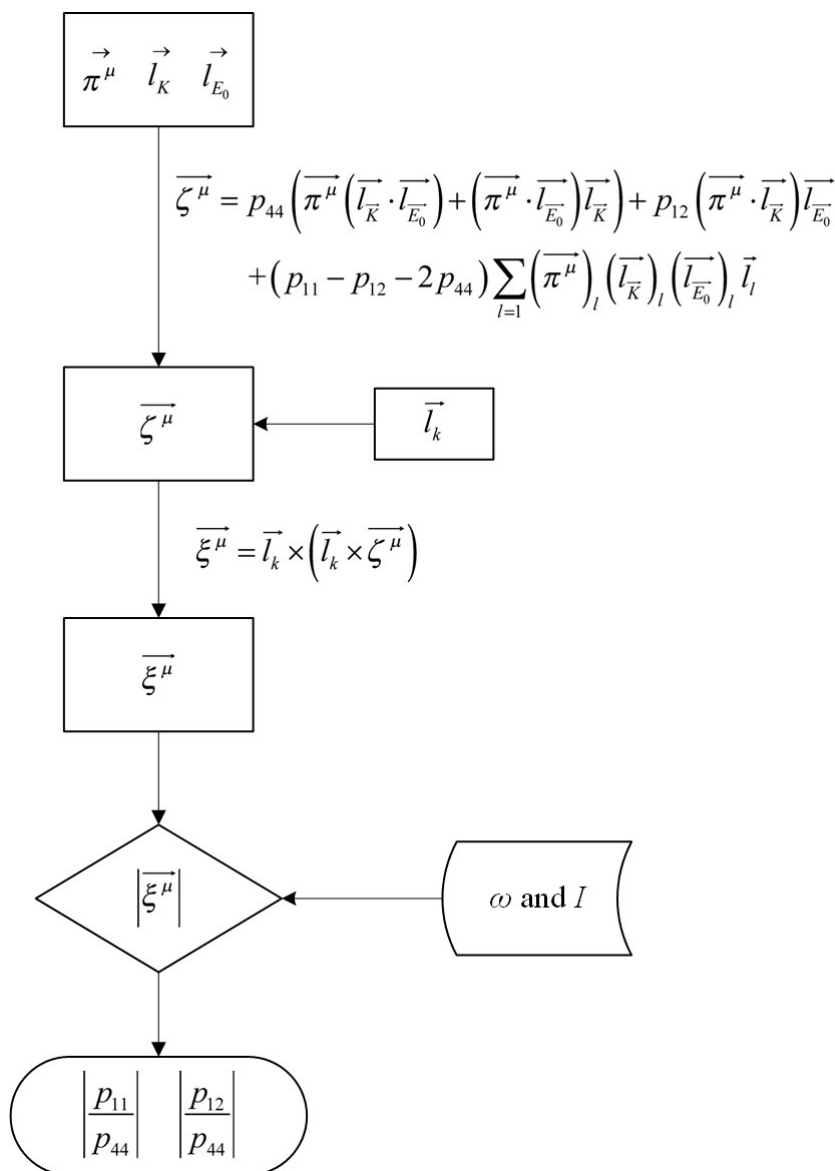


## Supplemental Information

Deduction of the expressions of some key vectors for the solution of the Brillouin scattering factors is summarized as following. The whole flowchart is shown in followed figure.



The flowchart for the deduction of Brillouin scattering factor.

The first thing to draw attention to is that all followed expressions of vectors are based on the crystallographic orientations and the physical properties of MgO single crystal. As for the deduction for CaF<sub>2</sub>, the only difference is the values of physical properties.

(1). Determination of the vibrational vector  $\vec{\pi}^\mu$ , acoustic wave propagation vector  $\vec{l}_K$ , incident light polarization vector  $\vec{l}_{E_0}$ , and the scattered light propagation vector  $\vec{l}_k$ , for V-polarized and H-polarized incidents, respectively.

