Heptanuclear {Dy₂Cu₅} complex as a single-molecule magnet Romana Mičová, Cyril Rajnák, Ján Titiš, Ján Moncol', Jana Nováčiková, Alina Bieńko and Roman Boča

Supporting information (SI)

Physical measurements. ¹H NMR and ¹³C NMR spectra were recorded for the samples dissolved in DMSO- d_6 at 600 MHz for ¹H NMR and 150 MHz for ¹³C NMR by a Varian VNMRS 600 MHz instrument. Elemental analyses were measured by Flash 2000 CHNS apparatus (Thermo Scientific). The samples for FT-IR (ATR) measurements were not dried prior to its using and were used as freshly growth crystals. FT-IR spectra were measured by ATR method in region 400 – 4000 cm⁻¹ at room temperature FT-IR spectrometer (Shimadzu IRAffinity-1) with the Quest ATR holder. Absorption spectra of H₃L¹ in the UV–Vis region (9000–50 000 cm⁻¹) were recorded by UV–Vis–NIR spectrophotometer 50 Bio (Varian). UV-Vis spectra for solid sample (1) was dispersed in the Nujol oil were measured by Specord 250 Plus (Analytica Jena) with the DAD detector. Melting points were studied with thermo-microscopy by Kofler hot-stage microscope at 4°C·min⁻¹ and reported without corrections.

X-ray crystal structure determination. data collection and cell refinement of suitable single crystal of **1** were made by Stoe StadiVari diffractometer using PILATUS3R 300K HPAD detector and microfocused source Xenocs Genix3D with Cu HF at 100K. Corrections to Lorentz, polarization and multiscan absorption effects were applied. The structure was solved by charge-flipping method and refined anisotropically by common least-squares methods. The programs ShelXT, Superflip ShelXL, OLEX2 and MERCURY have been used for structure determination, refinement and drawing.^{S1-S5} The hydrogen atoms were refined with fixed distances from the parent carbon atoms. Crystal data are presented in Table S1 in detail.

Details on the synthesis/analysis

The organic reactants (2-hydroxy-1,3-propandiamine, 5-bromo-3-methoxysalicylaldehyde, 5-chloro-3-methoxysalicylaldehyde and inorganic salts (dysprosium(III) chloride hexahydrate, copper(II) nitrate trihydrate) of reagent grade were used as received. Ethanol, methanol and triethylamine were not dried and used without any further purification.

Synthesis and analytical data of H_3L^1 . 2-hydroxy-1,3-propandiamine (1 mmol, 0.901 g) was dissolved in ethanol (15 cm³) and 5-bromo-3-methoxysalicyl-aldehyde (2 mmol, 0.462 g) was added gradually. The resulting solution was stirred and heated under reflux for 3 h at 60° C. A yellow powder was immediately obtained, filtered and dried. Recipe was based on same manner to that describe from S.-P. Yan et al.⁵⁶ Yield: 86 %. Melting point: 180-185 °C. Anal Calc. for $C_{19}H_{20}Br_2N_2O_5$ (M = 516.19 g·mol⁻¹): C, 44.21; H, 3.91; N, 5.43. Fond: C, 43.61; H, 3.89; N, 5.60. ¹H NMR (600 MHz, DMSOd₆, 25 °C) δ (ppm) 13.76 (bs, OH, 2H), 8.46 (s, -CH=N-, 2H), 7.22 (d, J = 2.3 Hz, H-6', 2H), 7.08 (d, *J* = 2.3 Hz, H-4', 2H), 5.36 (bs, OH, 1H), 4.02-3.97 (m, H-2, 1H), 3.78 (s, OCH₃, 6H), 3.77 (ddd, *J* = 12.5, 4.2, 0.9 Hz, H-1, H-3, 2H), 3.58 (ddd, J = 12.5, 6.8, 0.7 Hz, H-1, H-3, 2H); ¹³C NMR (150 MHz, DMSO-d₆, 25 °C) δ (ppm) 166.0 (-CH=N-), 154.0 (C-2'), 149.9 (C-3'), 124.8 (C-6'), 118.5 (C-1'), 116.8 (C-4'), 107.2 (C-5'), 68.9 (C-2), 60.9 (C-1, C-3), 56.0 (OCH₃). Selected IR bands: 3027(w), 3066(w), 2897(w), 2850(w), 1629(s), 1519(s), 1479(s), 1436(s), 1370(m), 1353(m), 1230(s), 1213(sh), 1092(m), 1075(m), 1016(m), 988(m), 962(m), 864(m), 835(s), 760(m), 696(m), 638(m), 578(m), 565(m), 491(sh), 420(m) (s = strong). UV/Vis (EtOH) $v_{max}/10^3$ cm⁻¹ (c = 1.94 \cdot 10^{-5} mol·dm³): 23.50 $(0.097; \varepsilon = 5007 \text{ M}^{-1} \cdot \text{cm}^{-1}); 29.28 (0.084; \varepsilon = 4336 \text{ M}^{-1} \cdot \text{cm}^{-1}); 34.02 (0.182; \varepsilon = 9395 \text{ M}^{-1} \cdot \text{cm}^{-1}); 43.53$ $(0.920; \varepsilon = 47489 \text{ M}^{-1} \cdot \text{cm}^{-1}).$

