Supplementary Information for

Understanding xenon and vacancy behaviors in UO₂, UN and U₃Si₂: A comparative DFT+*U* study

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Table S1 Effect of supercell size on the formation energies of $V_{\rm U}$ in both UO₂ and UN as well as $V_{\rm U-2}$ in U₃Si₂. The cutoff energy of 500 eV with the *k*-point grid spacing of 0.04 Å⁻¹, 0.02 Å⁻¹ and 0.03 Å⁻¹ is used for UO₂, UN and U₃Si₂, respectively. The supercell sizes highlighted in bold are employed in current work.

UO ₂		UN		U_3Si_2	
Supercell size	$E_f(eV)$	Supercell size	$E_f(eV)$	Supercell size	$E_f(eV)$
$1 \times 2 \times 2$	12.75	$1 \times 2 \times 2$	6.05	$2 \times 2 \times 2$	1.65
$2 \times 2 \times 2$	11.60	$2 \times 2 \times 2$	5.35	$2 \times 2 \times 3$	1.49
$2 \times 2 \times 3$	11.28	$2 \times 2 \times 3$	5.72	$2 \times 2 \times 4$	1.51
				$2 \times 2 \times 5$	1.76

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		Standard reference	U-rich state	X-rich (X = O, N, Si) state
UO ₂	Vu	10.51	10.51	2.84
	Ref.	6.0 ¹	4.48 ²	-6.50 ²
	Vo	4.71	-0.91	2.93
	Ref.	5.6 ¹	-0.43 ²	5.06 ²
	Ui	2.59	2.59	10.26
	Ref.	8.2 1	2.50 ²	13.48 ²
	Oi	-0.34	5.28	1.44
	Ref.	-1.6 ¹	3.05 ²	-2.44 ²
UN	Vu	5.35	5.35	3.88
	Ref.	3.74 ³ ; 6.90 ⁴	3.66 5	2.09 5
	Vn	3.33	0.35	1.82
	Ref.	4.24 ³ ; 7.91 ⁴	0.62 5	1.42 5
	Ui	4.28	4.28	5.75
	Ref.	12.62 4	6.20 ⁵	7.77 ⁵
	Ni	0.39	3.37	1.90
	Ref.	4.54 4	3.82 5	3.01 ⁵
U ₃ Si ₂	VU-1	2.68	2.55	3.50
	Ref.	2.65 ⁶ ; 3.00 ⁷	3.00 7	2.96 7
	VU-2	1.51	1.38	1.36
	Ref.	1.64 ⁶ ; 1.69 ⁷	1.69 ⁷	1.65 7
	$V_{\rm Si}$	1.56	0.89	0.92
	Ref.	2.48 ⁶ ; 1.79 ⁷	1.77 7	1.84 7
	Ui	2.30	2.43	2.45
	Ref.	1.66 ⁶ ; 0.65 ⁶ , 1.17 ⁶ ; 0.87 ⁷	0.86 7	0.91 7
	Sii	0.14	0.80	0.77
	Ref.	0.10 ⁶ ; 0.85 ⁶ , -0.20 ⁶ ; 0.55 ⁷	0.57 7	0.50 7

Table S2 Predicted formation energies (E_f) of different types of mono-atomic vacancies and mono-atomic interstitials in UO₂, UN and U₃Si₂ by comparing with the existing data.

		Our work	Ref.
UO ₂	IS	9.45	11.11 ⁸ ; 9.48 ⁹
	Vu	2.85	2.5 ⁸ ; 3.84 ⁹ ; 5.18 ²
	Vo	8.80	9.5 ⁸ ; 6.89 ⁹ ; 9.01 ²
	{ <i>V</i> U: <i>V</i> O}	1.89	2.45 ⁸
	{ <i>V</i> U:2 <i>V</i> O}	1.09	1.38 ⁸ ; 1.18 ⁹ ; 2.90 ²
UN	IS	14.64	14.62 ¹⁰ ; 14.64 ¹¹
	Vu	4.03	3.59 10; 4.50 12; 3.74 11
	Vn	8.87	8.6 10; 8.28 12; 8.45 11
	$\{V_{\mathrm{U}}:V_{\mathrm{N}}\}$	2.72 (in [100] direction), 4.08 (in [111] direction)	2.79 ¹¹ ; 3.37 (in [100] direction) ¹² ; 4.90 (in [111] direction) ¹²
U ₃ Si ₂	IS	3.86	6.07 ⁶ ; 5.36 ⁷
	<i>V</i> U-1	0.17	3.39 ⁶ ; 3.14 ⁷
	<i>V</i> U-2	0.01	3.15 ⁶ ; 3.24 ⁷
	$V_{ m Si}$	0.27	3.39 ⁶ ; 3.26 ⁷

Table S3 Incorporation energies (E_i) between Xe atom and the typical trap sites in UO₂, UN and U₃Si₂ by comparing with other DFT-calculated results.