V-polarized incident			
	LA	TA1	TA2
$\vec{\pi}^\mu$	[100]	[001]	[010]
$\vec{l}_K$	[100]	[100]	[100]
$\vec{l}_{E_0}$	[001]	[001]	[001]
$\vec{l}_k$	[-0.284,0.959,0]	[-0.284,0.959,0]	[-0.284,0.959,0]
H-polarized incident			
	LA	TA1	TA2
$\vec{\pi}^\mu$	[100]	[001]	[010]
$\vec{l}_K$	[100]	[100]	[100]
$\vec{l}_{E_0}$	[-0.959,0.284,0]	[-0.959,0.284,0]	[-0.959,0.284,0]
$\vec{l}_k$	[-0.284,0.959,0]	[-0.284,0.959,0]	[-0.284,0.959,0]

(2). Determination of the intermediate vector  $\vec{\zeta}^\mu$  based on the followed equation for V-polarized and H-polarized incidents, respectively.

$$\zeta^\mu = \frac{p_{44}}{2} \left( \pi^\mu (l_K \cdot l_{E_0}) \right) + \left( \pi^\mu \cdot l_{E_0} \right) l_K + p_{12} \left( \pi^\mu \cdot l_K \right) l_{E_0} + (p_{11} - p_{12} - p_{44}) \sum_{l=1}^3 \left( \pi^\mu \right)_l (l_K)_l (l_{E_0})_l l_l$$

V-polarized incident			
	LA	TA1	TA2
$\pi^\mu (l_K \cdot l_{E_0})$	0	0	0

$(\vec{\omega} \cdot \vec{l}_{E_0})_K$	0	[100]	0
$(\vec{\omega} \cdot \vec{l}_K)_{E_0}$	[001]	0	0
$\sum_{l=1}^3 (\vec{\omega} \cdot \vec{l}_l) (\vec{\omega} \cdot \vec{l}_{E_0})_l$	0	0	0
$\vec{\zeta}^\mu$	[0,0, $p_{12}$ ]	[ $p_{44}$ ,0,0]	0
H-polarized incident			
	LA	TA1	TA2
$\vec{\omega} \cdot \vec{l}_K$	[-0.959,0,0]	[0,0,-0.959]	[0,-0.959,0]
$(\vec{\omega} \cdot \vec{l}_{E_0})_K$	[-0.959,0,0]	0	[0.284,0,0]
$(\vec{\omega} \cdot \vec{l}_K)_{E_0}$	[-0.959,0.284,0]	0	0
$\sum_{l=1}^3 (\vec{\omega} \cdot \vec{l}_l) (\vec{\omega} \cdot \vec{l}_{E_0})_l$	[-0.959,0,0]	0	0
$\vec{\zeta}^\mu$	[-0.959 $p_{11}$ ,0.284 $p_{12}$ ,0]	[0,0,-0.959 $p_{44}$ ]	[0.284 $p_{44}$ ,-0.959 $p_{44}$ ,0]

(3). Determination of the Brillouin scattering vector  $\vec{\xi}^\mu = \vec{l}_k \times (\vec{l}_k \times \vec{\zeta}^\mu)$  for V-polarized and H-polarized incidents, respectively.

V-polarized incident			
	LA	TA1	TA2
$\vec{l}_k \times \vec{\zeta}^\mu$	$p_{12}[0.959,0.284,0]$	$p_{44}[0,0,-0.959]$	0
$\vec{\xi}^\mu = \vec{l}_k \times (\vec{l}_k \times \vec{\zeta}^\mu)$	$p_{12}[0,0,-1]$	$p_{44}[-0.959^2,-0.959 \cdot 0.284,0]$	0
H-polarized incident			
	LA	TA1	TA2
$\vec{l}_k \times \vec{\zeta}^\mu$	$[0,0,0.959^2 p_{11} - 0.284^2 p_{12}]$	$p_{44}[-0.959^2, -0.959 \cdot 0.284, 0]$	0
$\vec{\xi}^\mu = \vec{l}_k \times (\vec{l}_k \times \vec{\zeta}^\mu)$	$[0.959^3 p_{11} - 0.959 \cdot 0.284^2 p_{12},$ $0.284 \cdot 0.959^2 p_{11} - 0.284^3 p_{12}, 0]$	$p_{44}[0,0,0.284^2 \cdot 0.959 + 0.959^3]$	0

(4). Final determination of the magnitude of Brillouin scattering vector  $|\vec{s}^\mu|$  for V-polarized and H-polarized incidents, respectively.

Polarization of incident	LA	TA1	TA2
V-polarized	$ p_{12} $	$0.959 p_{44} $	0
H-polarized	$ 0.959^2 p_{11} - 0.284^2 p_{12} $	$0.959 p_{44} $	0