Supplementary Information

Squeezed State in Hydrodynamic Focusing Regime for *Escherichia coli* Bacteria Detection

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1. Mathematical Formulations & Numerical Methodology

In order to analyze the mass transport processes and flow focusing mechanisms, we performed three-dimensional numerical studies on the sheath flow in this microfluidic channel. Assuming incompressible and laminar flows, governing equations for conservation of mass and momentum are described by

$$\nabla \cdot \vec{U} = 0 \tag{S1}$$

$$\frac{\partial \vec{U}}{\partial t} + \nabla \cdot (\vec{U} \otimes \vec{U}) = -\frac{\nabla p}{\rho} + \nabla \cdot \left[\nu(\nabla \vec{U} + \nabla^T \vec{U})\right] \tag{S2}$$

where, $\vec{U}$, p, ν and ρ represent velocity vector, local pressure, kinetic viscosity and density, respectively.

In addition, we introduce one scalar transport equation to track the mass transfer of fluorescent dye in the channel,

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\vec{U}\phi) = \nabla \cdot (\Gamma_\phi \nabla \phi) \tag{S3}$$

where ϕ is mass fraction of the dyed fluid and Γ_ϕ denotes the diffusivity of fluorescent dye in fluids, of which the dimension is area per unit time.

In this study, a solver based on the segregated approach was developed in the finite-volume CFD code package – OpenFOAM (**Open**-source **F**ield **O**peration **A**nd **M**anipulation). The flow is assumed to be laminar and incompressible. Although this problem is steady-state in nature, we employ the PISO (Pressure-Implicit with Splitting of Operators) algorithm to handle the pressure velocity coupling for elaborate control of numerical stability in the solution process. Solution of the transport equation for fluorescent dye is embedded into the PISO loop to establish a close coupling with flow equations. In addition, the run-time selection (RTS) mechanism supported by OpenFOAM is adopted which enables us to choose proper numerical schemes at a point of program execution. Typical numerical schemes can be implemented to discretize the transient (e.g. Euler implicit scheme), convection (e.g. Gauss upwind), gradient (Gauss integration) and diffusion term (Central Differencing Scheme) in the momentum and mass transport equations.

The main framework of the solver in this study is displayed in **Fig. S1**. U-equation and p-equation represent the solution of flow equations for the velocity-pressure system, and ϕ-equation represents the solution process of the mass transport equation. The coupled equations

are solved in a sequential manner for several iterations within each time step and the overall calculation is terminated until the criteria of convergence are satisfied. The simulation solver and framework will be shared and available upon request.

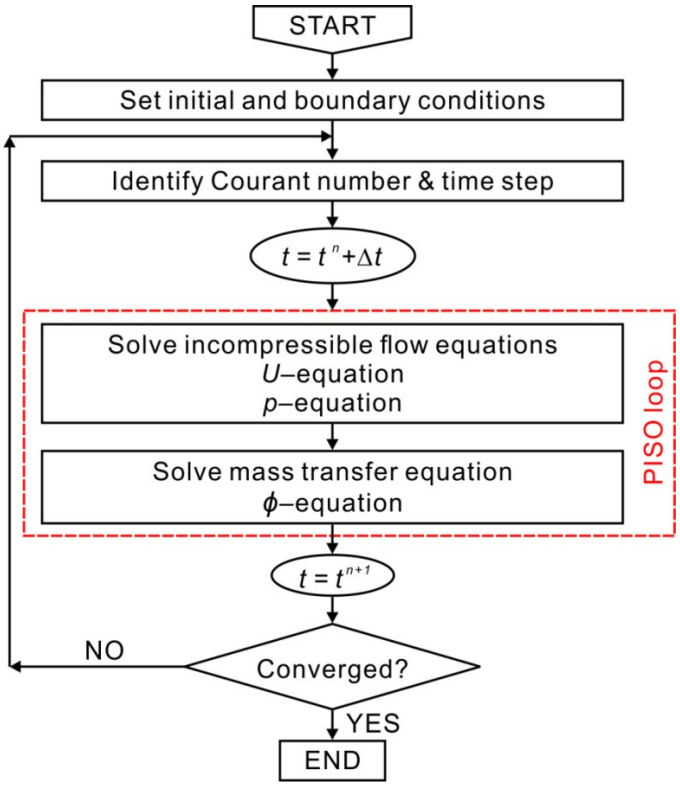

Figure S1. Numerical algorithm of the solver used in the present study.

