

## Supplementary Information

# Extension of Molecules with Inverted Singlet-Triplet Gap with Conjugated Branches to Alter the Oscillator Strength

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## Active space used for CASSCF

### Molecule 1

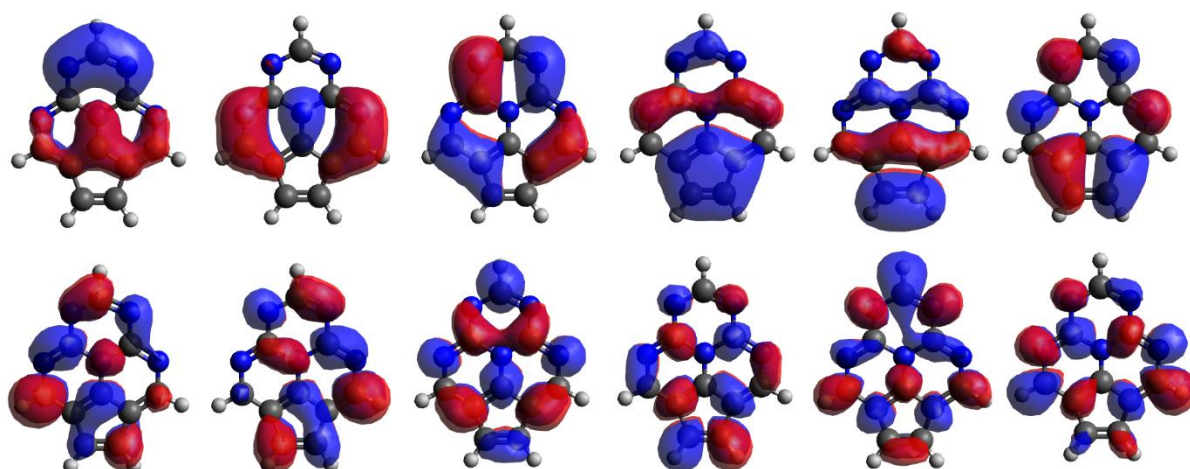


Figure S1. Active space used for Molecule 1

### Molecule 2

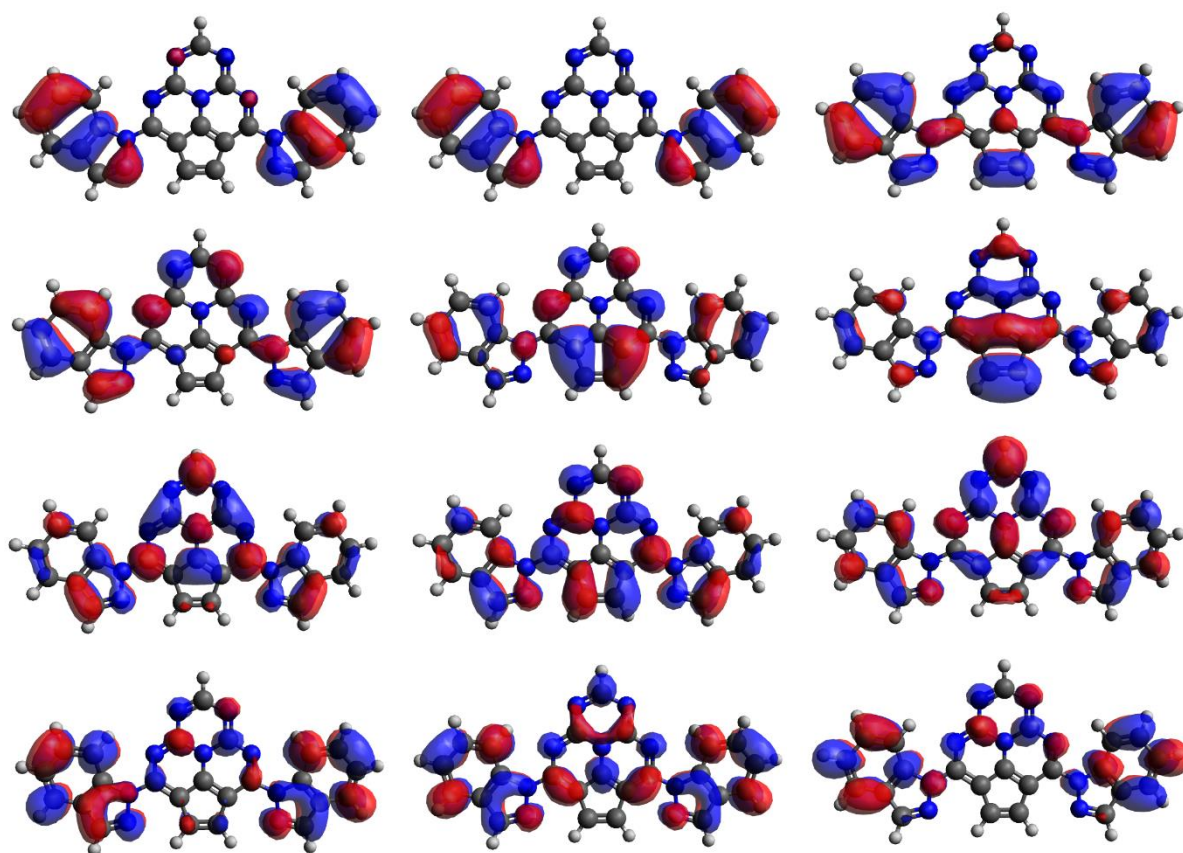


Figure S2. Active space used for Molecule 2