Synthesis and analytical data of $[(H_2O)_6Dy^{III}(\mu_2-L^1)_2(\mu_3-O)_4Cu^{II}_5(\mu_2-CI)_2]$, 1. H_3L^1 (1 mmol, 0.179 g) was dissolved in methanol (15 cm³) and triethylamine was added (2 mmol 140 cm³). The solution was stirred for 20 min, while in the second beaker, copper(II) nitrate trihydrate (1 mmol, 0.121 g) was dissolved in methanol (9.2 cm³). Finally, the two solutions were combined and stirred for 20 min before dysprosium(III) chloride hexahydrate was added. The resulting mixture was stirred at room temperature for two hours, filtered and allowed to crystallize. Green crystals were collected after 5 days. Yield: 2.1%. Melting point: > 300°C. *Anal Calc.* for C₃₈H₄₆Br₄Cl₂Cu₅Dy₂N₄O₂₀ ($M = 1912.03 \text{ g} \cdot \text{mol}^{-1}$): C, 23.87; H, 2.42; N, 2.93. Found: C, 22.87; H, 3.53; N, 3.89. UV/Vis (Nujol) $v_{max}/10^3 \text{ cm}^{-1}$ (relat. absorb.): 15.39 (0.257); 27.58 (0.748); 35.96 (0.874).

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Quantum-chemical calculations. Density functional theory (DFT) calculations were performed with ORCA 5.0.4 computational package at the experimental geometry of complex 1 established by the X-ray diffraction [S7]. However, the structure was truncated as needed or otherwise modified to make the calculations more efficient. The relativistic effects were included in the calculations with the second-order Douglas-Kroll-Hess (DKH) approximation [S8]. All calculations included SARC-DKH-TZVP (for Dy) and DKH-def2-TZVP (for for all other elements) basis sets and utilized the RIJCOSX approximation with the AutoAux generation procedure [S9]. Increased integration grids (DefGrid2, for Dy the SpecialGrid option was set to 8) and tight SCF convergence criteria were also used. Calculation of the electronic structure and the exchange coupling constant *J* were based on B3LYP functional [S10]. The *J* values have been calculated using the broken-symmetry solution for low-spin wave function [S11]. The time-dependent DFT method was used to model the electronic spectra of individual metal centers using the ω B97X-D3 functional [S12].

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Fig. S1. ¹H NMR spectrum of ligand H_3L^1 .







Fig. S3. FT-IR (ATR) spectrum of H_3L^1 .



Fig. S4. Electronic spectra of H_3L^1 (in EtOH, $c = 1.94 \text{ x } 10^{-5} \text{ g} \cdot \text{mol}^{-1}$) and 1 (Nujol suspension).



Fig. S5. Calculated (TD-DFT) electronic spectra for individual metal centres (four-, five- and six-coordinated Cu(II) and Dy(III)) in the complex **1**.



Fig. S6 Calculated powder diffraction pattern for 1 from cif-file (theor), and recorded pattern at Cu $\lambda = 1.54060$ Å (exp).

	Complex 1
Empirical formula	$C_{38}H_{46}Br_4Cl_2Cu_5Dy_2N_4O_{20}$
Formula weight /g mol ⁻¹	1912.03
Crystal system	triclinic
Space group	<i>P</i> -1
Temperature /K	150
Crystal size /mm	0.18 imes 0.15 imes 0.12
Ζ	1
<i>a</i> / Å	10.7667(2)
<i>b</i> / Å	12.5061(3)
<i>c</i> / Å	15.2672(3)
lpha /°	66.153(2)
β /°	69.597(2)
γ /°	74.811(2)
$V/Å^3$	1744.60(7)
$ ho_{ m calc}/ m g\ m cm^{-3}$	1.820
μ /mm ⁻¹	16.775
F(000)	913.0
Radiation	$CuK\alpha \ (\lambda = 1.54186)$
2Θ range for data collection/°	6.584 to 144.7
Index ranges	$-13 \le h \le 12, -12 \le k \le 15, -18 \le l \le 16$
Data/restraints/parameters	6663/0/345
Goodness-of-fit on F^2	1.111
Final R indexes [I>= 2σ (I)]	$R_1 = 0.0496, wR_2 = 0.1371$
R indices (all data)	$R_1 = 0.0594, wR_2 = 0.1430$
Colour	Dark green
CCDC code	2160393

 Table S1. Crystal data and structure refinement for 1.

Heptanuclear {Dy₂Cu₅} complex as a single-molecule magnet Romana Mičová, Cyril Rajnák, Ján Titiš, Ján Moncol', Jana Nováčiková, Alina Bieńko and Roman Boča

Table S2. Sele	cted bond lengths (Å)	in 1 .		
Dy1-Cu1	3.4989(4)	Cu1-O6	1.969(4)	
Dy1-Cu2	3.4113(10)	Cu1-O6 ¹	1.969(4)	
Dy1-Cu31	3.4844(10)	Cu1-O7	1.950(5)	
Dy1-O1	2.601(5)	Cu1-O7 ¹	1.950(5)	
Dy1-O2	2.336(4)	Cu1-Cl1 ¹	2.7889(17)	
Dy1-O4 ¹	2.357(5)	Cu1-Cl1	2.7889(17)	
Dy1-O5 ¹	2.544(5)	Cu2-O2	1.959(5)	
Dy1-O6	2.397(5)	Cu2-O3	1.960(5)	
Dy1-O71	2.462(5)	Cu2-O6	1.950(4)	
Dy1-O8	2.376(4)	Cu2-N1	1.917(6)	
Dy1-O9	2.380(6)	Cu2-Cl1 ¹	2.6738(19)	
Dy1-O10	2.400(5)	Cu3-O3	1.950(5)	
		Cu3-O4	1.954(5)	
		Cu3-O7	1.970(4)	
		Cu3-N2	1.918(6)	

