

Supporting Information

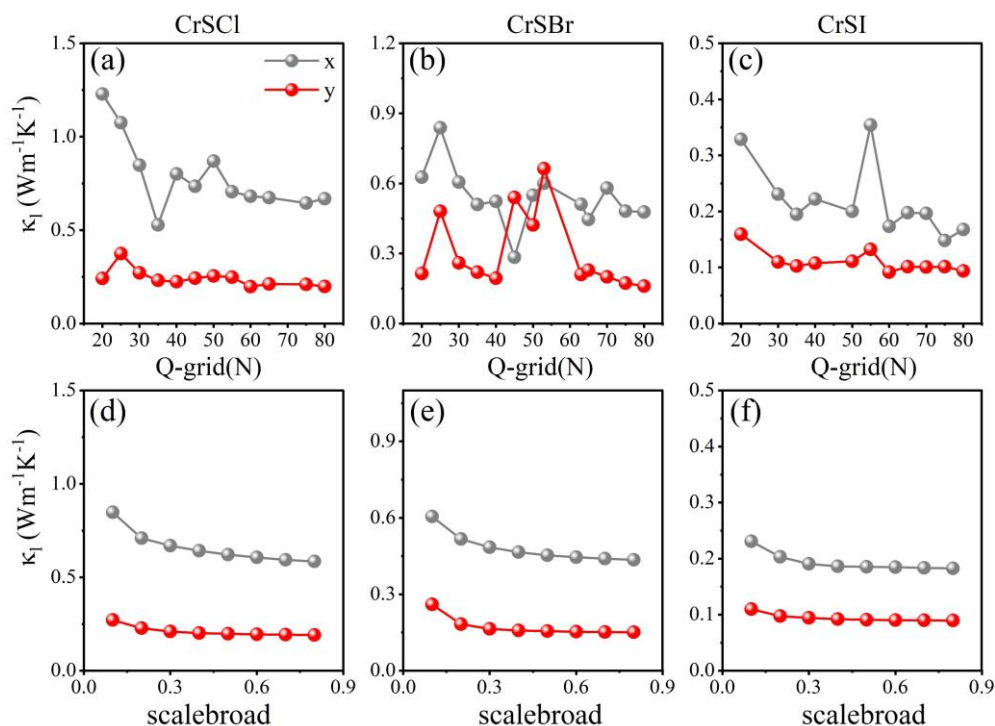


Fig. S1 The first is the experimental results of convergence of thermal conductivity with respect to the nearest Q point. The second section is the experimental results of convergence of thermal conductivity of CrSX monolayers with respect to daub (scale width) parameters.