Molecule 3t

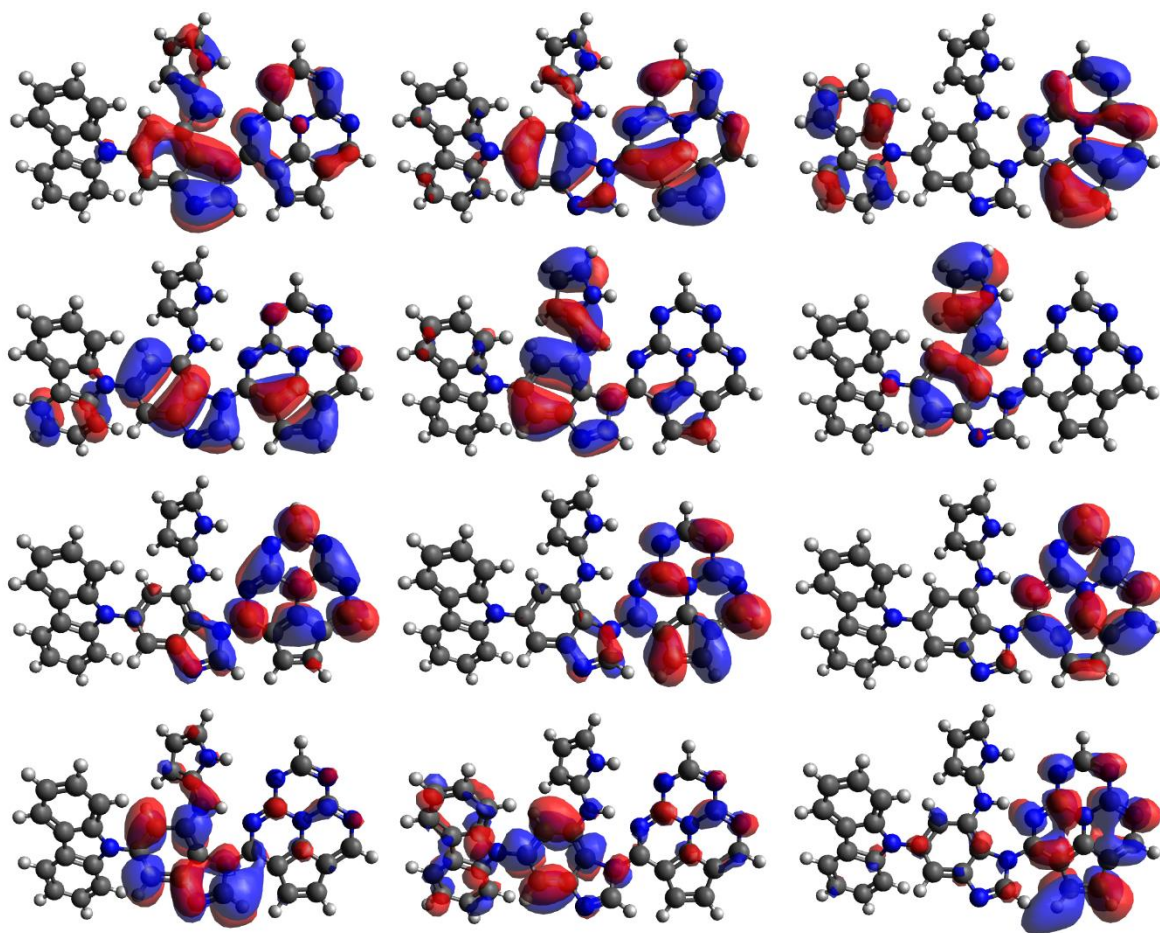


Figure S3. Active space used for Molecule 3t

Molecule 4

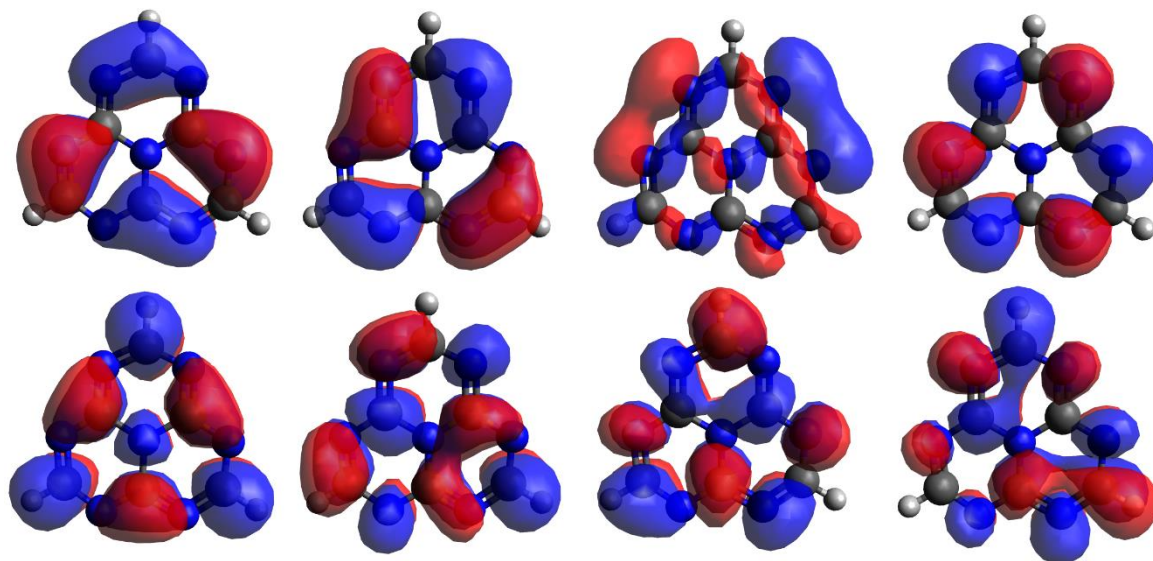


Figure S4. Active space used for the CASSCF (8,8) step of Molecule 4

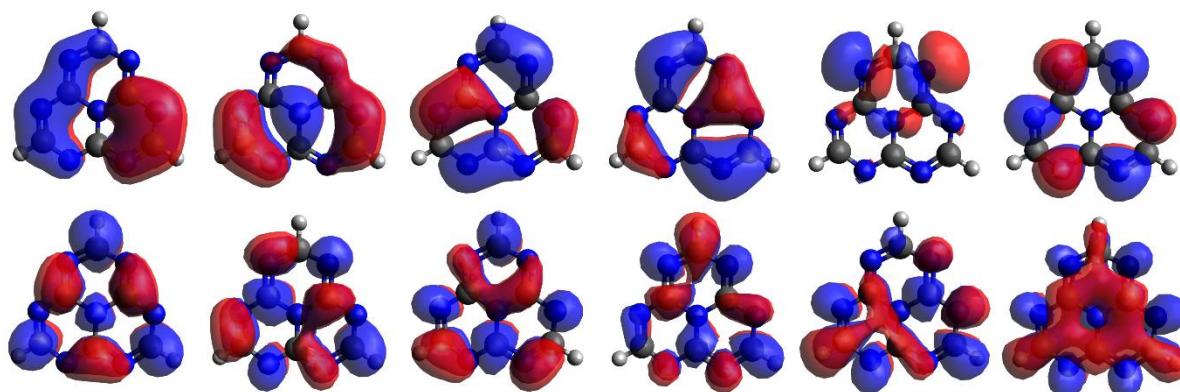


Figure S5. Active space used for the CASSCF (12,12) step of Molecule 4

Molecule 5t

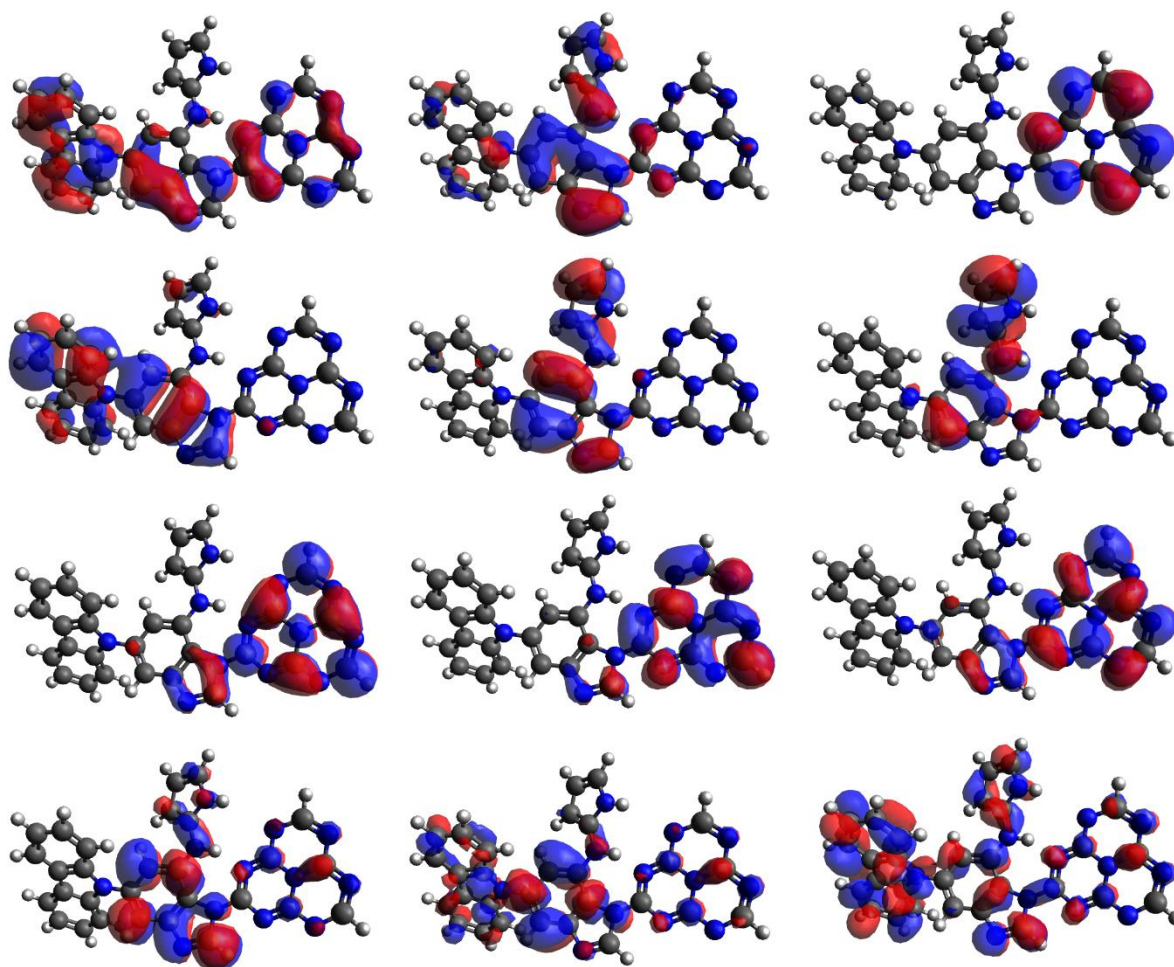


Figure S6. Active space used for Molecule 5t