Atom numbering:



Heptanuclear {Dy₂Cu₅} complex as a single-molecule magnet Romana Mičová, Cyril Rajnák, Ján Titiš, Ján Moncol', Jana Nováčiková, Alina Bieńko and Roman Boča

Table S3. Selected	bond angles (°)) in chromophores o	of 1 .		
Cu2-Dy1-Cu1	54.448(18)	O2-Dy1-O4 ¹	150.70(18)	O41-Dy1-O1	111.29(17)
Cu2-Dy1-Cu31	112.34(3)	O2-Dy1-O51	130.11(16)	O41-Dy1-O51	63.82(15)
Cu31-Dy1-Cu1	59.884(18)	O2-Dy1-O6	67.59(16)	O41-Dy1-O6	118.62(16)
O1-Dy1-Cu1	142.94(11)	O2-Dy1-O71	96.33(15)	O41-Dy1-O71	65.44(15)
O1-Dy1-Cu2	96.04(11)	O2-Dy1-O8	74.53(16)	O41-Dy1-O8	133.48(17)
O1-Dy1-Cu31	131.21(12)	O2-Dy1-O91	80.1(2)	O41-Dy1-O9	72.4(2)
O2-Dy1-Cu1	82.52(12)	O2-Dy1-O10	132.95(18)	O41-Dy1-O10	73.19(19)
O2-Dy1-Cu2	33.72(12)	O41-Dy1-Cu1	91.03(12)	O51-Dy1-Cu1	146.70(12)
O2-Dy1-Cu31	127.79(11)	O41-Dy1-Cu2	144.63(11)	O51-Dy1-Cu2	150.41(11)
O2-Dy1-O1	63.39(16)	O41-Dy1-Cu31	32.32(11)	O51-Dy1-Cu31	95.88(11)
O51-Dy1-O1	69.91(16)	O10-Dy1-Cu1	80.66(13)	Cl1-Cu1-Dy1	87.02(4)
O6-Dy1-Cu1	32.73(10)	O10-Dy1-Cu2	104.54(13)	Cl1 ¹ -Cu1-Dy1 ¹	87.02(4)
O6-Dy1-Cu2	33.87(11)	O10-Dy1-Cu31	78.31(12)	Cl1-Cu1-Dy11	92.98(4)
O6-Dy1-Cu31	90.69(10)	O10-Dy1-O1	133.05(17)	Cl1 ¹ -Cu1-Cl1 ¹	180.0
O6-Dy1-O1	128.79(15)	O10-Dy1-O51	71.64(18)	O2-Cu2-Dy1	41.45(13)
O6-Dy1-O51	143.60(17)	O10-Dy1-O71	92.55(17)	O2-Cu2-O3	167.85(19)
O6-Dy1-O71	65.31(16)	Dy1 ¹ -Cu1-Dy1	180.0	O2-Cu2-Cl11	98.17(15)
O6-Dy1-O10	74.78(17)	O61-Cu1-Dy1	138.85(14)	O3-Cu2-Dy1	136.87(14)
O71-Dy1-Cu1	32.69(11)	O6-Cu1-Dy1 ¹	138.85(14)	O3-Cu2-Cl1 ¹	93.98(14)
O71-Dy1-Cu2	79.60(11)	O6-Cu1-Dy1	41.15(14)	O6-Cu2-Dy1	43.22(14)
O71-Dy1-Cu31	33.42(10)	O61-Cu1-Dy11	41.15(4)	O6-Cu2-O2	84.67(19)
O71-Dy1-O1	132.94(17)	O61-Cu1-O6	180.0	O6-Cu2-O3	95.1(2)
O71-Dy1-O51	129.23(15)	O61-Cu1-Cl1	87.93(13)	O6-Cu2-Cl11	91.67(14)
O8-Dy1-Cu1	112.98(12)	O61-Cu1-Cl11	92.07(13)	N1-Cu2-Dy1	134.4(2)
O8-Dy1-Cu2	74.80(12)	O6-Cu1-Cl1 ¹	87.93(13)	N1-Cu2-O2	93.3(2)
O8-Dy1-Cu31	150.70(12)	O6-Cu1-Cl1	92.07(13)	N1-Cu2-O3	85.7(2)
O8-Dy1-O1	73.11(17)	O7-Cu1-Dy1 ¹	42.99(13)	N1-Cu2-O6	173.9(2)
O8-Dy1-O51	76.18(16)	O71-Cu1-Dy1	42.99(13)	N1-Cu2-Cl1 ¹	94.34(18)
O8-Dy1-O6	80.70(16)	O71-Cu1-Dy11	137.01(13)	Cl11-Cu2-Dy1	97.08(5)
O8-Dy1-O71	145.56(17)	O7-Cu1-Dy1	137.01(13)	O3-Cu3-Dy11	140.14(14)
O8-Dy1-O9	137.0(2)	O7-Cu1-O6	96.01(19)	O3-Cu3-O4	169.6(2)
O8-Dy1-O10	72.40(17)	O71-Cu1-O6	83.99(19)	O3-Cu3-O7	99.32(19)
O9-Dy1-Cu1	97.19(18)	O7-Cu1-O6 ¹	83.99(19)	O4-Cu3- Dy1 ¹	40.16(13)
O9-Dy1-Cu2	101.8(2)	O71-Cu1-O61	96.01(19)	O4-Cu3-O7	83.25(19)
O9-Dy1-Cu31	71.15(16)	O7 ¹ -Cu1-O7	180.0	O7-Cu3- Dy 1^1	43.50(13)
O9-Dy1-O1	64.5(2)	O71-Cu1-Cl11	92.43(14)	N2-Cu3- Dy1 ¹	132.61(17)
O9-Dy1-O51	95.4(2)	O7-Cu1-Cl1	92.43(14)	N2-Cu3-O3	85.5(2)
O9-Dy1-O6	120.5(2)	O7-Cu1-Cl1 ¹	87.57(14)	N2-Cu3-O4	92.5(2)
O9-Dy1-O71	70.6(2)	O71-Cu1-Cl1	87.57(14)	N2-Cu3-O7	174.3(2)
O9-Dy1-O10	145.4(2)	Cl1 ¹ -Cu1-Dy1	92.98(4)		