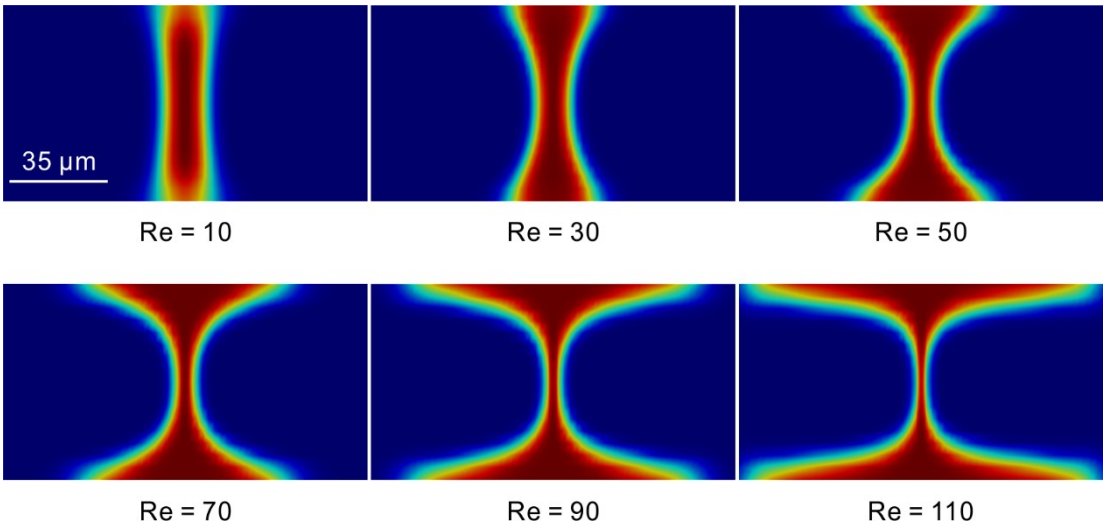

Figure S2. Cross-sectional flow patterns predicted by numerical simulation at different Reynolds numbers.