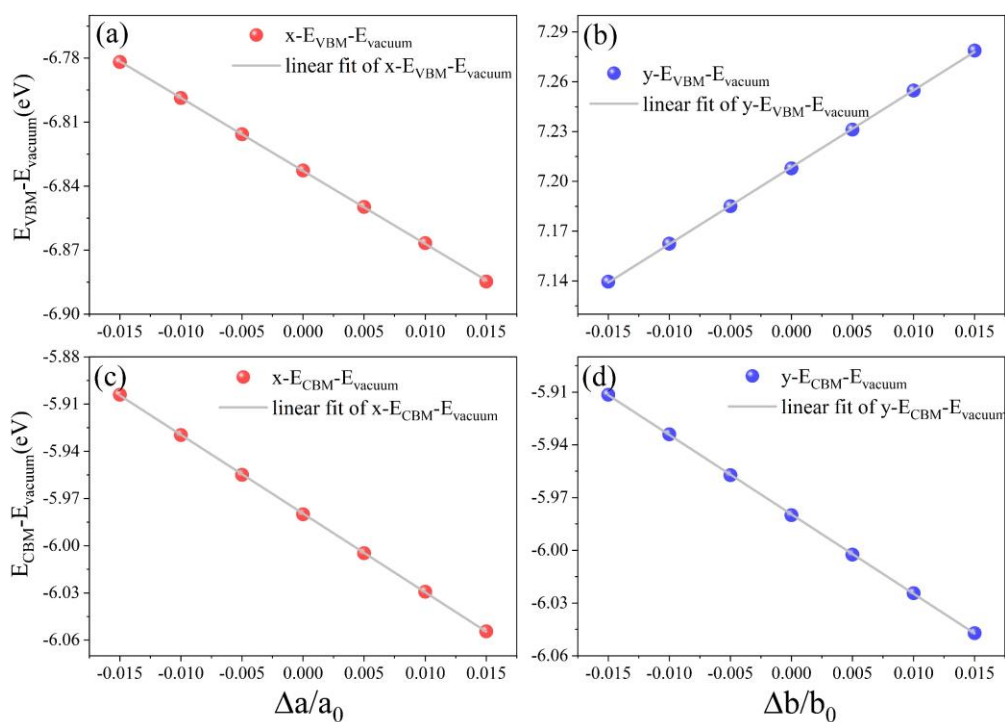


Fig. S2 Band energy with respect to the vacuum energy as a function of lattice dilation for the (a, b) CBM and (c, d) VBM of CrSCl monolayer. Gray solid lines are the fitting curves.

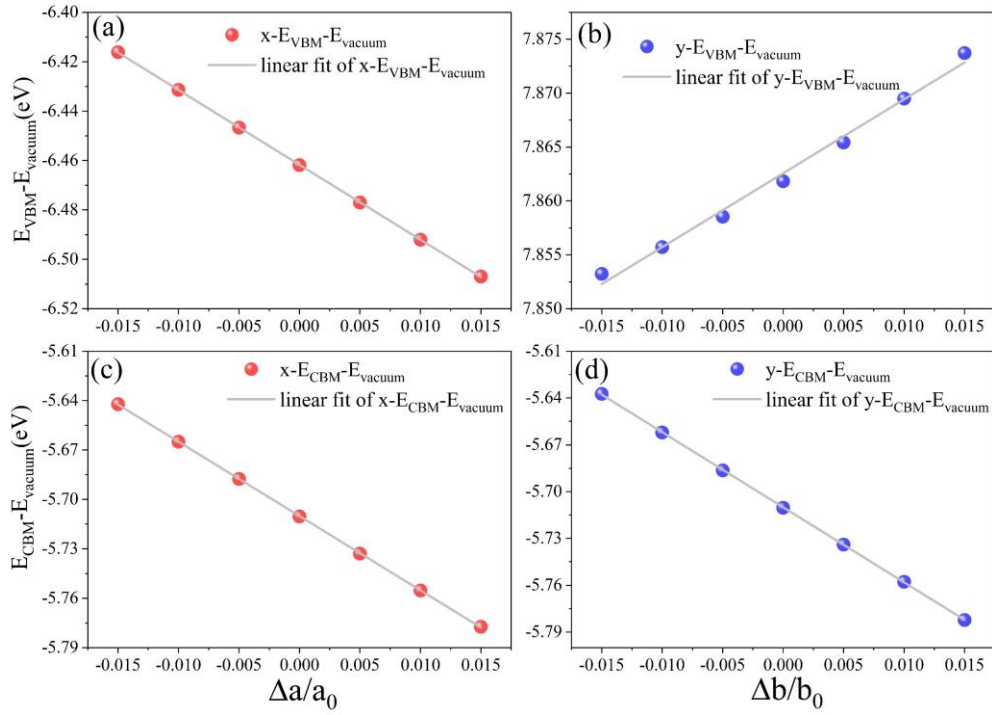


Fig. S3 Band energy with respect to the vacuum energy as a function of lattice dilation for the (a, b) CBM and (c, d) VBM of CrSBr monolayer. Gray solid lines are the fitting curves.