CCDC: CONZIQCCDC:DC: χ – not fitted; M – plotted, not digitized.DC: χ –AC: single mode relaxation (HF), on-set of LF –In Jianignored,Lin Jian $U_{eff}/k_B = 4$ K, $\tau_0 = 3 \times 10^{-6}$ s; $B_{DC} = 0.09$ TXin LiuV. Chandrasekhar, A. Dey, S. Das, M. Rouxieres, R.19, 181Clerac, *Inorg.Chem.* (2013), **52**, 2588.19, 181



CCDC: GASJUI DC: χ – not fitted; M – plotted, not digitized. AC: silent χ " Lin Jiang, Bin Liu, Hao-Wen Zhao, Jin-Lei Tian, Xin Liu, Shi-Ping Yan, *CrystEngComm* (2017), **19**, 1816. <u>https://doi.org/10.1039/C6CE02519F</u>



Dy: 9-coordination, Cu: 4-, 5-, 6-coordination



AC: single mode relaxation,	
$U_{\rm eff}/k_{\rm B} = 13 \text{ K}, \tau_0 = 1.2 \times 10^{-6} \text{ s}; B_{\rm DC} = 0;$	
D. Dermitzaki, V. Psycharis, Y. Sanakis, Th. C.	
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https://doi.org/10.1016/j.poly.2019.05.004	

 Table S5. Coordination polyhedra in 1.

Atom	$N_{\rm k}$	Chromophore	Shape index	Geometry	Abbr.
Cu1	6	${CuO_4Cl_2}$	3.179	Octahedron	Oh
Cu2	5	$\{CuO_3NCl\}$	1.146	Square pyramid	Spy
$Cu2^1$	5	$\{CuO_3NCl\}$	1.146	Square pyramid	Spy
Cu3	4	$\{CuO_3N\}$	0.699	Square	Sq
$Cu3^1$	4	$\{CuO_3N\}$	0.699	Square	Sq
Dy1	9	${DyO_9}$	7.048	Capped cube	Ccu
$Dy1^1$	9	${DyO_9}$	7.053	Capped cube	Ccu

DFT calculations



Fig. S7. Single-occupied quasi-restricted orbitals (DFT) for 1.

DC magnetic data

Factoring of large-dimensional matrices according to the spin S and/or total angular momentum J

The principal problem associated with a computational approach for large exchange coupled systems lies in the size of the interaction matrices. The Heisenberg Hamiltonian appropriate to the exchange coupled system reads

$$\hat{H}^{\text{ex}} = -\sum_{A < B}^{N} J_{AB} (\mathbf{S}_{A} \cdot \mathbf{S}_{B})$$

and it generates an interaction matrix $H_{LK} = \langle L | \hat{H}^{ex} | K \rangle$ in the basis set of spin kets. The size of such a matrix

grows rapidly with the number of constituent spins, i.e.

$$M = \prod_{A=1}^{N} (2S_A + 1)$$

In the {Dy₂Cu₅} system with J(Dy) = 15/2 and S(Cu) = 1/2 the total number of basis set functions is $N = 16^2 \cdot 2^5$ = 8192. Such a big interaction matrix can be factored, using the apparatus of the irreducible tensor operators, according to the molecular J = 1/2 through 35/2 to smaller matrices of the dimension 20, 30, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 32, 31, 26, 16, 6 and 1. In this model the molecular J is a "good quantum number" and the Zeeman term need be applied assuming uniform g-factors, $g_{\rm eff}$.

The Heisenberg exchange Hamiltonian commutes with the total spin of the system

$$\left[\hat{H}^{\text{ex}},\hat{S}^{2}\right]=0, \quad \left[\hat{H}^{\text{ex}},\hat{S}_{z}\right]=0$$

Therefore a set of eigenstates common for $\{\hat{H}^{ex}, \hat{S}^2, \hat{S}_z\}$ operators exists; this means that the states spanning different total spin (or J) are orthogonal

$$H_{LK} = \left\langle L : \dots S'M' \left| \hat{H}^{\text{ex}} \right| K : \dots SM \right\rangle = \delta_{S'S} \delta_{MM} (2S+1)^{-1/2} \left\langle L : \dots S \right\| \hat{H}^{\text{ex}} \| K : \dots S \right\rangle$$

It allows a factoring of the interaction matrix into blocks of a much lower size

$$\mathbf{H}^{\text{ex}} \rightarrow \begin{bmatrix} S = S_{\min} & 0 & \dots & 0 & 0 \\ 0 & S = S_{\min} + 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & S = S_{\max} - 1 & 0 \\ 0 & 0 & \dots & 0 & S = S_{\max} \end{bmatrix}$$

which can be treated (diagonalized) independently.