### Molecule 6

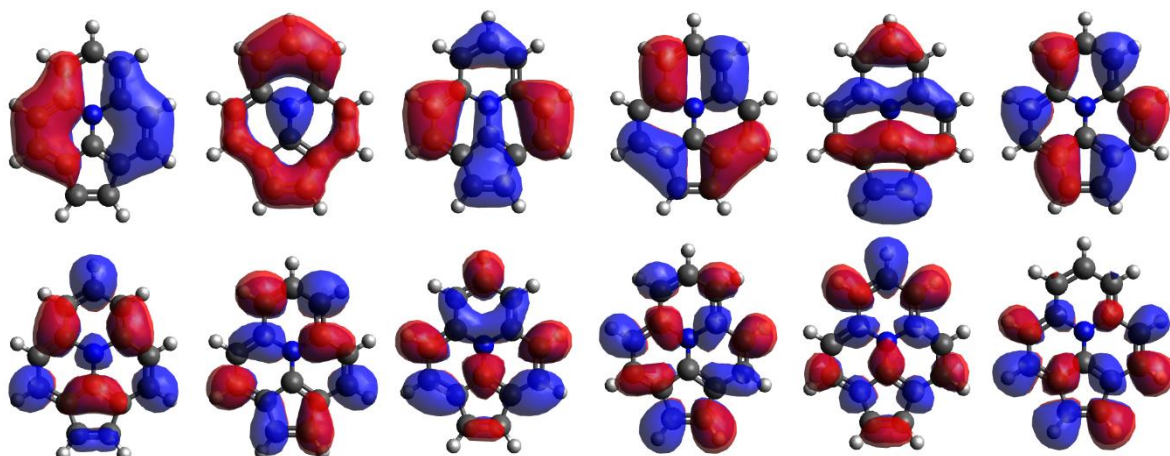


Figure S7. Active space used for Molecule 6

### Molecule 7t

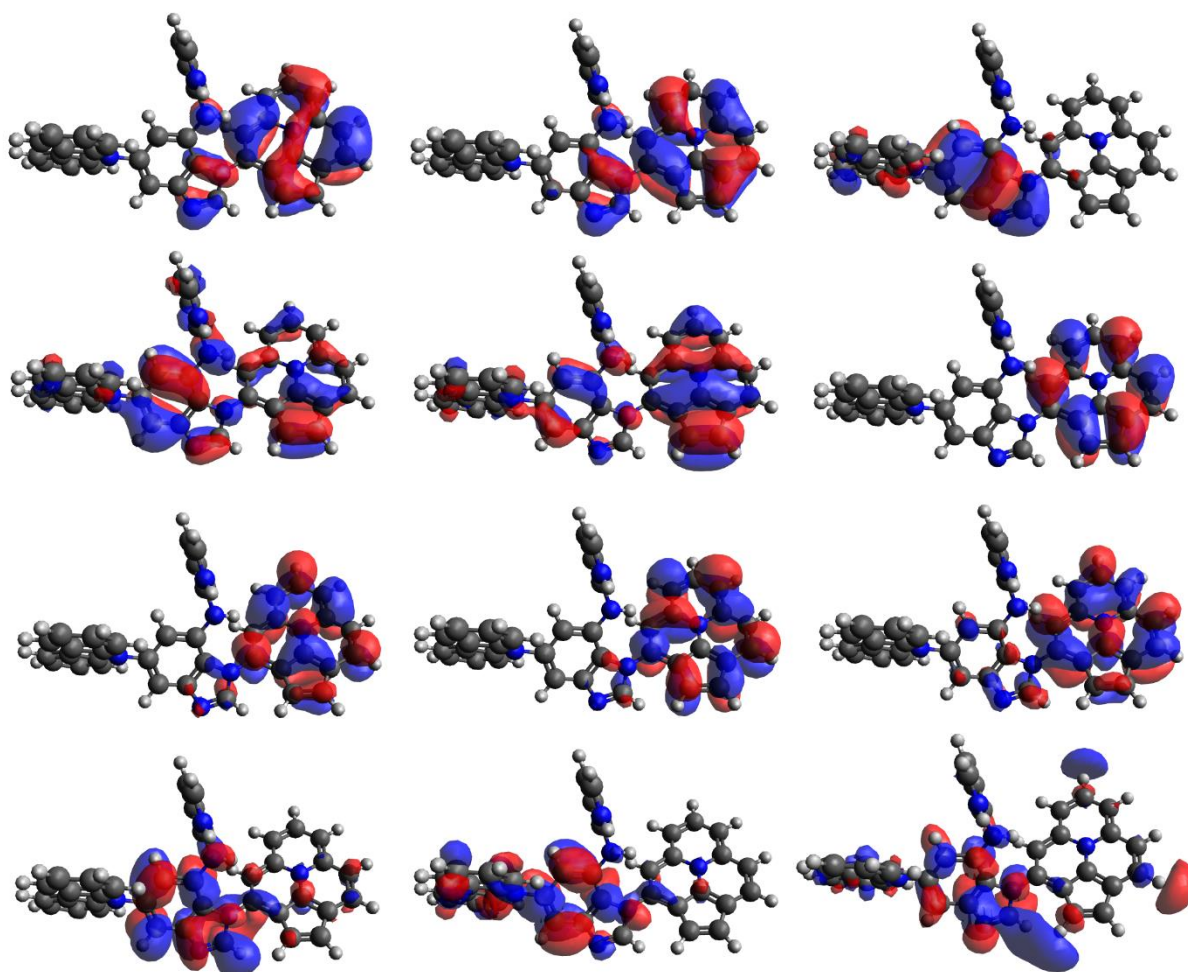


Figure S8. Active space used for Molecule 7t

## Interaction with 2UE DEC

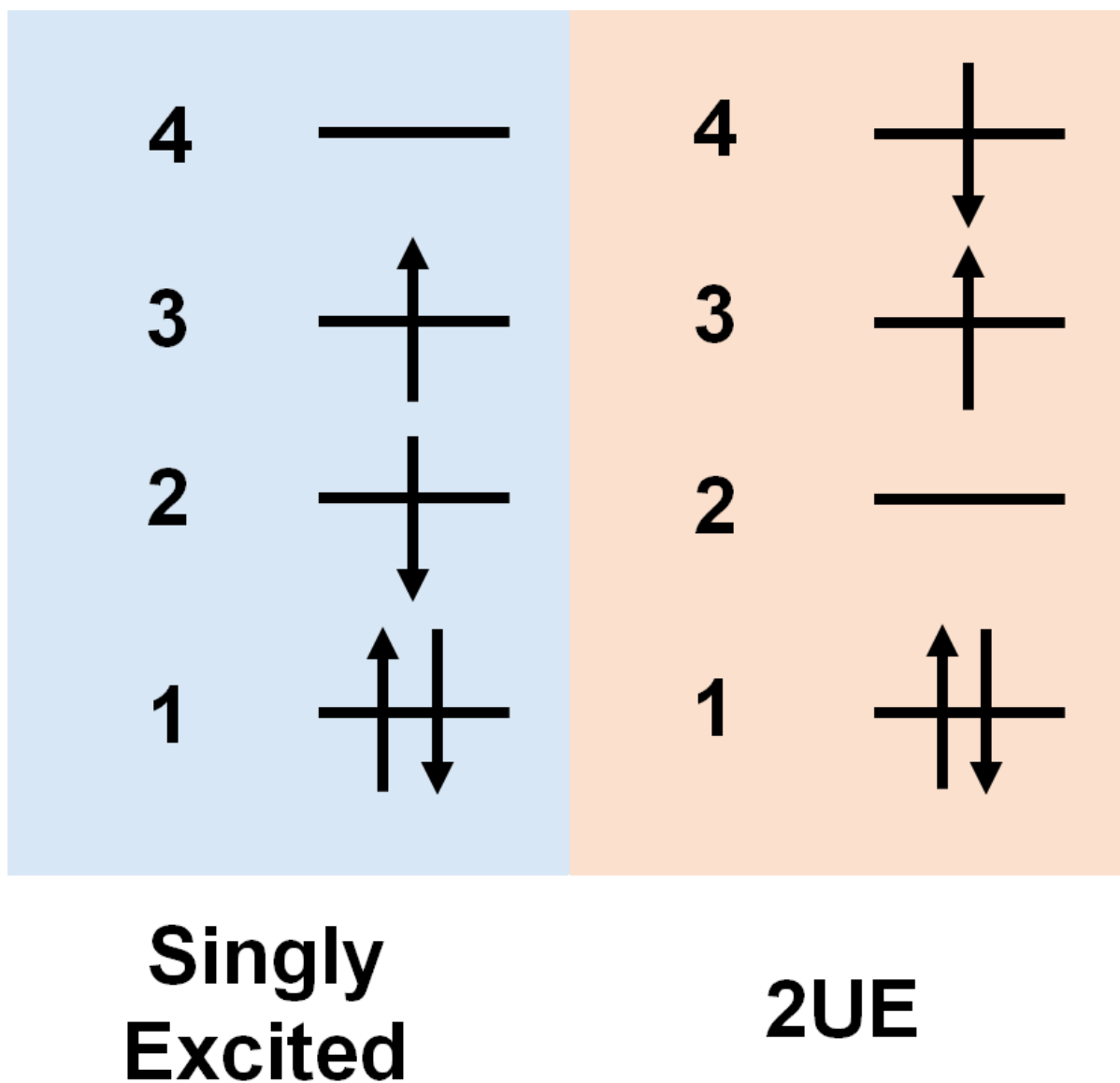


Figure S9. The electron configuration for the zeroth-order singly excited state with HOMO-LUMO excitation and the 2UE doubly excited state