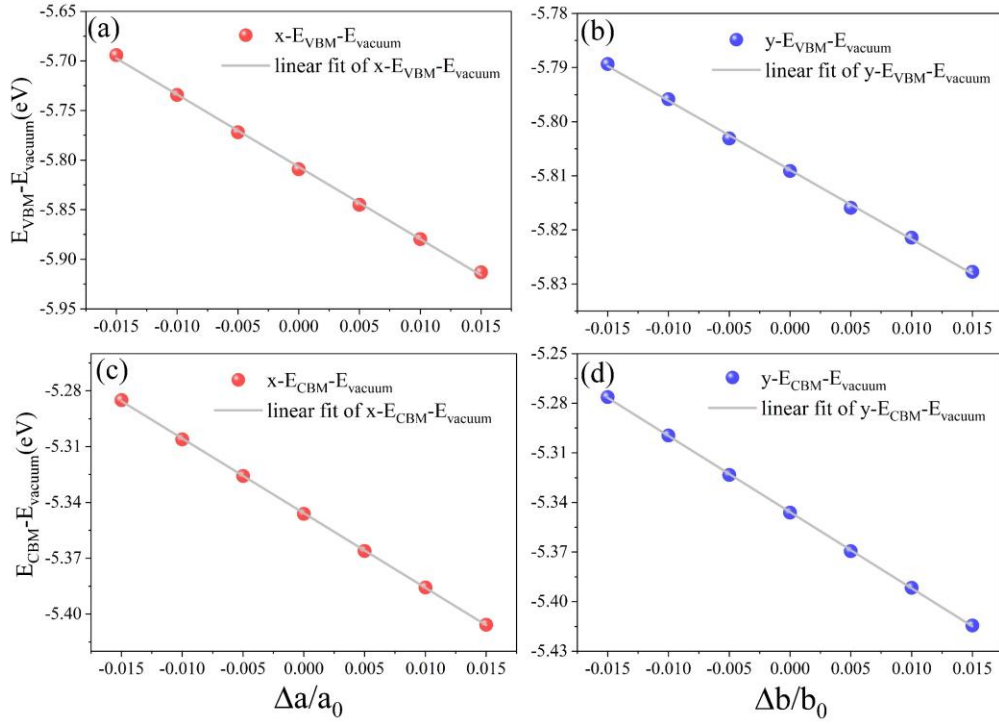


Fig. S4 Band energy with respect to the vacuum energy as a function of lattice dilation for the (a, b) CBM and (c, d) VBM of CrSI monolayer. Gray solid lines are the fitting curves.

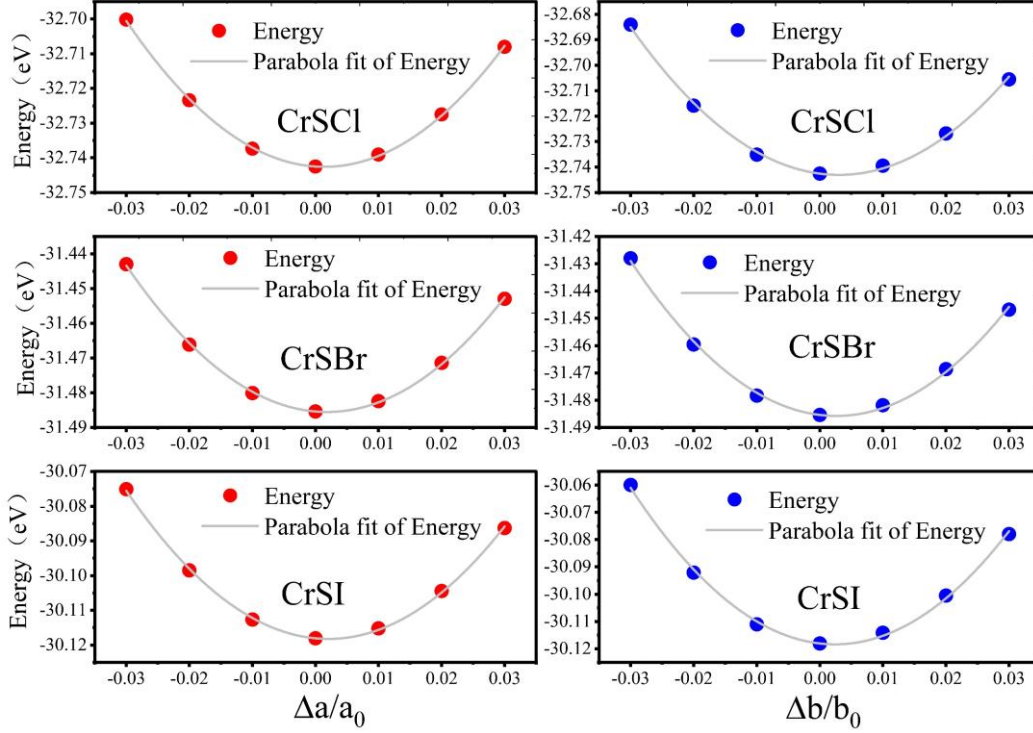


Fig. S5 Total energy-strain curve of CrSX monolayers along the (a, c and e) a and (b, d and f) b directions, respectively.