The matrix elements of the interaction operators in the coupled basis set of spin kets $|K\rangle = |(S_1, S_2, \dots, S_N), (S_{12}, S_{1-3}, \dots, S_{1-N}), S\rangle$ have been generated with the help of the irreducible tensor operators for a loop for $S = S_{\min}$ until $S = S_{\max}$. The intermediate spins (IS) are abbreviated as $S_{1\dots N} = S_N^{\prime 0}$ and the full set of IS is (\mathscr{Y}_N) . The general form of such matrix elements utilizes a consecutive decoupling of spins until the elementary spin operators with the help of the 9j-symbols (recoupling coefficients of the angular momenta) according to the formula

$$\left\langle S_{1}S_{2}...S_{N}(\mathscr{G}S') \| \hat{H}^{\text{ex}} \| S_{1}S_{2}...S_{N}(\mathscr{G}S) \right\rangle$$

$$= \sum_{k_{1}k_{2}...k_{N}} \left\langle S_{N} \| \hat{T}_{k_{N}}(\overset{\mathbf{r}}{S}_{N}) \| S_{N} \right\rangle \times \sum_{\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{ \rightarrow} \overset{\mathcal{R}}{ \rightarrow}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{\not\sim}\overset{\mathcal{R}}{ \rightarrow}\overset{\mathcal{R}}{ \rightarrow}\overset{\mathcal{R}} \overset{\mathcal{R}}{ \rightarrow} \overset{\mathcal{R}}{ \rightarrow}\overset{\mathcal{R}}{ \rightarrow}\overset{\mathcal{R}}{ \rightarrow}\overset{\mathcal{R}}{ \rightarrow}\overset{\mathcal$$

The only non-zero proportionality factor accounting for the relationship between the scalar and tensor product of spin operators is

$$G[k_1k_2(k_2')k_3(k_3')...k_{N-1}(k_{N-1}')k_N] = \sqrt{3}J_{ij}$$

where the tensor ranks are $k_i = k_j = 1$ for $i \neq j$, $k_f = 0$ for $f \neq i, j$, and $k = k_N^0 = 0$. The matrix elements of the elementary spin operators are trivial

$$\left\langle S_{f} \left\| \hat{T}_{k_{f}=0} \left(S_{f} \right) \right\| S_{f} \right\rangle = (2s_{f}+1)^{1/2} \left\langle S_{f} \left\| \hat{T}_{k_{f}=1} \left(S_{f} \right) \right\| S_{f} \right\rangle = [s_{f}(s_{f}+1)(2s_{f}+1)]^{1/2}$$

with $(s_f) = S_1 S_2 ... S_N$.

A computational problem arises when the magnetic field is applied: the matrix elements of the Zeeman term in the basis set of the coupled kets <u>are off-diagonal</u> in the total spin number. There is one exception: when all *g*-factors are equal, then the off-diagonal matrix elements of the Zeeman operator vanish exactly. This is really a fortunate case, since then the Zeeman contributions can be simply added to the roots of the zero-field Hamiltonian

$$\varepsilon(S,B) = \varepsilon_0(S) + \mu_{\rm B}g_{\rm iso}BM_S$$

Then the magnetic functions (magnetization and susceptibility) can be <u>exactly</u> expressed with the help of the thermodynamic partition function

$$M_{\rm mol} = N_{\rm A} \frac{1}{Z} T_1$$
$$\mathscr{H}_{\rm mol} = \frac{N_{\rm A} \mu_0}{k_{\rm B} T} \frac{1}{Z^2} (T_2 Z - T_1^2)$$

The terms entering the magnetization and the differential magnetic susceptibility are

$$Z = \sum_{i} \exp(-\varepsilon_{i} / k_{\rm B}T) = \sum_{S=S_{\rm min}}^{S_{\rm max}} \sum_{M_{\rm S}=-S}^{+S} \exp[(n_{S}J - \mu_{\rm B}gBM_{S}) / k_{\rm B}T]$$

$$T_{\rm I} = \sum_{i} \left(-\frac{\partial\varepsilon_{i}}{\partial B}\right) \exp(-\varepsilon_{i} / k_{\rm B}T) = \mu_{\rm B}g \sum_{S=S_{\rm min}}^{S_{\rm max}} \sum_{M_{\rm S}=-S}^{+S} M_{S} \exp[(n_{S}J - \mu_{\rm B}gBM_{S}) / k_{\rm B}T]$$

$$T_{\rm 2} = \sum_{i} \left(\frac{\partial\varepsilon_{i}}{\partial B}\right)^{2} \exp(-\varepsilon_{i} / k_{\rm B}T) = (\mu_{\rm B}g)^{2} \sum_{S=S_{\rm min}}^{S_{\rm max}} \sum_{M_{\rm S}=-S}^{+S} M_{S}^{2} \exp[(n_{S}J - \mu_{\rm B}gBM_{S}) / k_{\rm B}T]$$

with $n_s = S(S+1) / 2$.

