmatrix}$$

$$\text{Am}_2\text{O}_3 = \begin{bmatrix} 225.266 & 126.198 & 87.798 & 35.660 & 0.000 & 0.000 \\ 126.198 & 225.266 & 87.798 & -35.660 & 0.000 & 0.000 \\ 87.798 & 87.798 & 141.140 & 0.000 & 0.000 & 0.000 \\ 35.660 & -35.660 & 0.000 & 66.908 & 0.000 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 & 66.910 & 35.660 \\ 0.000 & 0.000 & 0.000 & 0.000 & 35.660 & 49.534 \end{bmatrix}$$

$$\text{AmO}_2 = \begin{bmatrix} 355.760 & 107.274 & 102.814 & 0.000 & 0.000 & 0.108 \\ 107.274 & 354.489 & 113.638 & 0.000 & 0.000 & -0.049 \\ 102.814 & 113.638 & 315.913 & 0.001 & 0.000 & -0.496 \\ 0.000 & 0.000 & 0.001 & 44.473 & 0.272 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.272 & 56.156 & 0.000 \\ 0.108 & -0.049 & -0.496 & 0.000 & 0.000 & 61.704 \end{bmatrix}$$

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TABLE S2. The structural details (Lattice type, Space group, and Lattice parameters) of the three predicted ground states of  $\text{AmO}_{2-x}$ .

Predicted GS	Lattice Parameters	Atomic Positions
$\text{AmO}_2$	$a=b=3.883, c=5.593 \text{ \AA}$	Am (2a) 0 0 0
I4/mmm (139)	$\alpha=\beta=\gamma=90.0$	O (4a) 0 1/2 1/4
$\text{Am}_{10}\text{O}_{19}$	$a=6.6665, b=7.7658,$	Am(1a) 0.60715 0.39800 0.79539, Am(1a) 0.19947 0.28390 0.60840
P1 (1)	$c=8.6812,$	Am(1a) 0.41378 0.59954 0.18192, Am(1a) 0.00400 0.49823 -0.00120
	$\alpha=76.83,$	Am(1a) 0.20009 0.80912 0.61272, Am(1a) 0.76912 0.70712 0.39669
	$\beta=75.36,$	Am(1a) 0.00002 -0.00224 0.00728, Am(1a) 0.61028 0.89813 0.79966
	$\gamma=72.92,$	Am(1a) 0.80084 0.20186 0.40002, Am(1a) 0.39324 0.10244 0.19826
	$V_0=409.9057 \text{ \AA}^3$	O(1a) 0.86831 0.40908 0.54729, O(1a) 0.25120 0.50198 0.71742
		O(1a) -0.05226 0.28716 0.85082, O(1a) -0.04067 0.80837 0.85432
		O(1a) 0.65025 0.60456 -0.06225, O(1a) 0.06381 0.70352 0.16755
		O(1a) 0.74388 0.49234 0.24587, O(1a) 0.72425 0.01723 0.25384
		O(1a) 0.40465 0.78734 0.35690, O(1a) 0.87485 0.88132 0.54712
		O(1a) 0.53953 0.70516 0.65578, O(1a) 0.57182 0.19152 0.64525
		O(1a) 0.25821 0.00148 0.75594, O(1a) 0.64860 0.09734 -0.04954
		O(1a) 0.36090 -0.09729 0.04590, O(1a) 0.36733 0.39408 0.04245
		O(1a) 0.05107 0.18683 0.15889, O(1a) 0.41706 0.32914 0.36148
		O(1a) 0.14923 0.10274 0.45584
$\text{Am}_8\text{O}_{15}$	$a=6.72692, b=6.73147,$	Am(1a) 0.24272 0.25118 0.49351, Am(1a) 0.14913 0.62881 0.25099
P1 (1)	$c=8.63044,$	Am(1a) 0.48766 0.49463 -0.02054, Am(1a) 0.37208 0.87797 0.75038
	$\alpha=105.16247,$	Am(1a) 0.75307 0.76455 0.51749, Am(1a) 0.62033 0.10059 0.24783
	$\beta=104.98819,$	Am(1a) -0.00195 0.00791 0.00589, Am(1a) 0.87472 0.37740 0.75114
	$\gamma=109.29372,$	O(1a) -0.02604 0.26639 0.24144, O(1a) 0.07452 0.82360 0.46797
	$V_0=329 \text{ \AA}^3$	O(1a) 0.38464 0.63474 0.49665, O(1a) -0.00460 0.76097 0.77378
		O(1a) 0.23232 0.49559 0.74751, O(1a) 0.41678 0.17051 0.03144
		O(1a) 0.62712 0.86723 0.00108, O(1a) 0.25510 -0.01241 0.24188
		O(1a) 0.52134 0.72438 0.24629, O(1a) 0.65457 0.39207 0.47673
		O(1a) 0.85897 0.11543 0.50204, O(1a) 0.50321 0.26072 0.73907
		O(1a) 0.76179 0.01460 0.76348, O(1a) 0.86489 0.60919 0.02534
		S8 O(1a) 0.12764 0.37395 -0.00139