The configuration state function (CSF) made with the configurations shown in Figure S10 is

$$|\psi_{Singly\ Excited}^{Singlet}\rangle = \frac{1}{\sqrt{2}}|\varphi_1^\alpha\varphi_1^\beta\varphi_2^\alpha\varphi_3^\beta\rangle - \frac{1}{\sqrt{2}}|\varphi_1^\alpha\varphi_1^\beta\varphi_2^\beta\varphi_3^\alpha\rangle$$

$$|\psi_{Singly\ Excited}^{Triplet}\rangle = \frac{1}{\sqrt{2}}|\varphi_1^\alpha\varphi_1^\beta\varphi_2^\alpha\varphi_3^\beta\rangle + \frac{1}{\sqrt{2}}|\varphi_1^\alpha\varphi_1^\beta\varphi_2^\beta\varphi_3^\alpha\rangle$$

$$|\psi_{2UE}^{Singlet}\rangle = \frac{1}{\sqrt{2}}|\varphi_1^\alpha\varphi_1^\beta\varphi_3^\alpha\varphi_4^\beta\rangle - \frac{1}{\sqrt{2}}|\varphi_1^\alpha\varphi_1^\beta\varphi_3^\beta\varphi_4^\alpha\rangle$$

$$|\psi_{2UE}^{Triplet}\rangle = \frac{1}{\sqrt{2}}|\varphi_1^\alpha\varphi_1^\beta\varphi_3^\alpha\varphi_4^\beta\rangle + \frac{1}{\sqrt{2}}|\varphi_1^\alpha\varphi_1^\beta\varphi_3^\beta\varphi_4^\alpha\rangle$$

The coupling term between each spin multiplicity is

$$\begin{aligned} \langle \psi_{Singly\ Excited}^{Singlet} | \hat{H} | \psi_{2UE}^{Singlet} \rangle \\ = \langle \varphi_2 | \hat{h} | \varphi_4 \rangle + 2\langle \varphi_1\varphi_2 | \varphi_1\varphi_4 \rangle - \langle \varphi_1\varphi_2 | \varphi_4\varphi_1 \rangle + \langle \varphi_3\varphi_2 | \varphi_3\varphi_4 \rangle \\ + \langle \varphi_2\varphi_3 | \varphi_3\varphi_4 \rangle \end{aligned}$$

$$\begin{aligned} \langle \psi_{Singly\ Excited}^{Triplet} | \hat{H} | \psi_{2UE}^{Triplet} \rangle \\ = -\langle \varphi_2 | \hat{h} | \varphi_4 \rangle - 2\langle \varphi_1\varphi_2 | \varphi_1\varphi_4 \rangle + \langle \varphi_1\varphi_2 | \varphi_4\varphi_1 \rangle - \langle \varphi_3\varphi_2 | \varphi_3\varphi_4 \rangle \\ + \langle \varphi_2\varphi_3 | \varphi_3\varphi_4 \rangle \end{aligned}$$

from which cannot be determined which coupling is stronger as two are not multiples of each other.



## Characters of S<sub>1</sub> and T<sub>1</sub> states from SOS-B2GP-PLYP21 functional

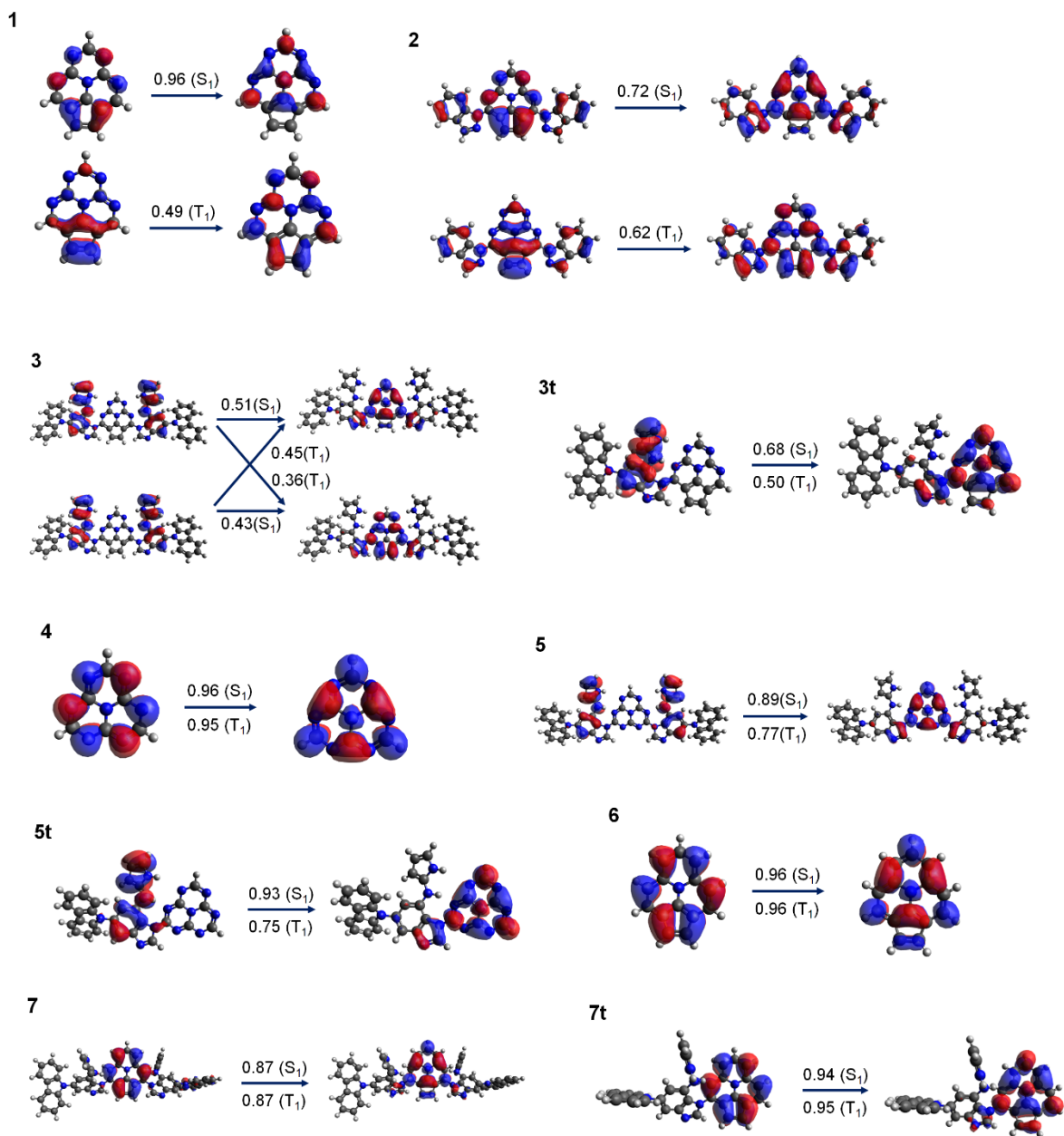


Figure S10. The characters of the molecules with the amplitudes of each transition, calculated with SOS-B2GP-PLYP21 functional

