# Supplementary Material for "Ideal topological Weyl complex phonons in two Dimensions"

Wei-Wang Yu, Ying Liu*, Zeqing He, Lirong Wang, Xiaoming Zhang, and Guodong Liu*
*School of Materials Science and Engineering, Hebei University of Technology, Tianjin 300130, China.*

## S1. The distinction between 2D Dirac and Weyl points

It is generally accepted that a Weyl point is a point degeneracy between two bands in a three-dimensional (3D) band structure, with linear dispersion in all three dimensions in its vicinity. One should admit that the most feature of Weyl points is that they host a linear dispersion, characterized by chirality. It could be generally described by the effective model given in the following:

$$H = v\, k \cdot \sigma,$$

In our case, the LNPs also can be characterized by such an effective model, but squeezing in 2D. However, it still has a linear dispersion and chirality. Analogous to the 3D Weyl point, the 2D one is still characterized by quantized Berry phase or topological invariants, like the ones we have defined in main text.

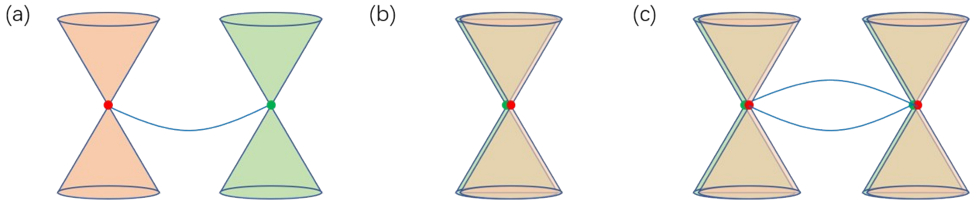
On the other hand, a conventional Dirac point is a composition of two Weyl points of opposite chirality, which can be generally described by,

$$H = \begin{pmatrix} v\, k \cdot \sigma & 0 \\ 0 & -v\, k \cdot \sigma \end{pmatrix}.$$

Such forms can be extended to 2D cases, with squeezing in dimensionality. Consequently, the most features of Weyl points or Dirac points still persist in 2D systems, and even in 2D phononic systems.

For concreteness, the hall marks of Weyl points are Fermi-arc-like edge states, connecting Weyl points of opposite chirality. In contrast, the bulk-surface correspondence of 2D Dirac points depends on the symmetry. For example, the Dirac points protected by the inversion and time-reversal symmetry has no Fermi arc-like edge states at the boundary. Because, the Berry curvature disappear throughout the whole BZ under the PT symmetry [as shown in F1. (b)]. In comparison, Dirac points without the inversion symmetry are characterized by nonvanishing Berry curvature, thus giving rise to two Fermi arcs, as shown in F1. (c).

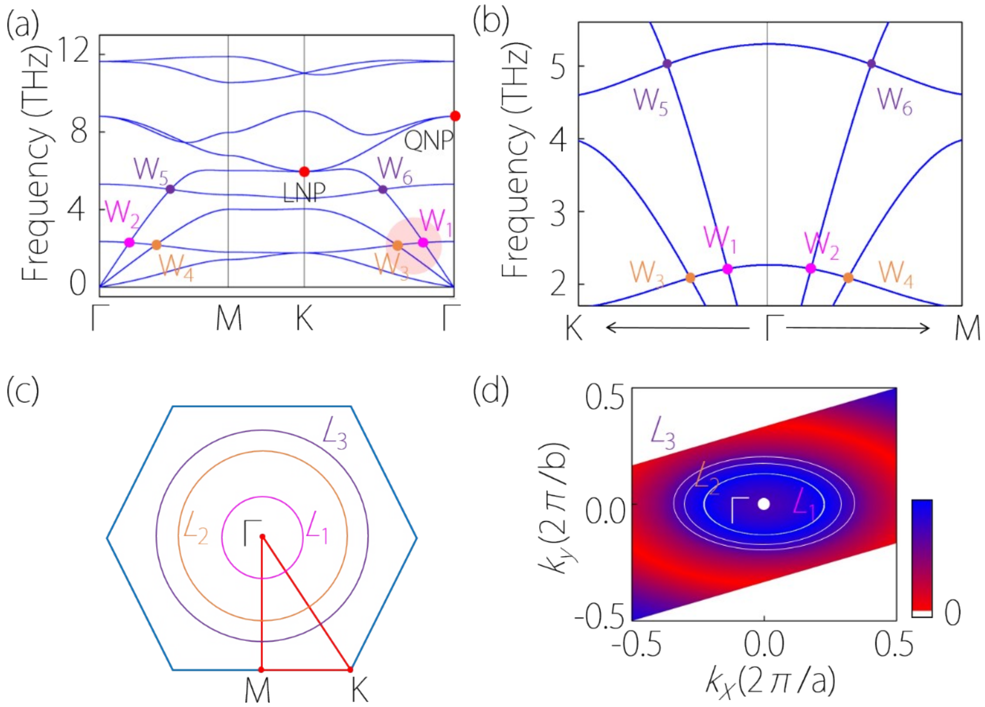
It is well known that the Weyl/Dirac phonons are the direct extensions of concepts from electronic systems, they are expected to exhibit the same features as their electronic ones.