Supplementary Note 1: Chemical potential range

With the experimental formation enthalpy of UO₂ (-11.25 eV ^{13, 14}), the chemical potential variation of U and O is restricted in a range of -11.25 eV $< \Delta \mu_U^{UO_2} < 0$ eV and -5.63 eV $< \Delta \mu_O^{UO_2} < 0$ eV, respectively. Further, more experimental formation enthalpies of binary U-O phase ¹⁴, including UO₂, U₄O₉, U₃O₈ and UO₃, are involved to assess the variation range of $\Delta \mu_U^{UO_2}$ and $\Delta \mu_O^{UO_2}$, which can be expressed as

$$-7.67 \text{ eV} \le \Delta \mu_{\mathrm{U}}^{\mathrm{UO2}} \quad 0 \tag{s1}$$

$$-5.62 \text{ eV} \le \Delta \mu_0^{\text{UO}_2} -1.78 \text{ eV}$$
 (s2)

It should be pointed out that the chemical potential values of -1.78 eV and -7.67 eV are related to the critical points of intersection between UO₂ and U₄O₉ (i.e., UO_{2.25}) lines as shown in the $\Delta \mu_{\rm U}^{\rm UO_2} - \Delta \mu_{\rm O}^{\rm UO_2}$ diagram (see Fig. S1(a)).

However, there is no available experimental data for the formation enthalpies of hyper-stoichiometric phases in binary U-O, U-N and U-Si phase diagram. The DFT-calculated total energies of uranium compounds are subsequently considered to obtain the U-rich boundary of chemical potentials ¹⁵. All the used U-O and U-N phases in the first-principles calculations are performed using the same value of U_{eff} for the stoichiometric UO₂ and UN. The DFT-calculated μ_O as a function of μ_U in the U-O system are displayed in Fig. S1(b). It is found that the phase transition from UO₃ to U₂O₅, UO₂ and UO can occur by adding more U atoms. The DFT-calculated chemical potentials are shown as follows:

$$-9.77 \text{ eV} \le \Delta \mu_{\mathrm{U}}^{\mathrm{UO}_2} -1.19 \text{ eV}$$
 (s3)

$$-5.54 \text{ eV} \le \Delta \mu_0^{002} -1.25 \text{ eV}$$
 (s4)

which can be fitted as $\Delta H_{UO_k}^{GGA+U} - \Delta H_{UO_k}^{Exp.} = -0.002k - 0.024$ according to the dispersed points in Fig. S2(a). The corrected range of chemical potential of UO₂ is limited by the critical points of intersection among the chemical potential lines of UO₂, U₂O₅ and UO in Fig. S1(c), which can be expressed as

$$-11.07 \text{ eV} \le \Delta \mu_{\mathrm{U}}^{\mathrm{UO2}} - 0.90 \text{ eV}$$
 (s5)

$$-5.80 \text{ eV} \le \Delta \mu_0^{\text{UO}_2} - 0.71 \text{ eV}$$
 (s6)

From a comparison of chemical potential ranges, the difference assessed by experimental μ_U and μ_O values in Fig. S1(a) is smaller than that by the corrected

ones in Fig. S1(c). Additionally, a large deviation between the dispersed points and the fitting line is presented in Fig. S2(a). As a result, the experimental formation energy derived chemical potential is used as a consequence for UO₂, and the variation ranges of $\mu_{\rm U}^{\rm UO_2}$ and $\mu_{\rm O}^{\rm UO_2}$ are separately determined as follows:

$$-14.54 \text{ eV} < \mu_{\text{U}}^{\text{EO}_2} - 6.87 \text{ eV}$$
 (s7)

$$-10.55 \text{ eV} \le \mu_0^{UO_2} - 6.71 \text{ eV}$$
 (s8)

Here, the values of $\mu_U^{UO_2}$ and $\mu_O^{UO_2}$ in U-rich UO₂ are equal to -6.87 eV and -10.55 eV, respectively. While for the O-rich condition, the values of $\mu_U^{UO_2}$ and $\mu_O^{UO_2}$ are calculated as -14.54 eV and -6.71 eV, respectively. The same approach in estimating chemical potential using experimental formation enthalpies ¹⁶⁻¹⁸ is applied for UN to assess the variation range of μ_U^{UN} and μ_N^{UN} as follows:

$$-8.34 \text{ eV} \le \mu_{\text{U}}^{\text{UN}} - 6.87 \text{ eV}$$
(s9)

$$-11.28 \text{ eV} \le \beta_{N}^{UN} - 9.81 \text{ eV}$$
 (s10)

It is worthy to note that the values of μ_U^{UN} and μ_N^{UN} for U-rich UN are -6.87 eV and -11.28 eV, respectively, and in the case of N-rich UN, the values of μ_U^{UN} and μ_N^{UN} equal to be -8.34 eV and -9.81 eV, respectively. A comparison of experimentally measured, DFT-calculated and corrected chemical potentials of UN shown in Figs. S2(d)-(f) indicates that the ranges of corrected μ_U^{UN} and μ_N^{UN} are much close to those of the experimentally measured ones. Therefore, the experimentally measured chemical potentials of U and N are employed in current work.