The critical step of the procedure is the evaluation of the reduced matrix elements of the compound operator $\mathbf{r} = \mathbf{r} + \mathbf{r}$

$$R_{IS';IS}^{AB(k)} = \left\langle I'S' \left\| \hat{T}_{k} \left(S_{A} \otimes S_{B} \right) \right\| IS \right\rangle$$

which depends upon all intermediate spins (I'S'; IS); then the decoupling formula over the uncoupled spins adopts a form

where $S_i = S_{12...i}$ denote the intermediate spins and $k_i = k_{12...i}$ the intermediate operator ranks.

In the above procedure two kinds of the operators are met a) bilinear isotropic exchange

$$\hat{H}^{\text{iso}} = \sum_{A}^{N} \sum_{B>A}^{N} (-J_{AB}) (\hat{S}_{A} \cdot \hat{S}_{B}) h^{-2} = \sum_{A}^{N} \sum_{B>A}^{N} (-J_{AB}) (-\sqrt{3}) \left\{ \hat{T}_{0,0} (\hat{S}_{A} \otimes \hat{S}_{B}) \right\} h^{-2}$$

b) the Zeeman operator

$$\hat{H}^{Z} = \mu_{\mathrm{B}} \sum_{A}^{N} (\stackrel{\mathbf{f}}{B} \cdot \overline{\overline{g}}_{A} \cdot \stackrel{\mathbf{f}}{S}_{A}) \mathbf{h}^{-1} = \mu_{\mathrm{B}} \sum_{A}^{N} \sum_{q=-1}^{+1} (-1)^{q} (\stackrel{\mathbf{f}}{B} \cdot \overline{\overline{g}}_{A})_{1,-q} \left\{ \hat{T}_{1,q} (\stackrel{\mathbf{f}}{S}_{A}) \right\} \mathbf{h}^{-1}$$

For details see: R. Boča, A Handbook of Magnetochemical Formulae, Elsevier, Amsterdam, 2012.

Table S6. Topological matrix for exchange interactions referring to the core of 1.

	Dy	Cu4	Cu5	Cu6	Cu5'	Cu4'	Dy'
Dy	-	J1	J2	J3	-	-	-
Cu4		-	J4	J5	-	-	-
Cu5			-	J6	-	-	-
Cu6				-	J6	J5	J3
Cu5'					-	J4	J2
Cu4'						-	J1
Dy'							-



Fig. S8 Calculated spectrum of zero-field energy levels for 1 (482 entries).

AC magnetic data



Fig. S9 Temperature dependence of the AC susceptibility for 1.



Fig. S10 Field and frequency dependence of the AC susceptibility at T = 2.0 K. Lines are guide for eyes. Verticals identify the B_{DC} used for the detailed frequency dependence.



Fig. S11 Argand diagram (left) and Arrhenius-like plot (right) for 1: $U_{\text{eff}}/k_{\text{B}} = 24.9 \text{ K}$, $\tau_0 = 1.1 \times 10^{-7} \text{ s}$ at $B_{\text{DC}} = 0.2 \text{ T}$.

Heptanuclear {Dy₂Cu₅} complex as a single-molecule magnet Romana Mičová, Cyril Rajnák, Ján Titiš, Ján Moncol, Jana Nováčiková, Alina Bieńko and Roman Boča

Table	f able S7. Temperature dependence of AC susceptibility parameters for 1 at $B_{DC} = 0.1$ T. ^a														
<i>Т/</i> К	$R(\chi')$	R(\chi'')	χs	$\chi_{ ext{LF}}$	$lpha_{ m LF}$	$ au_{ m LF}$	$\chi_{ ext{if}}$	$lpha_{IF}$	$ au_{IF}$	$\chi_{ extsf{hf}}$	$lpha_{ m HF}$	$ au_{ m HF}$	$x_{\rm LF}$	X _{IF}	$x_{\rm HF}$
	/%	/%				/ s			/10⁻³ s			/10⁻ ⁶ s			
1.8	0.64	0.97	45(15)	52(31)	.37(9)	2(4)	79(70)	.15	3(8)	147(9)	.46(2)	170	.07	.26	.67
2.0	0.17	0.71	43(3)	51(7)	.40(2)	2(1)	74(16)	.14(7)	2(1)	138(2)	.45(6)	127(47)	.09	.24	.67
2.2	0.43	0.65	44(9)	53(20)	.45	2(3)	74(48)	.15	1(3)	129(7)	.41(2)	104	.11	.25	.64
2.4	0.28	0.81	46(7)	55(13)	.49(3)	1(1)	73(40)	.14	1(3)	120(4)	.36(2)	105(96)	.11	.25	.64
2.6	0.21	0.74	50(8)	54(7)	.25(1)	.56(1)	90(20)	.26(2)	.50(1)	109(4)	.14	56(15)	.07	0.61	.33
2.8	1.0	0.60	47	50	.18	.59(4)	80	.27(2)	.41	101(1)	.21	61	.06	.54	.40
3.2	0.18	0.52	50(4)	52(4)	.21(1)	.57(1)	84(26)	.27(1)	.19(1)	90(2)	.0027	57(23)	.05	.78	.17
3.6	1.88	1.32	49	50	.20	.46	83	.26	.10	82(2)	.29	30	.04	.99	03
3.6b	0.26	1.26	47(9)	-	-	-	48(1)	.19(1)	.46(1)	81(2)	.27(1)	98(5)	-	.04	.96
4.0b	0.20	0.68	45(9)	-	-	-	46(1)	.28(1)	.49(1)	73(2)	.24(1)	74(4)	-	.04	.96
4.6b	0.13	1.48	42(8)	-	-	-	42(8)	.04	.31(6)	63(6)	.24(1)	49(3)	-	.02	.98
5.2b	0.38	2.04	40(2)	-	-	-	41(2)	.23	.54	56(3)	.22(4)	40(13)	-	.02	.98
5.8b	0.25	1.29	40(1)	-	-	-	40(2)	.25	.41	50(1)	.21(4)	35(12)	-	.02	.98

^a Obtained by the three-set Debye model.