## Characters of $S_1$ and $T_1$ states from SCS-PBE-QIDH functional

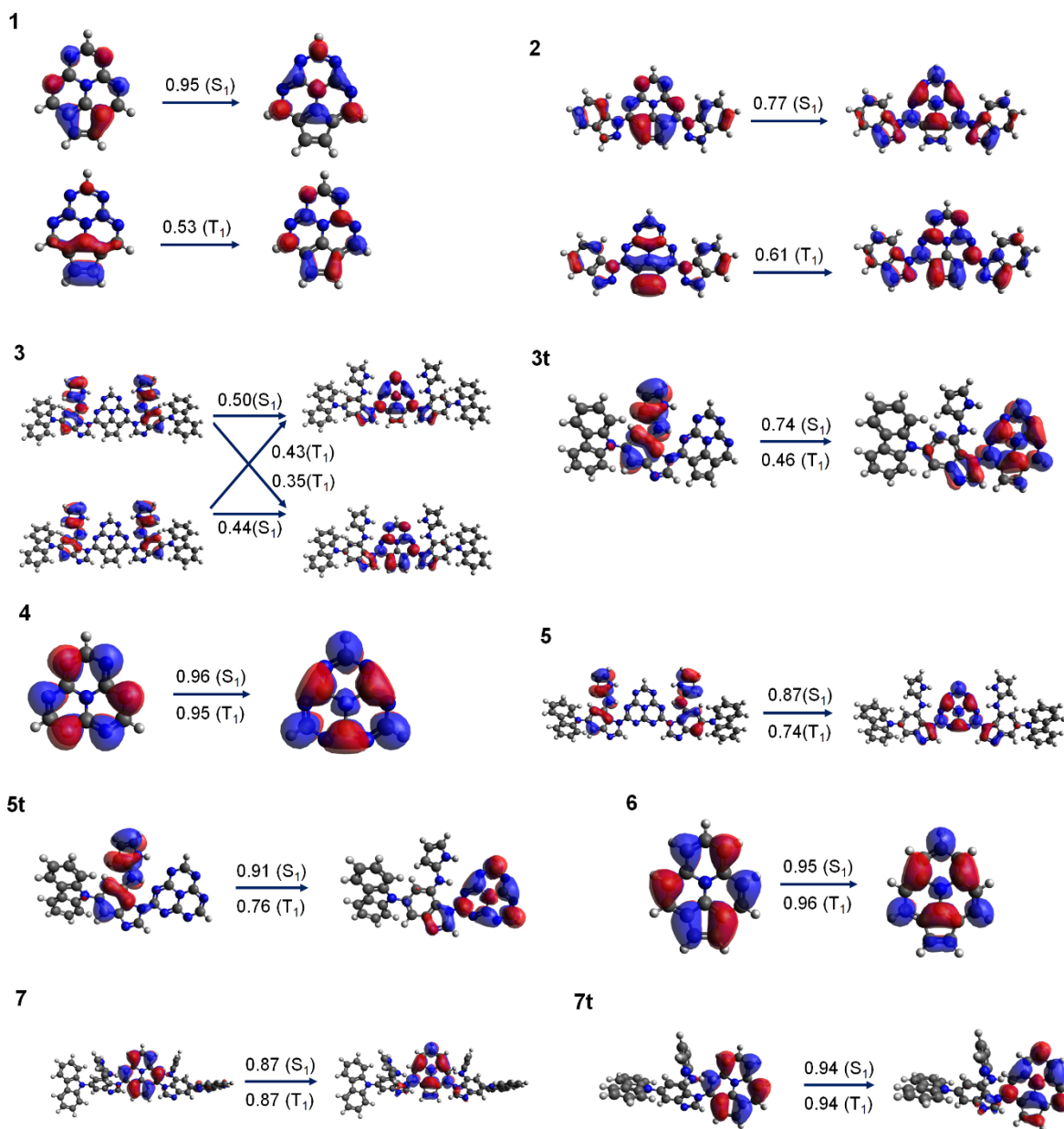


Figure S11. The characters of the molecules with the amplitudes of each transition, calculated with SCS-PBE-QIDH functional

### CT Distance ( $D_{CT}$ )

|           | SOS-B2GP-PLYP21       | SCS-PBE-QIDH          | SOS-RSX-QIDH          | CASSCF                |
|-----------|-----------------------|-----------------------|-----------------------|-----------------------|
| <b>1</b>  | 0.276                 | 0.268                 | 0.257                 | $2.55 \times 10^{-2}$ |
| <b>2</b>  | 0.179                 | 0.182                 | 0.182                 | 0.623                 |
| <b>3</b>  | 0.591                 | 0.574                 | 0.422                 | N/A                   |
| <b>3t</b> | 1.26                  | 1.23                  | 0.678                 | 1.53                  |
| <b>4</b>  | $6.89 \times 10^{-3}$ | $6.67 \times 10^{-3}$ | $6.62 \times 10^{-3}$ | $6.90 \times 10^{-2}$ |
| <b>5</b>  | 0.388                 | 0.381                 | 0.293                 | N/A                   |
| <b>5t</b> | 1.69                  | 1.66                  | 1.37                  | 2.09                  |
| <b>6</b>  | 0.434                 | 0.434                 | 0.430                 | $2.55 \times 10^{-2}$ |
| <b>7</b>  | $3.01 \times 10^{-2}$ | $3.10 \times 10^{-2}$ | $3.50 \times 10^{-2}$ | N/A                   |
| <b>7t</b> | 0.123                 | 0.123                 | 0.124                 | 0.105                 |

Table S1. The distance between the  $S_0$  and  $S_1$  state's barycenter of electron density. The unit of the distance is in picometers.

### Transition Dipole

|           | SOS-B2GP-PLYP21 | SCS-PBE-QIDH | SOS-RSX-QIDH | CASSCF |
|-----------|-----------------|--------------|--------------|--------|
| <b>1</b>  | 0.786           | 0.841        | 0.956        | 1.02   |
| <b>2</b>  | 2.69            | 2.87         | 3.07         | 2.97   |
| <b>3</b>  | 6.85            | 6.99         | 8.19         | N/A    |
| <b>3t</b> | 4.32            | 4.45         | 5.21         | 3.36   |
| <b>4</b>  | 0.00            | 0.00         | 0.00         | 0.200  |
| <b>5</b>  | 6.73            | 6.85         | 7.94         | N/A    |
| <b>5t</b> | 3.93            | 4.02         | 4.78         | 3.88   |
| <b>6</b>  | 0.474           | 0.489        | 0.505        | 0.504  |
| <b>7</b>  | 1.14            | 1.16         | 1.16         | N/A    |
| <b>7t</b> | 0.665           | 0.684        | 0.690        | 0.232  |

Table S2. The transition dipole of the  $S_0$ - $S_1$  transitions. The unit is in Debye.