F1. (a) A pair of Weyl points with opposite chirality. (b) Dirac point with the inversion symmetry, (c) without the inversion symmetry

## S2. The topological properties of other phonon band crossings

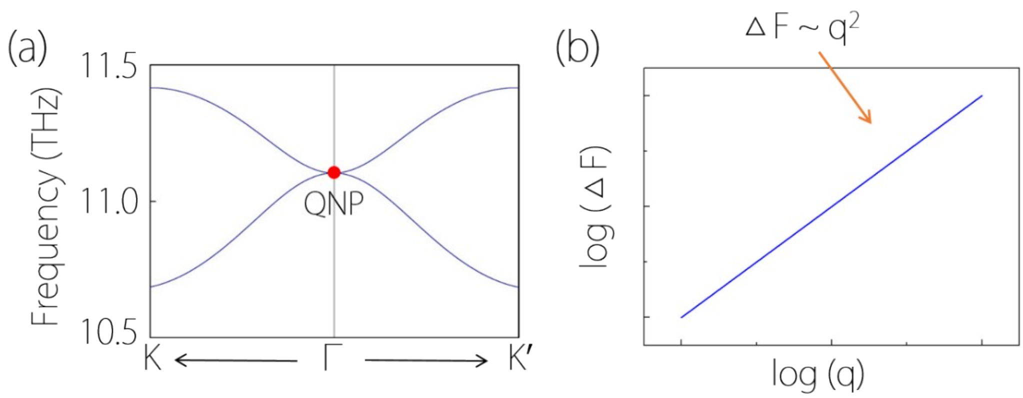
In addition to the ideal Weyl complex, we find that $Cu_2Si$ has quite interesting topological states in other frequency regions. As shown in F2. (a), in the range of 6 - 8 THz, it has another Weyl complex, which is composed of three bands. In the range 0 - 6 THz you will find three concentric rings around the $\Gamma$ point, which will be analyzed below. In F2. (b), we plot an enlarged image of the phonon dispersion K - $\Gamma$ - M for the concentric rings. A schematic diagram of the position of this concentric ring in the Brillouin zone is shown in F2. (c). In addition, in order to clearly display the shape of the concentric rings, we also calculated the corresponding 2-dimensional frequency dispersion diagram of the concentric ring as shown in F2. (d). The nodal rings are centered at $\Gamma$ point due to the presence of PT symmetry.




F2. (a) Phonon spectrum of $Cu_2Si$. (b) Enlarged image of K - Γ - M phonon dispersion, (c) Illustration of three nodal rings in the Brillouin zone. (d) Two-dimensional dispersion diagram of phonon concentric rings.

## S3. The order of the quadratic Weyl point

In order to further verify the dispersion of the QNP, we draw the enlarged graph of the quadratic node, and then process its data. We take the logarithmic derivative of the frequency (F) - momentum (q). F3. (b) shows that △F and $q^2$ are linearly proportional, that is to say, the frequency is a quadratic function of momentum $q$. Therefore, the QNP indeed has a quadratic dispersion.



F3. (a) Enlarged view of the band dispersion around $\Gamma$ point of phonon spectrum of $Cu_2Si$. (b)The logarithmic derivative of the frequency (F) - momentum (q).