As for the chemical potential of hyper-stoichiometric U_3Si_2 , the U-rich U_3Si_2 in equilibrium with γ -U₃Si and the Si-rich U_3Si_2 in equilibrium with U_3Si_5 are taken into account. By referring to the experimental formation enthalpies of U_xSi_y ¹⁵, the ferromagnetism property and $U_{eff} = 1.0$ eV are used in the formation enthalpy calculation of U_xSi_y . The variation ranges of corrected chemical potentials $\mu_U^{U_3Si_2}$ and $\mu_{Si}^{U_3Si_2}$ are predicted as follows:

$$-7.02 \text{ eV} < \mu_{\text{U}}^{\text{U}_{3}\text{Si2}} -7.00 \text{ eV}$$
(s11)

$$-6.08 \text{ eV} < \beta_{\text{Si}}^{U_3 \text{Si}_2} - 6.05 \text{ eV}$$
(s12)

In the case of U-rich U₃Si₂, $\mu_U^{U_3Si_2} = -7.00$ eV and $\mu_{Si}^{U_3Si_2} = -6.08$ eV are separately obtained, and for Si-rich U₃Si₂, $\mu_U^{U_3Si_2} = -7.02$ eV and $\mu_{Si}^{U_3Si_2} = -6.05$ eV are identified, respectively. In Figs. 2(g)-(i), it is obviously found that the variation in $\mu_U^{U_3Si_2}$ and $\mu_{Si}^{U_3Si_2}$ are nearly the same as those of the DFT-calculated ones. The corrected GGA+*U* approximation could evidently reduce the range of chemical potential in U₃Si₂. Hence the corrected chemical potentials of U and Si are applied. As is known, the phases in binary U-Si phase diagram are really complex for DFT calculations, and more formation enthalpies of U_xSi_y species should be experimentally measured in the future.



Fig. S1 (a)-(c) $\Delta\mu_0$ of the U-O phases, (d)-(f) $\Delta\mu_N$ of the U-N phases and (g)-(i) $\Delta\mu_{Si}$ of the U-Si phases as a function of $\Delta\mu_U$. The formation enthalpies are evaluated using (a)(d)(g) the experimental data, (b)(e)(h) the DFT-calculated data and (c)(f)(i) the data corrected from the Jain's approach. The superscript ^{a, b, c} respectively indicate the results from the work of Katsura and Serizawa ¹⁶, Katsura and Sano ¹⁷, and Hiroaki ¹⁸.



Fig. S2 Difference between the calculated and experimental formation enthalpies as a function of the ratio of X (X = O, N, Si) to U in (a) UO_2 , (b) UN and (c) U_3Si_2 . The red lines are fitted via least-squares to the binary U-O, U-N and U-Si systems, respectively. *k* represents the ratio of X atom number to U atom number.

Supplementary Note 2: Verification of formation energies

Most of the formation energy values of point defects in UO₂, UN and U₃Si₂ agree well with the reference ones. In the standard reference, the formation energy values of defects in UO₂ are ordered as: $V_U > V_O > U_i > O_i$, while Gupta et al. ¹ found that the formation energy of mono-interstitial U is the highest one among the investigated point defects (i.e., $U_i > V_U > V_O > O_i$). In the U-rich environment of UO₂, the predicted and other DFT-calculated values of formation energy decrease in the order of $V_U > O_i > U_i > V_0^2$, and our energetic trend of $U_i > V_U > O_i > V_0$ in O-rich UO₂ confirms well with that in Ref. ².

As for UN in the standard reference, the order of current formation energies of $V_{\rm U}$, $V_{\rm N}$, U_i and N_i change to $V_{\rm U} > U_i > V_{\rm N} > N_i$, which substantially differs from those of $U_i > V_{\rm N} > V_{\rm U} > N_i$ ⁴. Such different orders of $V_{\rm U}$ and U_i in not only UO₂ but also UN come from different choice of $U_{\rm eff}$ for α -U. A slight difference in the E_f order of U-rich UN between our result (i.e., $V_{\rm U} > U_i > N_i > V_{\rm N}$) and other DFT-calculated data (i.e., $U_i > V_{\rm U} > N_i > V_{\rm N}$ ⁵) originates from the fact that our experimental data corrected chemical potentials shown in Fig. S1(i) are lower than the DFT derived ones used in Ref. ⁵. Meanwhile, the values of E_f in the N-rich case are in the order of $U_i > V_U > N_i > V_N$, in accordance with the overall trend predicted by Kocevski et al. ⁵.

Comparing the results of U_3Si_2 under the standard reference with those in Refs. ⁶ and ⁷, the evolution trend matches well with each other in the E_f^V order of VU-1 > VSi > VU-2 and the E_f^i order of $U_i > Si_i$. It should note that our interstitial U (U_i) inserted at the U octahedron site (i.e., 2b site ^{19, 20}) remains in the octahedral site after relaxation. The formation energies of mono-vacancy and mono-interstitial U and Si are ordered as VU-1 > VU-2 > VSi and $U_i > Si_i$, respectively, in not only U-rich but also Si-rich U_3Si_2 samples, which slightly deviate from the prediction ⁷. It can be explained by different magnetic considerations in these work.

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