TABLE S3. The structural details (Lattice type, Space group, and Lattice parameters) of the three predicted ground states of  $\text{AmO}_{2-x}$ .

Predicted GS	Lattice Parameters	Atomic Positions
$\text{Am}_{10}\text{O}_{18}$	$a=6.780, b=7.769,$	$\text{Am}(1a) 0.61930 0.40711 0.78888, \text{Am}(1a) 0.19605 0.28506 0.61416$
P1 (1)	$c=8.709 \text{ \AA}$	$\text{Am}(1a) 0.42221 0.59905 0.17313, \text{Am}(1a) 0.00999 0.48691 0.00834$
	$\alpha=76.92,$	$\text{Am}(1a) 0.20686 0.81249 0.59399, \text{Am}(1a) 0.77254 0.70720 0.39219$
	$\beta=74.72,$	$\text{Am}(1a) 0.00321 0.00795 0.01983, \text{Am}(1a) 0.58372 -0.09411 0.79595$
	$\gamma=73.26,$	$\text{Am}(1a) 0.79712 0.20691 0.40534, \text{Am}(1a) 0.39654 0.10455 0.20661$
	$V_0=418.18 \text{ \AA}^3$	$\text{O}(1a) 0.85727 0.40866 0.55036, \text{O}(1a) 0.21008 0.53784 0.72709$
		$\text{O}(1a) -0.06594 0.29049 0.86955, \text{O}(1a) 0.67540 0.61994 -0.07524$
		$\text{O}(1a) 0.05365 0.71179 0.14385, \text{O}(1a) 0.74968 0.48713 0.25351$
		$\text{O}(1a) 0.73000 0.02243 0.26076, \text{O}(1a) 0.42214 0.79365 0.35071$
		$\text{O}(1a) 0.86594 0.88110 0.56820, \text{O}(1a) 0.53603 0.70435 0.64959$
		$\text{O}(1a) 0.55357 0.20314 0.65698, \text{O}(1a) 0.22324 -0.02139 0.76825$
		$\text{O}(1a) 0.71708 0.02756 -0.06324, \text{O}(1a) 0.35466 -0.07402 0.04268$
		$\text{O}(1a) 0.38978 0.37629 0.02243, \text{O}(1a) 0.05634 0.19586 0.16953$
		$\text{O}(1a) 0.41691 0.31773 0.35198, \text{O}(1a) 0.14663 0.09433 0.45457$
$\text{Am}_8\text{O}_{13}$	$a=16.056, b=7.734,$	$\text{Am} (8d) 0.38567 0.0 0.13730, \text{Am} (8b) 0.25 0.25 0.35875$
Fmm2 (42)	$c=11.164 \text{ \AA},$	$\text{Am} (8c) 0.0 0.26653 0.86702, \text{Am} (8d) 0.87036 0.00 0.13703$
	$\alpha=\beta=\gamma=90$	$\text{O} (8d) 0.25272 0.0 0.24060, \text{O} (16e) 0.86239 0.21986 0.00528$
	$V_0=1386 \text{ \AA}^3$	$\text{O} (16e) 0.37693 0.26314 0.25311, \text{O} (4a) 0.0 0.0 0.76819$
		$\text{O} (4a) 0 0 0.50238, \text{O} (4a) 0 0 0.21445$
$\text{Am}_2\text{O}_3$	$a=b=3.919, c=5.718 \text{ \AA}$	$\text{Am} (2g) 0 1/2 0.279$
P-4m2 (115)	$\alpha=\beta=\gamma=90.0$	$\text{O} (1d) 0 0 1/2, \text{O} (1b) 1/2 1/2 0$
		$\text{O} (1c) 1/2 1/2 1/2$