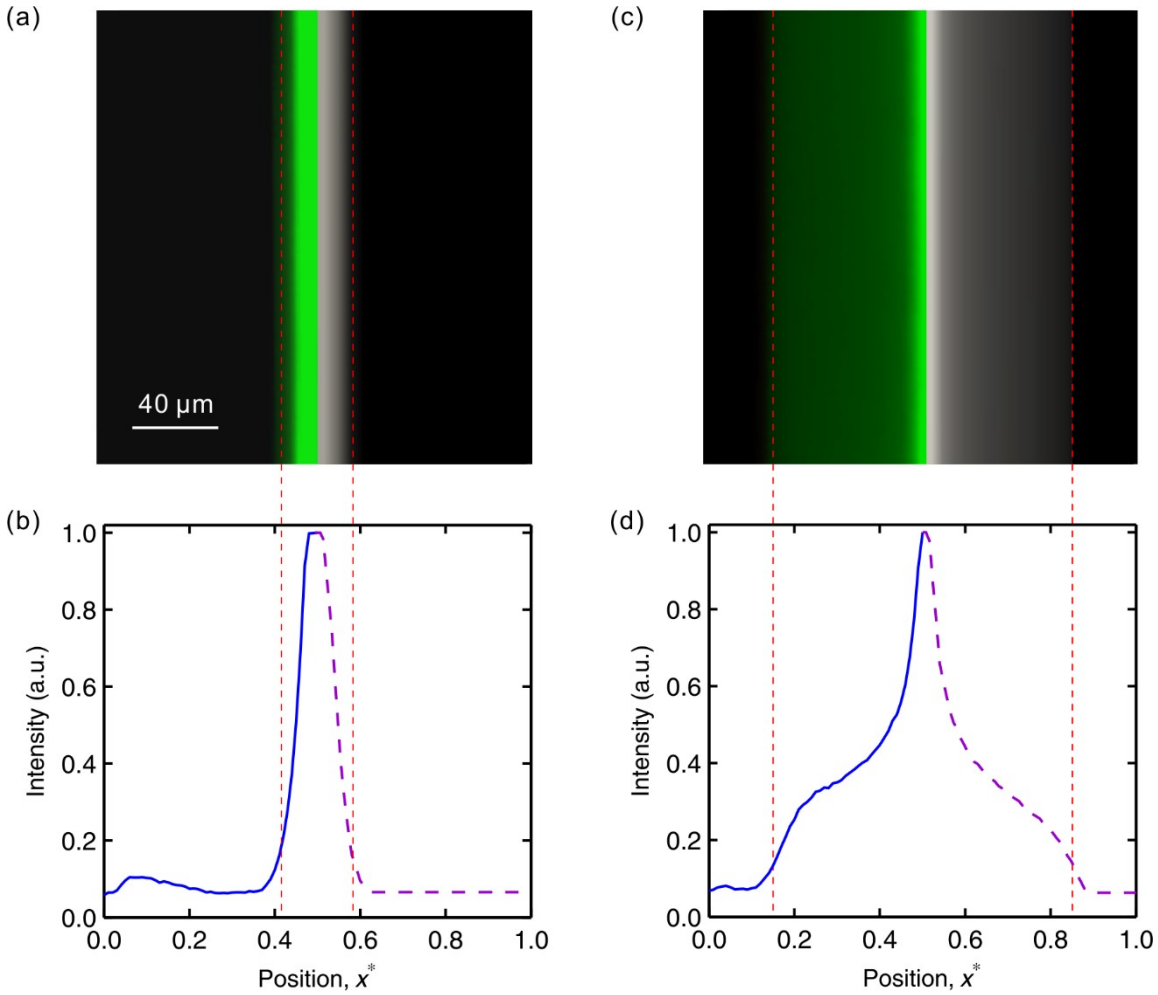

Figure S3. Experimental (left) and numerical (right) results of top-view mass transfer patterns at Re = (a) 30 and (c) 110; Experimental (solid lines) and numerical (dashed lines) results of transverse profiles of normalized fluorescence intensity at Re = (b) 30 and (d) 110.

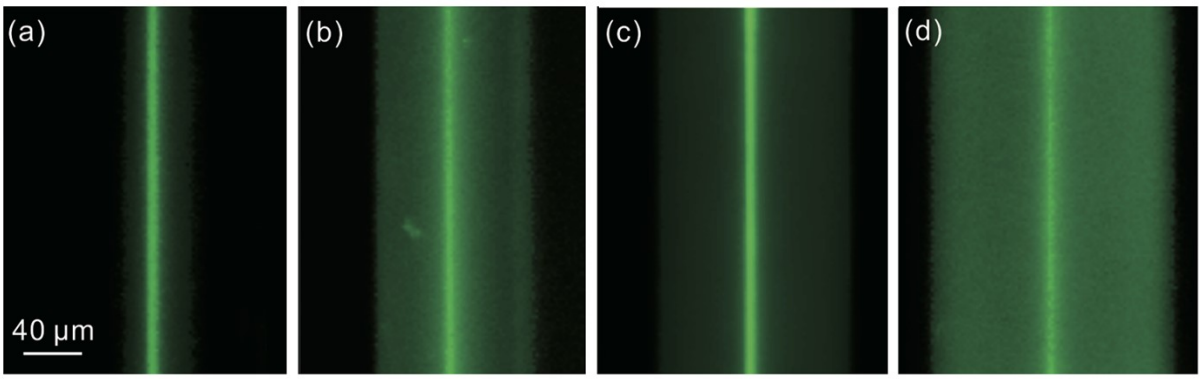

Figure S4. Top view of mass transfer pattern analysis at Re = 100 using fluorescent dye for the microchannel height of (a) 110 μm, (b) 70 μm, (c) 50 μm and (d) 28 μm.