Table S8. Temperature dependence of AC susceptibility parameters for 1 at $B_{DC} = 0.2$ T.^a

<i>Т/</i> К	$R(\chi')$	R(\chi')	χs	ХIF	$lpha_{IF}$	$ au_{IF}$	χ́нғ	$lpha_{HF}$	$ au_{HF}$	$X_{\rm LF}$	$x_{\rm HF}$
	/%	/%				/ s			/10⁻ ⁶ s		
1.8	0.81	4.2	36(2)	44(5)	.16	1.5(8)	113(3)	.50(2)	642(54)	.11	.89
2.0	0.79	4.0	37(2)	51(6)	.25(1)	1.5(7)	114(3)	.48(2)	516(46)	.18	.82
2.2	0.41	3.2	37(1)	52(3)	.26(6)	1.2(2)	111(2)	.47(1)	397(23)	.20	.80
2.4	0.32	2.9	38(1)	51(2)	.22(4)	.77(7)	106(7)	.45(1)	304(16)	.19	.81
2.6	0.27	2.1	39(9)	52(2)	.28(3)	.75(7)	103(6)	.42(1)	241(11)	.21	.79
2.8	0.21	1.8	38(8)	49(1)	.23(2)	.59(3)	98(3)	.41(9)	185(8)	.18	.82
3.2	0.21	1.4	37(1)	46(1)	.23(2)	.48(2)	90(2)	.39(1)	109(6)	.16	.84
3.6	0.22	1.3	36(1)	43(1)	.26(3)	.42(3)	82(2)	.36(1)	69(6)	.15	.85
4.0	0.31	2.0	36(2)	42(2)	.25(5)	.41(5)	76(3)	.34(2)	50(8)	.13	.87
4.6	0.28	2.4	35(3)	38(3)	.24(6)	.39(6)	67(2)	.32(2)	30(7)	.09	.91
4.6b	0.31	2.9	32(5)	35(5)	.31(8)	.38(8)	67(3)	.33(3)	25(8)	.09	.91
5.2	0.29	2.0	27(9)	29(9)	.31(1)	.39(1)	60(2)	.34(4)	12(7)	.06	.94
5.8	0.36	4.1	26(20)	28(20)	.37(2)	.32(2)	54(3)	.31(8)	8	.04	.96

^a Obtained by the two-set Debye model; χ in units of 10⁻⁶ m³ mol⁻¹; $R(\chi')$ and $R(\chi'')$ – discrepancy factors of the fit.

Table S9. Broken symmetry magnetic coupling analysis for 1

	J B		
	J1	J2	J3
Cu1-Cu2	535.27	267.64	528.64
Cu1-Cu3	482.29	241.14	475.61
Cu2-Cu3	-74.80	-37.40	-71.64
Dy-Cu1	1.59	0.87	1.66

J1: (a) A.P. Ginsberg J. Am. Chem. Soc. 102 (1980), 111, (b) L. Noodleman J. Chem. Phys. 74 (1981), 5737, (c) L. Noodleman E.R. Davidson Chem. Phys. 109 (1986), 131.

J2: A. Bencini D. Gatteschi J. Am. Chem. Soc. 108 (1980), 5763.

J3: (a) K. Yamaguchi Y. Takahara T. Fueno in: V.H. Smith (Ed.) Applied Quantum Chemistry. Reidel, Dordrecht (1986), pp 155, (b) T.Soda et al. Chem. Phys. Lett., 319, (2000), 223.

Cartesian coordinates of fragments corresponding to individual metal centers used for TD-DFT calculations

Fragment of	f the Cu-4 center:		
Br	12.77000	9.81500	16.16900
Cu	12.36700	8.38100	8.93300
0	13.26700	9.07100	7.34600
0	11.38100	8.02800	10.58200
0	9.61000	7.74800	12.45800
0	10.75500	7.63000	8.08400
Ν	13.92200	8,96900	9.88800
С	14.23500	11.23100	7,76900
C	14.50700	9.74400	7.66700
н	15.16500	9.58500	6.93200
C	15.05200	9.13500	8,96200
н	15.73500	9.73200	9.35900
н	15 47200	8 25900	8 77500
C	14 05500	9 19200	11 13300
н	1/ 90300	9 18000	11 //900
C	12 97900	9 03200	12 10100
C	13 27000	9.03200	13 12000
ц	1/ 12200	9.42000	13 63/00
n C	12 22100	9.78900	14 20100
	12.52100	9.29100	14.39100
	10 44700	0.75900	14.12000
	10.44/00	8.00000	14.82100
C	10.79700	8.36200	12.84300
C	11.74200	8.46/00	11.80500
C .	8.61100	7.67200	13.51300
н	8.43200	8.5/300	13.85900
Н	7.78300	7.29100	13.15400
Н	8.94500	7.10500	14.23900
Н	10.50385	8.19445	7.27239
Н	9.98492	7.63467	8.75284
Н	13.85814	11.58961	6.83397
Н	15.14303	11.74411	8.00796
Н	13.51175	11.40943	8.53710
Fragment of	f the Cu-5 center:		
C1	10.44500	10.94200	7.84600
Br	14.29300	15.77600	1.72700
Cu	12.28700	10.18200	6.06400
0	10.91400	11.74900	2.07300
0	11.60100	11.16800	4.51600
0	13.26700	9.07100	7.34600
0	11.14900	8.73600	5.41700
Ν	13.52600	11.56700	6.53600
С	12.23400	12.22100	3.96900
C	11.86700	12.57700	2.63900
С	12.45500	13.65100	1.98300
Н	12.18400	13.89200	1.10500
C	13,43400	14.35700	2.63000
C	13.85200	14.03700	3.89600
н	14.53500	14.54600	4.31700
C	13.26300	12.94900	4,56500
Č	13.80200	12.63100	5.86300
- Н	14.40700	13.25200	6,25200
C	14.23500	11.23100	7,76900
с Н	15.08100	11.74100	7.83700
н	13 67600	11 43300	8 56100
	14 50700	0 74400	7 6 7 7 0 0