## Scatter plot of $V_{DL}$ and Transition Dipole

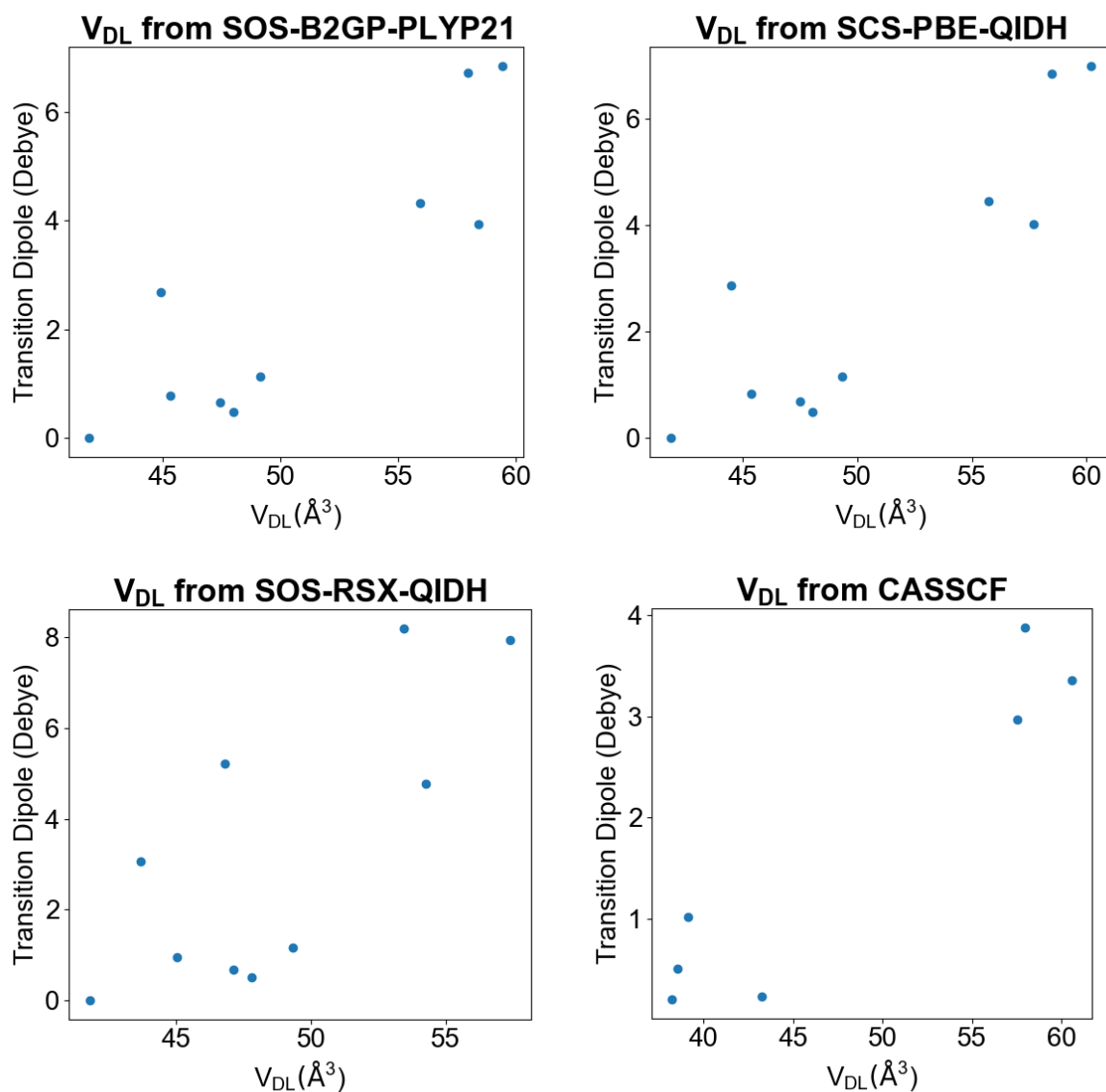


Figure S12. Scatter plot of  $V_{DL}$  and transition dipole from each functionals and the CASSCF calculation. CASSCF shows especially strong correlation with the four molecules with localized excitations (**1**, **4**, **6**, **7t**) having  $V_{DL}$  under  $45 \text{ \AA}^3$  and transition dipole of 1D or less and the three molecules with delocalized excitations (**2**, **3t**, **5t**) having  $V_{DL}$  above  $55 \text{ \AA}^3$  and transition dipole of 3D or more.

## Transition Density Matrix

The one-particle reduced transition density matrices (TDM) are provided in the basis of 12 optimized active orbitals from the CASSCF(12,12) calculations. The matrix elements are defined as  $T_{ij} = \langle S_1 | \hat{a}_i^\dagger \hat{a}_j | S_0 \rangle$ . Each block represents one row of TDM. The optimized orbitals are provided in the Supplementary\_Information.zip file in molden file format.

### Molecule 1

-2.64e-06 -1.26e-06 -1.85e-03 -3.67e-06 -4.00e-06 -1.17e-02 1.83e-06 5.75e-04 -3.14e-05  
1.06e-02 4.81e-06 8.11e-04

2.65e-06 -7.37e-06 8.16e-03 1.99e-05 -3.74e-06 4.37e-04 -8.73e-06 -2.00e-03 2.53e-05 3.38e-03  
-2.11e-05 6.72e-03

3.47e-03 9.10e-04 -2.04e-05 1.17e-02 2.69e-03 -8.19e-05 2.10e-02 -2.01e-04 -6.30e-02 -  
6.82e-05 -2.69e-03 -1.45e-05

2.55e-07 6.27e-06 -3.56e-03 2.09e-07 3.19e-05 3.39e-02 3.14e-04 5.14e-02 -1.39e-04 3.97e-02  
4.44e-06 1.09e-02

-1.07e-06 -1.63e-05 1.08e-02 4.51e-05 -2.00e-05 -6.54e-02 -3.60e-04 -7.44e-02 -6.36e-05 -  
1.96e-02 2.24e-05 8.95e-03

-6.71e-03 -1.79e-03 -2.55e-05 8.25e-03 -2.58e-02 2.53e-06 5.63e-02 -2.85e-04 -1.60e-02  
1.44e-05 1.16e-02 -4.18e-06

-1.03e-04 -9.14e-05 -6.05e-03 6.49e-04 -6.38e-04 1.19e+00 -4.64e-04 -7.70e-02 -1.76e-06  
8.21e-02 -4.44e-05 -3.25e-02

-8.47e-03 -2.33e-02 -1.84e-04 1.22e-01 -1.91e-01 -6.10e-03 -1.74e-02 4.93e-04 -5.04e-03 -  
4.19e-04 -5.73e-03 1.58e-04

-4.16e-05 2.08e-05 -1.14e-01 -2.25e-04 -1.17e-04 1.95e-01 -2.84e-05 -3.78e-03 5.60e-06  
1.67e-02 -1.93e-06 -1.24e-02

1.05e-02 3.25e-03 -8.46e-05 5.43e-02 -5.82e-02 8.93e-05 1.75e-03 -1.31e-05 1.09e-03 2.04e-06  
-1.17e-03 -5.05e-06

7.75e-06 -2.49e-05 -1.92e-02 -2.05e-05 5.59e-05 6.99e-02 -6.79e-06 -1.98e-03 3.62e-06  
7.56e-03 -6.48e-07 -8.62e-04

4.18e-03 1.77e-02 1.20e-05 1.79e-04 2.37e-02 -3.76e-05 -6.86e-04 -4.80e-07 3.51e-03 -6.31e-06  
-1.19e-03 1.09e-05

### Molecule 2

-1.66e-05 1.41e-03 -2.15e-06 6.46e-04 7.81e-03 -1.45e-05 -2.13e-03 -1.17e-05 2.33e-03 -  
5.68e-05 -5.31e-07 4.35e-04

1.06e-03 1.44e-05 4.77e-04 1.88e-06 4.28e-05 1.47e-03 -2.42e-06 2.08e-03 5.62e-05 2.40e-03  
-4.58e-04 3.21e-06

-8.36e-08 3.38e-05 -8.60e-07 -1.20e-03 -7.69e-03 -9.70e-06 4.19e-03 -9.63e-06 2.47e-04 -  
3.77e-06 -9.23e-06 4.13e-03

-3.95e-04 -1.84e-06 -1.50e-05 4.27e-06 1.90e-05 3.01e-03 2.52e-05 6.93e-03 -1.17e-05 -  
6.01e-04 3.33e-03 5.61e-06

-5.19e-03 -2.79e-05 7.96e-03 -4.97e-06 5.29e-05 6.61e-02 5.43e-05 8.37e-03 -1.22e-05 -  
1.02e-03 -6.73e-03 -3.26e-05

2.18e-05 -2.67e-03 -1.22e-05 -7.94e-03 -6.08e-02 -4.73e-05 1.37e-02 -4.35e-05 1.95e-03 -  
3.82e-05 6.84e-06 -1.71e-03

6.73e-02 5.57e-04 -1.15e-01 5.04e-04 1.96e-03 -1.16e+00 -2.67e-04 -8.36e-02 1.57e-04 8.84e-  
03 2.45e-02 7.21e-05

-6.02e-04 4.29e-02 6.08e-04 9.87e-02 6.30e-01 5.11e-03 2.10e-02 2.63e-04 8.78e-04 -5.14e-  
05 -8.37e-05 -4.88e-03

1.15e-02 5.79e-05 -2.17e-02 1.74e-05 -1.09e-03 -8.77e-02 -5.01e-05 -6.47e-03 2.41e-05  
1.08e-03 1.66e-03 1.62e-05

-2.12e-04 -4.06e-04 4.19e-04 3.09e-04 -6.29e-02 1.62e-03 -1.38e-03 1.27e-04 3.48e-04 -  
2.67e-05 -3.32e-05 6.06e-04

3.61e-05 1.07e-02 -1.72e-04 1.17e-02 -9.74e-02 -6.18e-04 -2.17e-03 -3.18e-05 -2.24e-04  
6.15e-06 3.23e-06 1.62e-03

-3.13e-02 -1.33e-04 4.71e-02 -2.67e-05 -2.66e-04 1.54e-01 3.73e-05 1.09e-02 -1.63e-05 -  
9.88e-04 -2.29e-03 -2.82e-06