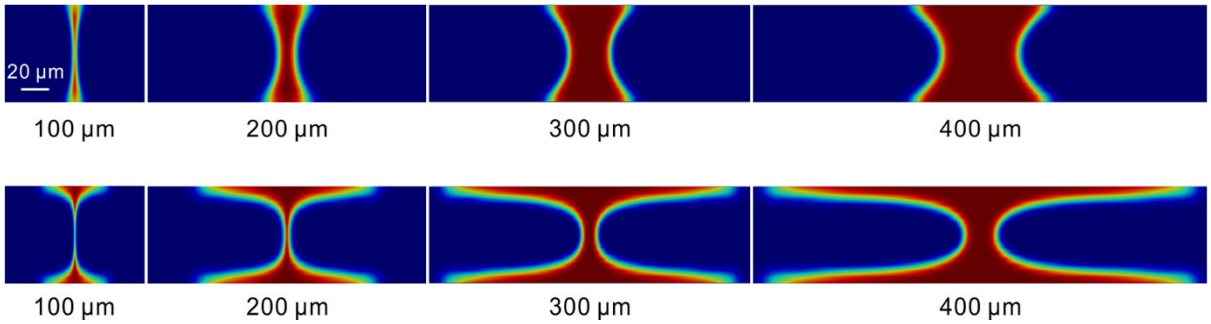

Figure S5: Flow patterns in the focusing channel with different widths at Re = 30 (top) and 110 (bottom).

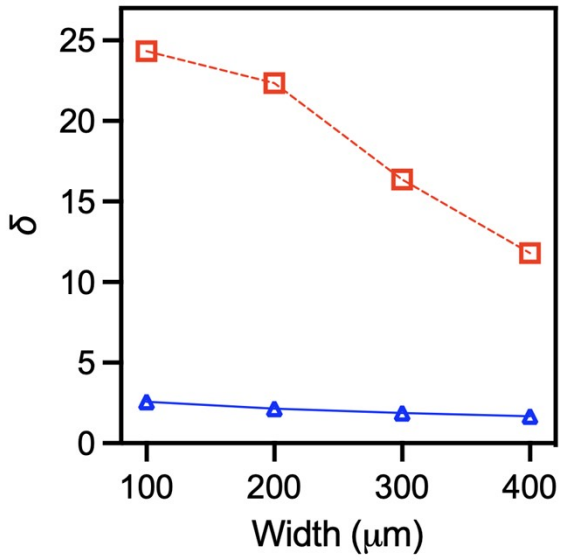

Figure S6: Focusing width ratio at Re = 30 (blue solid) and 110 (red dashed) under various channel widths.

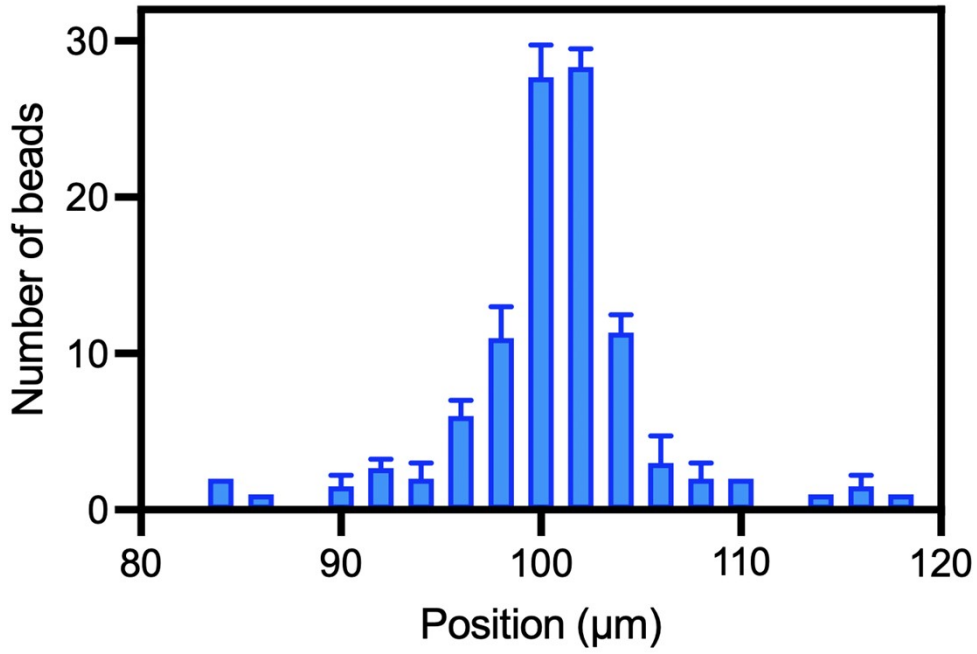

Figure S7. Statistical horizontal distribution of the microbeads in the transverse direction under Re = 30. For instance, the label 100 represents the transverse position of 98 to 100 μm measured from the left sidewall of the microchannel. Error bars represent the standard deviation of three technical replicates.