С

14.50700

9.74400

7.66700

н	15,16500	9.58500	6,93200	
C	15 05200	9 13500	8 96200	
c	10 11000	12 38000	1 02800	
L L	0 27000	11 70500	0 78000	
п	9.37000	12 22700	1 25500	
п	9.77500	12.23/00	1.33300	
н	10.6/900	12.53300	0.23900	
н	14.35352	9.30045	9.75551	
н	15.98611	9.59563	9.20/26	
Н	15.19/18	8.08338	8.82811	
Н	10.35937	8.61503	6.05123	
Н	11.68820	7.87112	5.37655	
Fragment	of the Cu-6 cent	er:		
Cu	9.68300	8.51000	6.71300	
0	8,21700	8,28400	8,00800	
0	8 61100	9 39000	5 34200	
C1	10 44500	10 94200	7 84600	
0	11 1/000	8 73600	5 41700	
0	10 75500	7 63000	9 09/00	
	2 02100	6.07900	5.00400 F F9000	
	0.92100	0.07000	5.50000	
н	8.20075	9.08007	8.64551	
н	8.36220	7.42564	8.53955	
Н	11.63909	9.61012	5.60707	
Н	10.76656	8.75923	4.47169	
Н	8.66470	10.40109	5.46537	
Н	7.64003	9.08856	5.42409	
Н	11.18249	8.33425	8.68541	
Н	11.48538	7.07378	7.63952	
Fragment	of the Dy center			
	9 20800	7 31600	9 97800	
Bn	5 07200	1 24400	11 60000	
	9 45200	I.24400 E 27200	11 25200	
0	0.45200	5.27200	11.55500	
0	7.70500	5.85500	8.90900	
0	8.21/00	8.28400	8.00800	
0	7.04400	7.83500	10.83200	
Н	6.68000	8.67000	11.03200	
Н	6.36900	7.22900	11.04300	
0	10.62900	5.44400	9.82600	
Н	11.55000	5.40600	9.95000	
Н	10.38700	4.56800	9.62500	
0	9.10500	9.70800	10.38700	
Н	9.09100	10.15700	9.61600	
Н	8.33200	9.92800	10.77200	
Ν	5.84000	5.45300	6.89000	
с	7.13200	4.79900	9.45700	
с	7.49900	4.44300	10.78600	
C	6,91100	3,36900	11,44300	
н	7 18200	3 12900	12 32000	
C C	5 93200	2 66300	10 79600	
c	5 51400	2 98300	9 53000	
с ц	1 82100	2.70500	0 10000	
с С	4.03100 6 10300	2.4/400 1 07100	9.10000	
	0.10200	4.0/100	0.00100	
	00000	4.30900	000COC /	
Н	4.95900	3./6800	/.1/300	
C	5.13100	5.78900	5.65600	
C	9.24700	4.64000	12.39700	
Н	9.99600	5.22500	12.63600	
Н	9.59300	3.78300	12.07100	

н	8.68700	4.48700	13.18700
Br	12.77000	9.81500	16.16900
0	11.38100	8.02800	10.58200
0	9.61000	7.74800	12.45800
0	10.75500	7.63000	8.08400
Ν	13.92200	8.96900	9.88800
С	15.05200	9.13500	8.96200
С	14.05500	9.19200	11.13300
Н	14.90300	9.48000	11.44900
С	12.97900	9.03200	12.10100
С	13.27000	9.42800	13.42000
Н	14.12200	9.78900	13.63400
С	12.32100	9.29100	14.39100
С	11.09000	8.75900	14.12800
Н	10.44700	8.66600	14.82100
С	10.79700	8.36200	12.84300
С	11.74200	8.46700	11.80500
С	8.61100	7.67200	13.51300
Н	8.43200	8.57300	13.85900
Н	7.78300	7.29100	13.15400
Н	8.94500	7.10500	14.23900
Н	14.73545	8.90004	7.96727
Н	15.84684	8.47818	9.24786
Н	15.39603	10.14758	8.99700
Н	5.51549	6.70664	5.26230
Н	4.08708	5.89941	5.86320
Н	5.27164	5.00680	4.93956
Н	7.22848	8.46804	8.17931
Н	8.31126	7.62650	7.23392
Н	10.27924	7.38253	7.21635
Н	11.56490	7.02344	8.21261