### Molecule 3t

1.58e-04 2.15e-03 6.77e-04 -1.99e-03 2.13e-04 3.39e-03 -4.16e-03 -2.21e-03 -5.02e-03 4.58e-  
03 -7.62e-03 4.82e-03

-4.32e-04 2.98e-03 1.64e-03 -8.83e-04 2.00e-03 7.61e-03 1.96e-03 1.09e-03 -7.08e-03 -1.35e-  
02 2.87e-03 -2.52e-03

2.20e-04 4.89e-04 -3.51e-04 7.78e-04 -9.49e-04 -2.01e-03 1.35e-03 9.70e-04 -8.84e-03 -  
2.34e-02 4.02e-03 -2.80e-03

-1.17e-03 1.68e-04 2.65e-04 -5.38e-03 1.02e-03 4.40e-03 -2.74e-03 3.88e-03 4.03e-03 5.02e-  
04 -4.61e-03 -8.76e-04

-1.83e-03 -3.64e-03 2.50e-04 -3.55e-03 -2.21e-03 -1.67e-02 -1.60e-03 1.70e-03 -6.35e-03  
1.81e-03 -1.17e-03 -2.67e-03

1.88e-03 -3.97e-03 -3.35e-04 1.39e-03 -5.77e-03 -1.74e-02 -5.72e-03 -1.89e-03 4.15e-03  
2.05e-03 -3.56e-03 2.80e-03

-3.08e-02 1.47e-01 -7.06e-02 -5.33e-02 2.94e-01 1.11e+00 6.38e-03 1.73e-02 -4.01e-03 3.18e-  
03 2.09e-02 3.76e-03

-2.00e-02 1.02e-01 -3.76e-02 9.69e-02 1.44e-01 6.34e-01 -1.79e-03 2.08e-02 -4.44e-03 2.48e-03 1.33e-02 7.19e-03

-1.85e-02 1.23e-02 -1.61e-02 3.40e-03 -8.70e-03 3.00e-02 -5.85e-04 -1.62e-06 -1.53e-03 -1.15e-04 6.00e-04 7.53e-04

1.03e-02 -3.45e-02 -2.62e-02 6.21e-03 -6.86e-03 -2.88e-02 5.25e-04 1.42e-04 -1.09e-03 -3.27e-03 1.44e-03 -1.47e-03

-8.63e-03 -1.21e-02 9.74e-03 -3.19e-02 2.00e-02 6.91e-02 1.83e-03 -1.22e-03 4.65e-04 6.05e-05 8.90e-05 3.91e-04

6.21e-03 1.70e-02 -1.50e-02 -6.25e-03 1.31e-03 3.77e-02 4.62e-04 1.72e-04 5.35e-04 3.60e-04 2.77e-03 -2.55e-04

#### Molecule 4

-1.92e-04 -1.38e-04 3.31e-03 -1.59e-03 -3.35e-08 2.96e-02 2.23e-03 2.07e-02 5.12e-03 -5.17e-03 4.16e-03 -4.70e-04

7.39e-04 4.54e-04 1.86e-03 6.97e-03 2.74e-07 8.00e-03 -1.18e-03 3.42e-03 -4.71e-03 4.82e-04 -2.49e-02 -3.65e-04

-2.35e-03 3.67e-04 2.14e-03 -6.54e-03 -3.19e-07 -1.65e-03 4.67e-03 1.34e-02 -1.04e-01 -3.30e-04 -1.28e-02 -2.89e-03

-1.13e-03 3.45e-03 1.29e-02 -1.99e-05 8.74e-08 -1.48e-02 -5.79e-04 9.81e-02 1.41e-02 5.69e-03 1.31e-03 6.83e-04

-2.64e-08 8.87e-08 2.12e-07 2.91e-09 -7.51e-09 -1.82e-07 2.10e-08 3.14e-06 -1.19e-07 7.11e-07 1.36e-09 1.53e-08

-1.06e-03 -4.54e-04 -2.23e-03 -9.29e-03 -3.28e-07 3.59e-03 -8.15e-02 7.17e-04 -7.16e-03 3.63e-05 3.77e-03 -1.66e-02

1.15e-02 -1.40e-02 2.16e-02 1.84e-03 2.89e-07 -1.22e+00 -5.92e-03 -7.12e-02 8.08e-03 1.69e-02 1.07e-02 6.60e-03

2.77e-02 5.12e-03 2.31e-02 1.54e-01 4.99e-06 -2.47e-03 -6.45e-03 -1.30e-03 6.13e-03 8.53e-04 -6.10e-04 2.50e-03

6.81e-03 -3.31e-03 -1.50e-01 2.03e-02 -1.56e-07 -8.42e-02 1.24e-04 -1.53e-02 1.51e-03 5.38e-03 2.54e-03 1.58e-03

-7.83e-03 1.52e-04 -4.25e-03 -7.60e-03 5.65e-07 4.65e-03 1.84e-03 1.75e-04 -3.99e-03 -1.16e-04 -5.06e-03 -9.03e-04

5.17e-03 -3.25e-02 -8.77e-03 -1.99e-03 -4.73e-08 8.18e-03 6.45e-04 8.27e-03 8.89e-04 -5.80e-04 -5.83e-04 -7.09e-05

-6.30e-05 -5.06e-04 -7.22e-03 1.19e-04 -2.38e-09 -8.16e-02 1.09e-04 -4.63e-03 1.09e-03 9.50e-04 7.13e-04 4.37e-04

### Molecule 5t

2.16e-04 -1.64e-05 1.87e-05 6.00e-03 5.72e-03 -5.47e-02 1.54e-02 1.32e-02 -5.61e-03 1.14e-03 -5.65e-03 -6.06e-03

1.21e-03 -5.87e-04 -5.80e-04 -5.24e-03 -8.08e-05 9.41e-03 -3.40e-03 -1.28e-03 9.43e-03 -1.10e-03 1.82e-03 1.72e-03

-8.37e-05 -4.55e-04 -1.92e-03 4.81e-04 6.96e-05 1.66e-03 1.90e-04 -3.05e-03 -6.62e-03 -2.84e-03 5.64e-04 -1.48e-03

-4.91e-04 -1.57e-03 9.47e-04 6.43e-03 -1.48e-04 -3.52e-02 7.63e-03 5.32e-04 -1.85e-02 5.01e-03 2.95e-03 -8.18e-03

1.47e-03 2.14e-03 -3.17e-04 -5.13e-03 -3.96e-03 1.14e-02 -4.49e-03 1.04e-02 4.70e-02 -1.29e-02 5.07e-03 4.36e-03

-1.59e-03 -2.37e-03 1.86e-03 1.95e-02 3.11e-03 -8.97e-02 1.71e-02 4.81e-03 -1.56e-03 2.33e-03 6.48e-03 1.10e-04

-1.88e-02 -2.44e-02 2.53e-02 2.77e-01 8.14e-02 -1.26e+00 9.66e-02 5.45e-02 3.42e-02 -9.92e-03 4.77e-03 1.09e-02

2.15e-02 1.82e-02 -1.27e-02 -6.37e-02 6.35e-03 2.43e-02 6.93e-04 1.82e-03 -4.65e-03 9.45e-04 -1.32e-03 -9.46e-04

-1.46e-03 8.04e-03 -1.62e-02 -2.29e-02 7.72e-02 3.16e-02 -2.37e-04 -3.14e-03 -8.62e-03 2.86e-03 -1.51e-03 -1.21e-03

-3.23e-03 -4.49e-03 -2.16e-02 4.26e-02 -1.28e-02 -7.45e-02 4.13e-03 3.34e-03 6.12e-03 4.33e-04 8.40e-04 1.39e-03

-1.87e-02 1.03e-03 2.89e-03 2.37e-02 2.05e-02 -5.54e-02 7.55e-03 5.26e-03 6.69e-04 -5.01e-04 2.44e-04 -2.20e-04

-1.07e-02 -1.68e-03 -1.46e-03 8.23e-03 1.08e-02 -3.86e-02 4.25e-03 2.39e-03 -3.86e-04 2.25e-04 4.52e-04 -9.58e-04

### Molecule 6

-7.36e-08 4.82e-04 3.55e-08 -1.03e-03 -1.05e-06 4.41e-02 -2.70e-06 2.26e-02 2.63e-06 3.93e-03 2.69e-03 -6.15e-07