Table S1 Optimized lattice constant a and b (Å), electronic band gap E_{gap} (eV) of CrSX monolayers.

| Name | a | b | E_{gap} |
|-------|------|------|-----------|
| CrSCl | 3.48 | 4.83 | 1.80 |
| CrSBr | 3.58 | 4.82 | 1.69 |
| CrSI | 3.75 | 4.80 | 1.26 |

Table S2. The calculated elastic constants (C_{ij}) (N/m).

| | C_{11} | C_{22} | C_{12} | C_{33} |
|-------|-------------|-------------|------------|------------|
| CrSCl | 82.4951791 | 101.9338301 | 9.09047097 | 1.58109603 |
| CrSBr | 78.46179433 | 100.1102892 | 9.17140177 | 1.63488964 |
| CrSI | 74.95065929 | 98.1040442 | 9.1621099 | 1.39012426 |

Based on the DPA theory¹, the μ_{2D} and τ in 2D systems can be calculated using the following formula²:

$$\mu_{2D} = \frac{e\hbar^3 C^{2D}}{k_B T m_e^* (E_1)^2}, \quad (1)$$

$$\tau = \frac{2\hbar^3 C^{2D}}{3k_B T m_e^* (E_1)^2}, \quad (2)$$

where e , \hbar , k_B , T , C^{2D} , E_1 and m_e^* are the electron charge, the reduced Planck

constant, the Boltzmann constant, the temperature, the 2D elastic modulus, the deformation potential constants and the effective mass along the transport direction.

Table S3

The calculated effective mass (m^*/m_0), deformation-potential constant (E_1 , eV), 2D elastic modulus (C^{2D} , Jm^{-2}), relaxation time (τ , fs) and carrier mobility (μ_{2D} , $\text{cm}^2\text{V}^{-1}\text{s}^{-1}$) for CrSX monolayers at 300 K using old and new formulas (where “old” and “new” denote the use DPA theory and modified DPA method).

| Name | type | Direction | m^*/m_0 | C^{2D} | E_1 | old- τ | new- τ | old- μ | new- μ |
|-------|------|-------------|-----------|----------|-------|-------------|-------------|------------|------------|
| CrSCl | h | Γ -X | 6.554 | 81.1 | 3.42 | 8.546 | 97.792 | 27.844 | 26.243 |
| | | Γ -Y | 0.080 | 101.0 | 4.63 | 475.734 | 88.940 | 1550.051 | 1955.372 |
| | e | Γ -X | 5.982 | 81.1 | 5.00 | 4.381 | 32.062 | 8.067 | 9.427 |
| | | Γ -Y | 0.343 | 101.0 | 4.52 | 116.425 | 35.621 | 214.393 | 182.657 |
| CrSBr | h | Γ -X | 7.118 | 77.1 | 3.03 | 9.530 | 123.511 | 28.155 | 30.519 |
| | | Γ -Y | 0.091 | 98.8 | 3.51 | 710.647 | 122.068 | 2099.428 | 2359.297 |
| | e | X- Γ | 7.962 | 77.1 | 4.51 | 3.846 | 24.610 | 5.200 | 5.436 |
| | | X-S | 0.478 | 98.8 | 4.81 | 72.166 | 25.355 | 97.576 | 93.295 |
| CrSI | h | Γ -X | 0.280 | 73.7 | 7.29 | 40.008 | 30.983 | 87.587 | 194.620 |
| | | Γ -Y | 4.888 | 96.9 | 1.28 | 97.739 | 62.733 | 213.973 | 22.573 |
| | e | X- Γ | 3.987 | 73.7 | 4.01 | 9.286 | 50.456 | 22.396 | 22.258 |
| | | X-S | 0.300 | 96.9 | 4.61 | 122.771 | 50.399 | 296.106 | 295.477 |

References

1. Bardeen, J. & Shockley, W. Deformation Potentials and Mobilities in Non-Polar Crystals. *Phys. Rev.* **80**, 72–80 (1950).
2. Zhang, L.-C. *et al.* Tinselenidene: a Two-dimensional Auxetic Material with Ultralow Lattice Thermal Conductivity and Ultrahigh Hole Mobility. *Sci. Rep.* **6**, 19830 (2016).