4.38e-04 5.31e-07 9.19e-03 1.42e-06 -7.59e-03 -9.57e-07 -4.07e-04 5.06e-07 1.43e-02 7.99e-07 -2.38e-06 1.43e-02

1.56e-07 4.75e-03 6.75e-07 1.07e-02 -3.04e-07 3.42e-03 -1.42e-06 -4.81e-02 -5.19e-06 -3.19e-02 -5.81e-03 -2.93e-06

3.40e-03 6.32e-07 -6.98e-03 6.12e-07 -9.22e-03 -1.24e-06 -5.14e-03 -5.41e-06 8.30e-02 -



2.27e-06 -4.34e-06 1.92e-02  
-1.94e-06 -1.07e-02 2.20e-06 -3.06e-02 -6.38e-07 1.26e-01 -4.45e-06 8.89e-02 -2.51e-06  
1.18e-02 -2.07e-02 -3.24e-06  
4.86e-03 -4.04e-07 -9.26e-03 -2.17e-06 1.38e-02 1.75e-05 4.77e-02 -3.11e-07 -2.26e-03 -  
1.89e-06 -1.84e-07 -6.97e-04  
-6.88e-06 -1.56e-02 8.17e-06 -8.51e-02 -1.67e-05 1.20e+00 -1.74e-05 -9.78e-02 -8.77e-06  
5.31e-02 -3.88e-02 -1.24e-05  
3.30e-02 -8.17e-07 -8.39e-02 -5.88e-06 7.56e-02 -3.13e-05 -1.22e-02 5.63e-07 6.10e-03 -  
1.50e-06 1.68e-06 -2.81e-03  
4.05e-06 1.15e-02 -9.06e-06 1.13e-01 -6.76e-07 -4.01e-02 2.91e-07 -7.21e-03 -5.17e-07 -  
4.67e-03 5.68e-03 7.01e-07  
6.12e-05 1.46e-06 -2.66e-02 4.00e-07 3.45e-02 -7.75e-06 4.05e-03 9.72e-07 -3.21e-03 -9.72e-  
07 6.24e-07 -2.97e-03  
-4.28e-05 -2.18e-06 8.21e-03 -1.62e-06 -2.03e-02 3.54e-06 -4.37e-03 -1.11e-06 -1.03e-03  
1.29e-07 -1.60e-07 -3.86e-03  
-5.40e-07 1.84e-02 -1.38e-06 9.31e-03 -3.05e-06 -1.96e-02 2.16e-07 1.28e-02 7.14e-07 1.83e-  
03 2.21e-05 -1.14e-07

#### Molecule 7t

-1.95e-04 1.45e-04 6.21e-04 5.80e-05 -1.90e-03 8.32e-03 8.28e-04 1.62e-03 1.15e-02 -5.83e-  
04 9.18e-04 1.05e-04  
9.34e-04 -2.06e-03 -2.66e-03 -3.04e-04 1.31e-02 4.62e-03 -5.61e-03 -1.83e-03 -6.56e-02  
4.30e-03 2.42e-03 1.13e-03  
-1.33e-03 8.50e-03 1.23e-03 8.48e-04 1.95e-03 -4.55e-02 1.62e-03 1.51e-02 -1.15e-02 -2.15e-  
05 4.03e-04 -8.14e-04  
-1.73e-05 7.60e-04 -4.57e-05 -5.21e-05 9.12e-04 -4.18e-03 7.69e-05 2.07e-03 -5.06e-03  
6.65e-04 1.95e-04 1.88e-04  
-5.80e-03 4.31e-02 6.46e-03 2.97e-03 3.28e-04 1.40e-01 7.06e-04 8.74e-02 1.44e-03 -4.96e-  
03 -3.75e-03 -1.68e-04  
6.76e-04 -1.34e-03 2.83e-03 4.52e-04 1.80e-02 -7.62e-03 -2.88e-02 2.34e-03 2.72e-03 1.09e-  
03 1.06e-03 4.01e-04  
1.49e-02 -1.17e-01 -1.31e-02 -4.40e-03 3.79e-03 -1.27e+00 7.57e-03 6.30e-02 3.36e-03 -  
3.37e-03 -1.72e-03 -1.44e-03  
-1.11e-03 -9.66e-03 5.76e-02 6.05e-03 1.19e-01 7.10e-03 1.35e-02 -3.58e-05 1.52e-03 -4.36e-  
04 -1.89e-04 -1.41e-04

1.02e-02 -7.34e-02 -1.48e-02 -6.29e-03 5.30e-03 3.96e-02 6.55e-04 -1.36e-02 6.07e-04 6.59e-04 3.90e-04 2.00e-04

-3.04e-05 7.81e-03 -1.44e-03 7.71e-04 -6.52e-03 2.30e-02 -9.79e-04 -6.67e-04 -1.71e-04 1.29e-04 1.03e-04 5.58e-05

1.88e-03 4.87e-03 -6.94e-05 -4.81e-04 -4.10e-03 8.72e-03 -6.08e-04 -2.24e-04 -9.20e-05 1.11e-04 1.12e-04 6.78e-05

3.27e-05 9.19e-04 -6.42e-04 3.47e-04 -6.61e-04 1.92e-03 -3.34e-04 -2.64e-04 2.54e-04 3.37e-05 7.56e-05 -1.36e-05

### Natural Transition Orbital

The natural transition orbitals (NTO) are obtained diagonalization the TDM( $T$ ) provided above. The electron NTOs are eigenvectors of the matrix  $TT^\dagger$ , while the hole NTOs are eigenvectors of the matrix  $T^\dagger T$ . Integrals from the PySCF<sup>1-3</sup> program were used to obtain the transition dipole and the exchange integral between the hole and electron NTO pair with the largest eigenvalue.

### $\Delta E_{ST}$ of 1, 2, and 3t

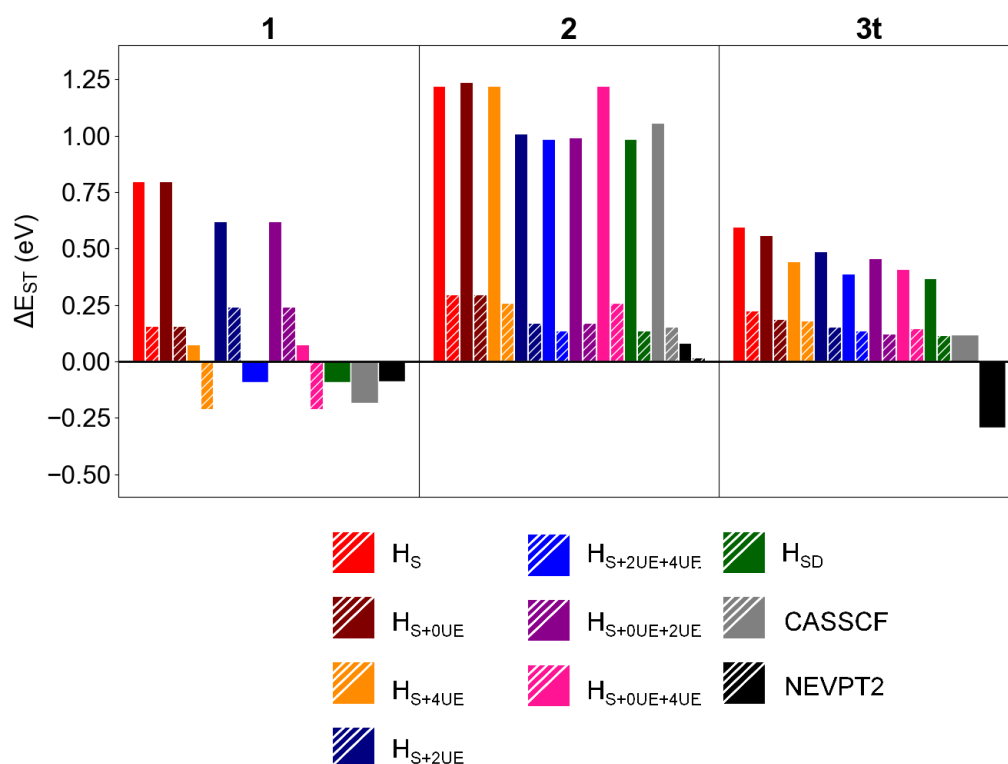


Figure S13.  $\Delta E_{ST}$  of molecules that have  $T_1$  states with differing characters from those of the  $S_1$  states. The hatched bars show the difference of energy between the  $S_1$  state and the triplet state that has the same configuration as its configuration with the largest coefficient.

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